Trade Adjustment Dynamics and the Welfare Gains from Trade*

George Alessandria†
University of Rochester and NBER

Horag Choi
Monash University

Kim J. Ruhl
University of Wisconsin – Madison and NBER

This Draft: June 2018
First Draft: September 2013

Abstract
We study how the transitions following a trade reform are shaped by the time it takes for new exporters to grow in the export market. We introduce time and risk into the fixed-variable cost tradeoff central to general equilibrium heterogeneous firm trade models: Investing in exporting gradually and stochastically lowers the costs of exporting. The model captures the tendency of new exporters to export on a small scale, to have low survival rates, and to take time to grow into large exporters. In the model, aggregate trade dynamics arise from producer-level decisions to invest in lowering their future variable export costs, and tariff reforms generate time-varying trade elasticities. We show that the gains from reducing tariffs arise from substituting away from firm creation and towards export capacity. This is in stark contrast to the static models that dominate the literature. The strength of this substitution is determined largely by the size of new exporters and their ability to grow into successful exporters. We calibrate the model and estimate the welfare gains from reducing tariffs, which differ substantially from the long-run changes in consumption or trade. We show that the welfare gain cannot be recovered from a static trade model or from formulas based on those models. Because aggregate trade grows slowly, the long-run effects are strongly discounted and, thus, are not the key determinants of the welfare gains from a change in trade policy. We also find that policy prescriptions based on static models can predict a loss from trade reform when our dynamic model predicts a gain.

JEL classifications: E31, F12.
Keywords: Welfare, trade, tariffs, establishment heterogeneity.

*We thank Mark Aguiar, Roc Armenter, Hal Cole, Lukasz Drozd, Virgiliu Midrigan, Veronica Rappoport, Natalia Ramondo, Richard Rogerson, Mike Waugh, Kei-Mu Yi, and audiences at the Board of Governors, BC, Clemson, CMSG, Michigan, NYU, NBER ITI, Philadelphia and Minneapolis Feds, Penn State, Princeton, SED, Toronto, UCLA, and Wharton for helpful comments. We thank Joseph Steinberg for an excellent discussion.

†George.Alessandria@rochester.edu, kim.ruhl@gmail.com, Horag.Choi@Monash.edu
1 Introduction

For the United States, and much of the world, the 1950s–2000s was a period of major trade reform. Standard models of trade liberalization (e.g., Eaton and Kortum 2002; Melitz 2003) predict that, as the transitional effects of these reforms wind down, the U.S. economy should be converging to higher levels of trade, consumption, and income. While international trade has grown rapidly, it is less clear that income and consumption have grown in the ways that the models predicted. The highly visible increase in trade, the recent stagnation in income, and the apparent failure of the expected benefits to materialize have provided fodder for a growing backlash against trade liberalization.

Against this backdrop, we argue that understanding the welfare gains from tariff reform requires accounting for the transition between the high- and low-tariff equilibria and that simply comparing the two steady states generates misleading predictions. In our model, the gains in consumption and output occur early in the transition, while trade is slow to adjust. Understanding the post-reform transition is crucial for welfare measurement. What is crucial for understanding the post-reform transition? We show that the substitutability between new firm creation and export capacity—a relationship that remains largely unstudied—is the key determinant of the gains from trade. A tariff decrease generates a gradual expansion in trade, but, as the economy cuts back on new firm creation, more resources are devoted to production, which generates a rapid expansion in consumption that overshoots the new steady state.

The substitutability between new firms and export capacity is determined mainly by the ease with which a firm can expand its exports. We study this relationship in a general equilibrium heterogeneous firm model in which we introduce time and risk into the fixed-variable cost tradeoff: Investing in exporting gradually and stochastically lowers the costs of exporting. The model captures the tendency of new exporters to export on a small scale, to have low survival rates, and to take time to grow into large exporters. In the model, aggregate trade dynamics arise from producers’ decisions to invest in lowering their future

\[\text{Our model integrates the structural, partial equilibrium literature that studies establishment-level export patterns (Das, Roberts, and Tybout 2007; Kohn, Leibovici, and Szkup 2015; Rho and Rodrigue 2016; Ruhl and Willis 2017) and the general equilibrium literature focused on measuring the gains from trade.}\]
variable export costs, and tariff reforms generate a trade elasticity that increases over time.

