Extended Abstract: Household Portfolio Accounting

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Abstract  
American households vary largely in their portfolio composition of safe and risky assets, defined as stocks, real estate, and non-corporate business. We consider a standard life-cycle model with labor income risk and portfolio choice (Cocco et al. 2005), augmented with a savings wedge that lowers the return on saving and a risky wedge that lowers the relative return on risky assets. Using the SCF (1989–2016), we compute household-level wedges that rationalize the data, in the spirit of Chari et al. (2007). This paper has three main contributions. First, we use the wedges to guide plausible frictions that researchers should consider. Second, we analyze the extent to which household characteristics can account for the wedges. For example, we find that risky wedges are decreasing in age and education, smaller for self-employed households and home owners, and larger for male and black households. Finally, in a hypothetical exercise of reducing the wedges, as in Hsieh and Klenow (2009), we investigate the changes to wealth levels and wealth inequality in the U.S.

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1 Introduction

American housing and stock markets provide a great opportunity for households to invest and accumulate wealth. However, more than 20 percent of households do not own any risky assets, defined as stocks, real estate, and non-corporate business, according to the Survey of Consumer Finances (SCF, 2016). A large literature has focused on this extensive margin. Less is understood about the heterogeneity in the portfolio composition. For example, 10 percent of risky asset owners invest less than 40 percent of their wealth in risky assets, while 15 percent invest more than 200 percent of their wealth in risky assets (SCF 2016). In this project, we focus on accounting for this intensive margin.

We consider a standard quantitative life-cycle model with labor income risk and portfolio choice (Cocco et al. 2005), augmented with a savings wedge that lowers the return on saving and a risky wedge that lowers the relative return on risky assets. In the first part of the paper, we show that many different types of models with frictions are equivalent to our simple model with wedges. In the second part of the paper, we develop a methodology to estimate these wedges from the data. We apply this method to the SCF (1989–2016) and compute household-level wedges that rationalize the data, in the spirit of Chari et al. (2007). The objectives of this research are threefold. First, we use the wedges to guide plausible frictions that researchers should consider. Second, we analyze the extent to which household characteristics can account for the wedges. For example, we find that risky wedges are decreasing in age and education, smaller for self-employed households and home owners, and larger for male and black households. Finally, in a hypothetical exercise of reducing the wedges, as in Hsieh and Klenow (2009), we investigate the changes to wealth levels and wealth inequality in the U.S.

2 Simple Model

In this section, we develop a simple model of portfolio with wedges to provide intuition about how the wedges affect portfolio choices. We also show that many models with frictions are equivalent to our model with wedges.

Consider a simple two-period model. The household receives a stochastic stream of labor income $y_1, y_2$. In the first period, the household chooses to invest in risk-free asset $b$, which delivers a constant return $R$, and a risky asset $x$, which delivers a stochastic return of $R_x$.\footnote{See, for example, Attanasio and Paiella (2011), Chambers et al. (2009), Guvenen (2009), Khorunzhina (2013), and Paiella (2007).}
The household with wealth $w_1$ and labor income $y_1$ chooses assets $b_2, x_2$ to solve:

$$V(w_1, y_1) = \max_{b_2, x_2} u(c_1) + \beta E[u(b_2 R + x_2 R + y_2)]$$

s.t. $c_1 + x_2 (1 + \tau_x) + b_2 (1 + \tau_s) = w_1 + y_1 + T(w_1, y_1)$

where $\tau_x$ is the relative wedge on the risky asset, $\tau_s$ is the wedge on total saving, and $T(w_1, y_1)$ is the transfer, which the household takes as given. Let $b^*(w_1, y_1)$ and $x^*(w_1, y_1)$ denote the optimal policies for risk-free and risky assets, respectively. We impose that $T = x^*(w_1, y_1) ((1 + \tau_x) (1 + \tau_s) - 1) + b^*(w_1, y_1) \tau_s$.