To see why exporter life cycles are important, consider the standard sunk-cost model of Das, Roberts, and Tybout (2007) studied in general equilibrium by Alessandria and Choi (2014). In this model, new exporters make a single, large upfront investment in export access in order to export on a large scale. Continuing exporters do not invest in expanding the scale of exporting—they pay only to maintain market access. With only a small number of firms investing in market access, tariffs create a smaller reduction in trade, making exporters and new firms not very substitutable. When tariffs are cut, the decrease in firm creation is relatively small and the growth in aggregate trade is relatively fast. In this model, there is some consumption and output overshooting, but much less than in the model with exporter life cycles that match the data. In our benchmark model, in contrast, new exporters make a small upfront investment to export on a small scale, and they must make repeated investments in market access to expand their exports. Thus, tariffs discourage investments on two margins—market access for new exporters and the scale of market access by all exporters—further distorting trade. In response to a cut in tariffs, the economy cuts back on firm creation and invests in increasing the export capacity of existing exporters. In the benchmark model, firm creation falls and exports rise by much more than in the sunk-cost model. The greater substitution away from firm creation means that consumption overshoots its steady-state level substantially, creating significant welfare gains along the transition.

We develop a parsimonious model of producer export entry, expansion, and exit within a two-country general equilibrium model with capital, roundabout production, and asset trade. The model nests the standard models of heterogeneous producers with fixed export costs (Krugman 1980; Ghironi and Melitz 2005; Das, Roberts, and Tybout 2007). To capture the observed new exporter dynamics, we allow the producer’s exporting technology—in particular, the fixed versus variable cost tradeoff—to be dynamic. As in the standard models, nonexporters have an infinite iceberg trade cost. By paying a fixed cost, nonexporters lower their iceberg cost to a finite level and become exporters. Existing exporters must pay another, potentially different, fixed cost to continue exporting. In our model, unlike in standard models, a producer’s iceberg cost is allowed to evolve dynamically. As long as a new exporter continues to export, its iceberg cost falls stochastically over time. As producers
become more efficient exporters, their export intensity increases, consistent with the data. It takes time, resources, and a bit of luck to become an efficient exporter.\(^2\)

With this general export technology, the aggregate volume of trade now depends on tariffs and the joint distribution of iceberg costs and productivity. Trade expands through accumulated exporters (extensive margin) and a better exporting technology (intensive margin). Because of the dynamic nature of the extensive and intensive margins, there is no simple mapping between the structural parameters of the model, tariffs, and the volume of trade along the transition. By disciplining our model of producer-level exporting technology with producer-level data, we avoid making any assumptions about how aggregate trade behaves. In particular, we are not forced to estimate a trade elasticity that will govern the aggregate behavior of trade—a difficult undertaking given that the trade elasticity is not constant.\(^3\) Indeed, a key advantage of our dynamic model is its ability to capture the well-known feature of the data that the trade elasticity increases with time.

The generality of our model allows us to estimate the exporting technology without imposing a priori restrictions. Following the literature, we divide the fixed export costs into an entry cost and a continuation cost. Consistent with Ruhl and Willis (2017) and subsequent partial equilibrium structural models of exporting, when the model generates a data-consistent exporter life cycle, the estimated entry cost is much smaller than those derived from models that ignore new-exporter dynamics.

One interpretation of the partial equilibrium literature is that the small producer-level export costs imply small aggregate export costs. In our general equilibrium model, this is not the case. In the benchmark model, the aggregate resources devoted to fixed export costs are larger than in a comparably calibrated sunk-cost model. The larger aggregate cost is due to the many producers that either exit before becoming successful or invest in lowering their variable trade costs. In Section 6.1, we show that the canonical sunk-cost model would need an entry cost almost eight times larger than the one in the benchmark model to generate the same aggregate expenditure on exporting costs. The benchmark model—despite having

\(^2\)While the focus is on generalizing the exporting technology, it is isomorphic to allow for export demand to increase with tenure in the foreign market.

\(^3\)Baier and Bergstrand (2007) and Baier, Bergstrand, and Fang (2014), for example, provide empirical evidence on the delayed impacts of trade liberalization and their lagged effect on trade volumes.
a small export entry cost—behaves, in the aggregate, much like a model with a very large sunk entry cost.