The solution is characterized by the following Euler equations

$$1 + \tau_x = \frac{E[R_x u'(b_2 R + x_2 R + y_2)]}{RE[u'(b_2 R + x_2 R + y_2)]},$$

$$1 + \tau_s = \beta R \frac{E[u'(b_2 R + x_2 R + y_2)]}{u'(c_1)}.$$

Notice that the saving wedges distorts the intertemporal margin and the risky wedge distorts the risky return relative to the risk-free return. Under certain conditions, we show that the risky portfolio share, defined as $x_2 / (b_2 + x_2)$, is decreasing in the risky wedge and that the savings rate, defined as $(b_2 + x_2) / (w_1 + y_1)$ is decreasing in the savings wedge. We now show that many models with frictions are equivalent to our simple model with wedges.

### 2.1 Borrowing limit

Consider a model with an ad hoc borrowing limit, $b_2 \geq -\bar{b}$. The household with wealth $w_1$ and labor income $y_1$ chooses assets $b_2, x_2$ to solve:

$$V(w_1, y_1) = \max_{b_2, x_2} u(c_1) + \beta E[u(b_2 R + x_2 R + y_2)]$$

s.t. $c_1 + x_2 + b_2 = w_1 + y_1$

$\quad b_2 \geq -\bar{b}$
Let $\hat{b}(w_1, y_1), \hat{x}(w_1, y_1)$, and $\hat{c}_1(w_1, y_1)$ denote the solution to (2). The models are equivalent if
\[
1 + \tau_s = \frac{u'(\hat{c}_1) - \lambda}{u'(\hat{c}_1)} \leq 1
\]
\[
1 + \tau_x = \frac{1}{1 + \tau_s} \geq 1
\]
where $\lambda$ is the Lagrange multiplier on the borrowing limit. Thus the borrowing limit acts as a subsidy to saving and a tax on the risky asset.

### 2.2 Short-sale constraint

Consider a model with short-sale constraint on the risky asset, $x_2 \geq 0$. The household with wealth $w_1$ and labor income $y_1$ chooses assets $b_2, x_2$ to solve:
\[
V(w_1, y_1) = \max_{b_2, x_2} u(c_1) + \beta \mathbb{E} [u(b_2 R + x_2 R_x + y_2)]
\]
\[\text{s.t. } c_1 + x_2 + b_2 = w_1 + y_1
\]
\[x_2 \geq 0
\]
Let $\hat{b}(w_1, y_1), \hat{x}(w_1, y_1)$, and $\hat{c}_1(w_1, y_1)$ denote the solution to (3). The models are equivalent if
\[
1 + \tau_s = 1
\]
\[
1 + \tau_x = \frac{u'(\hat{c}_1) - \lambda}{u'(\hat{c}_1)} \leq 1
\]
where $\lambda$ is the Lagrange multiplier on the short-sale constraint. Hence the short-sale constraint acts as a subsidy to the risky asset, but does not otherwise affect the intertemporal savings margin.

### 2.3 Collateral constraint

Consider a model with a collateral constraint, where the household can borrow against the collateral value of the risky asset, $b_2 \geq -\kappa x_2$. The household with wealth $w_1$ and labor
income $y_1$ chooses assets $b_2, x_2$ to solve:

$$V(w_1, y_1) = \max_{b_2, x_2} u(c_1) + \beta E\left[u(b_2 R + x_2 R_x + y_2)\right]$$

$$\text{s.t. } c_1 + x_2 + b_2 = w_1 + y_1$$

$$b_2 \geq -\kappa x_2$$

Let $\hat{b}(w_1, y_1)$, $\hat{x}(w_1, y_1)$, and $\hat{c}_1(w_1, y_1)$ denote the solution to (4). The models are equivalent if

$$1 + \tau_s = \frac{u'(\hat{c}_1) - \lambda}{u'(\hat{c}_1)} \leq 1,$$

$$1 + \tau_x = \frac{u'(\hat{c}_1) - \kappa \lambda}{u'(\hat{c}_1) - \lambda},$$

where $\lambda$ is the Lagrange multiplier on the collateral constraint. Notice that the collateral constraint acts as a subsidy to saving, but its effect on the risky investment depends on $\kappa$. If $\kappa$ is less than one, then the collateral constraint acts as a tax on risky investment.