We use the calibrated model to measure the aggregate effects of a unilateral and global reduction of a ten-percent tariff, taking into account the transition period. Even though new exporter dynamics cause the aggregate trade volume to grow slowly along the transition, the welfare gain exceeds the change in steady-state consumption. In the global tariff reduction experiment, consumption peaks in year seven, at which point the trade elasticity has grown to only 75 percent of its long-run value. When we take the transition into account, the welfare gain is 15 times larger than the change in steady-state consumption. The transition is even more important when we consider unilateral liberalization: Welfare in the liberalizing country increases by 0.5 percent, even though its steady-state consumption falls by 2.4 percent.

In our model, two competing forces shape the transition and long-run effects of a cut in tariffs. First, trade adjusts slowly as producers make investments in export-specific capacity that may boost future exports and profits. This force reduces the resources available for production and consumption in the short run, while improving the efficiency of the economy in the long run. These investments in exporting in the transition period act to reduce welfare. The second force is a desire to reduce investments in new varieties. Lowering tariffs increases the varieties available from foreign exporters. This extra competition reduces entry because firms strongly discount the future opportunities to export. This frees resources for production and consumption in the transition and increases welfare. These two forces summarize the trade-off between new firm creation and export capacity expansion that is at the heart of the model.

In Section 9.1, we show how the export cost process shapes the substitution between new firms and export capacity. Counterintuitively, when new exporters are small, trade grows more in response to a cut in tariffs: Investing in expanding export capacity is a more efficient way to increase exports than to create new exporters. In this way, the economy responds to a policy change (a tariff cut) by endogenously improving its export technology.

Our model predicts that we should observe firm creation rates falling as trade volumes rise. In Section 7, we present evidence from the United States that supports this prediction. The growing literature that addresses the decline in firm creation rates in the United States
has paid little attention to international factors (e.g., Decker, Haltiwanger, Jarmin, and Miranda 2014), but our calibrated model captures much of the decline in firm creation. While we view this as a preliminary success, a more complete answer requires considering the other mechanisms that work to decrease firm creation. We see this as important future research.

The dynamic features of our model—trade in financial assets, capital accumulation, and the gradual adjustment of trade—make it ideally suited for studying the impact of a unilateral trade liberalization. We find that both countries gain from unilateral reform, but the loss of tariff revenue implies that the reforming country’s gain is relatively small compared to its trading partner’s (0.51 percent versus 5.7 percent). We show that focusing on the steady-state change or evaluating the policy in a model without an export decision would lead to the conclusion that welfare in the reforming country would decrease. The reform also leads to interesting current account dynamics that depend on the nature of export costs. The reforming country becomes a net lender to the rest of the world, accumulating net assets of almost seven percent of GDP. The initial trade surpluses are substantial, peaking at 0.7 percent of GDP two years after the liberalization. In a model without export costs, borrowing and lending are much smaller or even non-existent.

The non-linear relationship between the trade elasticity and consumption along the transition implies that the sufficient statistic approach pioneered by Arkolakis, Costinot, and Rodríguez-Clare (2012) (hereafter, ACR) does not extend to our model. We make this point quantitatively and theoretically. To make the quantitative point, we construct a model without export entry costs, so that all producers export, as in Krugman (1980). To make the aggregate trade dynamics in this model consistent with our benchmark model, we introduce an adjustment friction. While the aggregate trade dynamics in the benchmark model and this no-cost model are identical, the consumption dynamics are very different. Without an exporter decision, consumption grows smoothly during the transition, and the welfare gain is only 37 percent of that in the baseline model, even though the steady-state increase in consumption is larger.

Given the nonlinear nature of the transitions, we are unable to characterize the welfare gain in our benchmark model analytically, but we can still say quite a bit about why the
simple ACR formula does not hold. First, for a simplified version of our model, we derive a long-run relationship between new exporters, consumption, entry, and trade. Second, to quantify the effects of a tariff reform along the transition, we derive a simple decomposition of consumption into several margins and relate it to the ACR formula. The decomposition is simple enough to distinguish between the change in steady-state consumption and welfare. We find that the welfare gain from trade liberalization is higher than the change in steady-state consumption because the available varieties, labor used in production, capital labor ratios, and average plant productivity are all relatively high along the transition.

This paper is part of the growing literature that quantifies the aggregate effects of trade on welfare. Atkeson and Burstein (2010) and Arkolakis, Costinot, and Rodríguez-Clare (2012) find that producer-level exporting details matter little for welfare, while Head, Mayer, and Thoenig (2014) and Melitz and Redding (2015) find a role for producer-level exporting in welfare when the trade elasticity depends on the level of trade costs. Melitz and Ottaviano (2008) and Edmonds, Midrigan, and Xu (2015) consider the effects of trade on competition. While these papers focus on trade and markups, tariffs in our model affect competition through entry and alter the share of income from profits.

Our model’s trade elasticity is time-varying because time and resources are required to expand trade after a decrease in tariffs. This is consistent with the slow adjustment of trade to changing trade barriers or relative prices documented in the empirical literature. A number of researchers have studied the transition following tariff reform. Alvarez, Buera, and Lucas (2013) study the transition following a change in trade frictions; however, this study does not model the intensive export margin. Ravikumar, Santacreu, and Sposi (2018) study trade imbalances following liberalization in a model with capital accumulation and financial assets. They also find that welfare gains accrue gradually and that the dynamic gains are larger than the static gains. In their model, gradual adjustment arises primarily from capital accumulation, whereas our gradual adjustment arises as firms adopt better

---

4 Simonovska and Waugh (2014) show that micro details are important for model parameters.
5 A large empirical literature identifies different short-run and long-run trade responses to aggregate shocks (Hooper, Johnson, and Marquez 2000; Gallaway, McDaniel, and Rivera 2003). Many theoretical studies of the role of trade adjustment explicitly or implicitly calibrate the trade elasticity differently, according to the horizon considered (Obstfeld and Rogoff 2007). Some recent theoretical work has endogenized the dynamics of the trade elasticity; these include Alessandria and Choi (2007), Drozd and Nosal (2012), Engel and Wang (2011), Ramanarayanan (2007), Ruhl (2008), and Alessandria, Pratap, and Yue (2013).
exporting technology.

Lastly, in contrast to much of the literature, our model draws a meaningful distinction between tariffs (a policy choice) and trade costs (a technological constraint). In our model, producers increase investment in their export technologies in response to a change in trade policy. In static multi-country models, Alvarez and Lucas (2007), Caliendo and Parro (2015), and Ossa (2014) also study the aggregate implications of policy frictions.

In Section 2, we review the data on the exporter life cycle, laying out key producer-level facts used to discipline our model of the producer’s exporting technology. In Section 3, we present the model, and in Section 4, we describe our strategy for calibrating the model. In Sections 5 and 9, we report the results from the baseline model and show how the gain from trade liberalization is much larger than the steady-state comparisons would suggest. In Section 8, we present alternative versions of the model, highlighting the importance of producer heterogeneity in understanding the welfare gain from trade. In Section 10, we consider a unilateral tariff reform.

2 The Exporter Life Cycle

At the center of our model is a novel generalization of the producer’s exporting technology that includes a risky time-to-build element. We show that this general technology generates an exporter life cycle that is consistent with a growing body of empirical work showing that: 1) exporting intensively takes many years of sustained foreign market participation; 2) many new exporters exit before achieving this status; 3) and many new exporters are not actually new to the foreign market.

To set ideas and discipline the importance of these margins, we summarize some salient features of the exporter life cycle using manufacturing sector plant-level data from Colombia and Chile and firm-level data from the United States. The results from unbalanced panels are in the top halves of Tables 1 and 2, while the bottom halves report results for balanced panels and some statistics from the literature.\footnote{Our aim is to reorganize facts that others have emphasized, so we do not report the regression tables. These are available upon request.}
Our focus is on the aggregate implications of the exporter life cycle. We begin by summarizing the importance of new exporters in overall exports at annual and eight-year horizons. New exporters are common, but relatively unimportant, at one-year intervals but grow in importance over longer horizons. Columns 1 and 3 of Table 1 show that, at the end of an eight-year window, 57 percent of exporters entered the export market during the sample, and they accounted for just under 40 percent of total exports. These numbers are similar in both Colombia and Chile. New exporters are less important annually—new exporters accounted for 17.6 percent of exporters but only 4.2 percent of total exports from Colombia.