### 2.4 Heterogeneous discount factors

Consider a model where households have heterogeneous discount factors. The household with discount factor $\beta$, wealth $w_1$, and labor income $y_1$ chooses assets $b_2, x_2$ to solve:

$$V(w_1, y_1) = \max_{b_2, x_2} u(c_1) + \beta_1 E\left[u(b_2 R + x_2 R_x + y_2)\right]$$

$$\text{s.t. } c_1 + x_2 + b_2 = w_1 + y_1$$

The models are equivalent if

$$1 + \tau_1 = \frac{\beta}{\beta_1}$$

$$1 + \tau_x = 1$$

Thus less patience, $\beta_1 < \beta$, acts as a tax to saving, but does not otherwise affect risky investment.
3 Model

We now describe the full model that we use to estimate wedges from the data. Consider the Cocco et al. (2005) life-cycle model with portfolio choice, uninsurable labor income risk, and mortality risk. Preferences are given by

$$E_1 \sum_{j=1}^{J} \beta^{j-1} \left( \prod_{k=1}^{j-1} \psi_k \right) \frac{c_j^{1-\gamma}}{1-\gamma}$$

(6)

where $c_j$ denotes age $j$ consumption, $\beta$ is the time discount factor, $\psi_j$ is the survival probability from age $j$ to $j+1$, and $\gamma$ is the coefficient of relative risk aversion.

Households can live up to $J$ periods, with exogenous retirement after age $j^*$. Working age labor income $y$ is given by

$$\log(y) = f(j, z) + v + \varepsilon$$

(7)

where $f$ is a deterministic function of age $j$ and other household characteristics $z$, which may include education, race, and gender, $\varepsilon \sim N(0, \sigma^2_\varepsilon)$ is a transitory shock, and $v$ is a persistent shock, which follows

$$v' = v + u'$$

(8)

where $u \sim N(0, \sigma^2_u)$. For computational tractability, we assume that retirees ($j > j^*$) receive a constant fraction $\lambda$ of permanent labor income in the last working age $j^*$

$$\log(y) = \log(\lambda) + f(j^*, z) + v_{j^*}.$$  

(9)

As in the simple model, households choose a portfolio of the risk-free asset $b$, which delivers a constant return $R$, and a risky asset $x$, which delivers a stochastic return of $R_x$. Let $w = Rb + R_xx$ denote household wealth. The budget constraint is given by

$$c + (x'(1 + \tau_x) + b')(1 + \tau_s) = w + y(j, z, v, \varepsilon) + T$$

(10)

where $\tau_x$ is the relative wedge on the risky asset, $\tau_s$ is the wedge on total saving, $T$ is the transfer, which the household takes as given. Similar to the labor income process, the wedges are assumed to have deterministic and stochastic components. Specifically, wedges follow

$$\tau_m = g_m(j, z) + h_m(v) + \varepsilon_m \text{ for } m = x, s$$

(11)
where \( g_m \) is a deterministic function of age \( j \) and other household characteristics \( z \), \( h_m \) is a function of the persistent labor shock, with the convention that \( v = v_j \) if \( j > j^* \), and \( \varepsilon_m \sim N(0, \sigma^2_{\varepsilon_m}) \) is an i.i.d. shock.

The household ’s problem can be written as:

\[
V_j(z, w, \varepsilon, \varepsilon_x, \varepsilon_s) = \max_{c', x'} u(c) + \beta \psi_j E_{R^s_x, u', \varepsilon', \varepsilon_s'} \left[ V_{j+1}(z, w', v + u', \varepsilon', \varepsilon_x', \varepsilon_s') \right] \tag{12}
\]

\[
\text{s.t. } c + (x'\left(1 + \tau_x(j, z, v, \varepsilon_x)\right) + b')\left(1 + \tau_s(j, z, v, \varepsilon_s)\right) = w + y(j, z, v, \varepsilon) + T(j, z, w, \varepsilon, \varepsilon_x, \varepsilon_s)
\]

\[
w' = b'R + x'R_x
\]

\[
b' + x' \geq 0
\]

Let \( s \equiv \{j, z, w, \varepsilon, \varepsilon_x, \varepsilon_s\} \). As before, we impose the condition that

\[
T(s) = x^*(s)((1 + \tau_x(j, z, v, \varepsilon_x))(1 + \tau_s(j, z, v, \varepsilon_s)) - 1) + b^*(s)\tau_s(j, z, v, \varepsilon_s)
\]  

(13)

where \( x^*(s) \) and \( b^*(s) \) denote the policy functions for risky and risk-free assets, respectively. Notice that households take contemporaneous transfers as given, but are able to internalize the fact that investment decisions today affect future transfers. This is because we require that the wedges be budget-neutral for each household.

If we restrict our sample to households who have positive savings, the solution to (12) can be characterized by the Euler equations:

\[
1 + \tau^x(j, z, v, \varepsilon_s) = \frac{\beta \psi_j E_{R^s_x, u', \varepsilon', \varepsilon_s'} \left[ \frac{\partial V_{j+1}(z, w', v + u', \varepsilon', \varepsilon_x', \varepsilon_s')}{\partial w'} \right] R}{u'(c^*(s))},
\]

(14)

\[
1 + \tau^x(j, z, v, \varepsilon_x) = \frac{E_{R^s_x, u', \varepsilon', \varepsilon_s'} \left[ \frac{\partial V_{j+1}(z, w', v + u', \varepsilon', \varepsilon_x', \varepsilon_s')}{\partial u'} \right] \left[ R_x \right]}{E_{R^s_x, u', \varepsilon', \varepsilon_s'} \left[ \frac{\partial V_{j+1}(z, w', v + u', \varepsilon', \varepsilon_x', \varepsilon_s')}{\partial w'} \right] R_x},
\]

(15)

Notice that as long as we have household data on risky assets \( x' \), safe assets \( b' \), and other household characteristics that affect the process for labor income, we can easily compute the risky wedge using equation (15). For this reason, we start with the SCF and focus on the risky wedge.
4 Preliminary Findings

We use standard parameters and functional forms from the literature. The household utility function is assumed to have a constant relative risk aversion of 2. We assume that the labor income process consists of a deterministic component that is a function of household characteristics and a stochastic component with permanent and transitory shocks. We estimate this process on disposable labor income from the Panel Survey of Income Dynamics (PSID), similar to Krueger et al. (2016).

In estimating the risky wedges, we take the following steps. First, guess that the wedges are only a function of the idiosyncratic shocks $\varepsilon_x$ and $\varepsilon_s$. Second, solve the value functions by backward induction. Third, using equation (15), compute the risky wedge for each household $i$. We face the problem that we are not able to distinguish between transitory and persistent shocks in the data, since that the SCF provides repeated cross-sections. We assume that the transitory shock is zero, so that

$$v = \begin{cases} 
\log y_{i}^{\text{data}} - f(j, z_{i}^{\text{data}}), & j \leq j^* \\
\log \frac{y_{i}^{\text{data}}}{\lambda} - f(j^*, z_{i}^{\text{data}}), & j > j^*
\end{cases}$$

Fourth, regress the risky wedges on age $j$ household characteristics $z$, and persistent labor shock $v$ to estimate $g$ and $h$. Finally, repeat steps 2–4 until the regressions converge.

We find that risky wedges are sizable and vary significantly across households with positive risky assets, as shown in Figure 1. In addition, more than 95 percent of households have a positive wedge, implying that these households are under-investing in risky assets.

We find that risky wedges decline with education, suggesting that informational frictions may be important. Surprisingly, risky wedges are higher for male heads, a result that is at odds with the claim that women are more risk-averse than men (Jianakoplos and Bernasek 1998). Home ownership and self-employment decrease the wedges by 1.9 and 0.7 percent, respectively, suggesting that these are important channels that need be considered. We also find that black households have higher risky wedges, supporting the literature on discriminatory lending practices, reviewed by Ladd (1998). It is worth noting, however, that the racial gap in risky wedges has diminished over time.
5 Next Steps

First, we plan to use the consumption and wealth supplements of the PSID, which also has information on consumption expenditures, to compute both wedges in equations (14) and (15). Second, we plan to use the model to examine (a) what are the welfare consequences of the wedges, (b) what are the policy implications, if any, and (c) how does wealth inequality in the U.S. change if we reduce or eliminate the wedges.
References