2.1 Export Intensity

New exporters account for a small share of annual total exports because new exporters are relatively small, in terms of both their total sales and their export-sales ratios, what we call their export intensity. Columns 5 and 6 of Table 1 show that the total sales of new exporters are 40 to 50 percent of those of an average exporter, and their export intensity is about 45 percent of the average exporter’s.

New exporters become more important in aggregate trade at longer horizons because continuing starters gradually expand their overall sales and export intensity. To quantify the dynamics of export intensity, we regress export intensity of establishment \( i \), \( exs_{it} \), on lagged export intensity and dummies for incumbent exporters, \( I_{it}^{\text{incumbent}} \), and new exporters, \( I_{it}^{\text{starter}} \):

\[
(1) \quad exs_{it} = \alpha_0 + \rho_{exs} exs_{it-1} + \alpha_1 I_{it}^{\text{starter}} + \alpha_2 I_{it}^{\text{incumbent}} + \varepsilon_{it}.
\]

New exporters are more important for total trade over longer periods because export intensity is quite persistent, with an autocorrelation of about 90 percent annually (Table 1, column 7). The low initial value and slow build-up of a new exporter’s export intensity is evident in Figure 1A (Ruhl and Willis 2017), which plots the average export intensity of new exporters in Colombia, conditional on continually exporting for five years. The average incumbent

---

7 We focus on eight-year windows due to data limitations and for comparability with the results for the United States, which are reported in other studies.

8 As our interest is in the aggregate importance of the exporter life cycle, in all of our regressions, we do not control for industry or years and weight by establishment sales.
exporter ships 13 percent of its total sales abroad, while a new exporter ships about six percent of total sales abroad in its first year. It takes five years for the new exporter to reach the same export intensity as the existing exporters.

Using the estimated coefficients from (1), we can predict the export intensity of an average exporter that has exported continuously for $a$ years as

$$\hat{e}_{xs_a} = (\alpha_0 + \alpha_1) \rho_{exs}^a + (\alpha_0 + \alpha_2) \frac{1 - \rho_{exs}^a}{1 - \rho_{exs}}. $$

Our estimates imply that it takes between nine and 11 years for a new exporter to export as intensively as the average exporter (Table 1, column 8). If a new exporter grew for 20 consecutive years, it would export 20–50 percent more intensively than an average exporter (Table 1, column 9).

### 2.2 Exporter Survival

The slow growth of new exporters documented above is, of course, conditional on their remaining in the export market; many new exporters exit. To evaluate the persistence of export participation, we estimate a linear probability model:

$$I_{it}^{\text{exporter}} = \beta_0 + \beta_1 I_{it-1}^{\text{starter}} + \beta_2 I_{it-1}^{\text{exporter}} + \beta_3 \left(1 - I_{it-1}^{\text{exporter}}\right) I_{it-2}^{\text{exporter}} + \varepsilon_{it}. $$

The coefficients capture the probability that a producer is exporting at $t$ if it entered the export market at $t - 1$, was an incumbent exporter at $t - 1$, or did not export in $t - 1$ but exported in $t - 2$. This last coefficient captures the importance of export reentry.

In column 3, we report the results from (3). The continuation rate for an incumbent exporter is between 80 and 90 percent, while the continuation rate for an exporter starter is 15–25 percentage points lower. The rising exporter survival rate can be seen in Figure 1B (Ruhl and Willis 2017), in which we plot the one-period survival rate of exporters, conditional on their time in the export market.

In column 3, we report that plants that last exported two years prior are 26 to 30 percent more likely to export—thus, many new exporters are not truly new to the market. This is an aspect of the data that is rarely incorporated into models. We show in Section 9 that allowing for reentry greatly improves the model’s ability to account for the relative size of
new exporters.

### 2.3 Other Countries

The patterns we document in the Colombian and Chilean data are also present in data from other countries and other periods. New exporter dynamics are documented, for example, in Portuguese data by Bastos, Dias, and Timoshenko (2015), in Irish data by Fitzgerald, Haller, and Yedid-Levi (2017), and in French data by Piveteau (2017).

We will calibrate our quantitative model to data from the United States. While we do not have access to the U.S. Census of Manufacturing, we find similar patterns for U.S. manufacturing firms in Compustat, which is, perhaps, surprising given the selective sample of firms listed in Compustat. This is a balanced panel, so, for comparison, we also report the results from a balanced panel of Chilean and Colombian producers. As in the unbalanced panels, the relative importance of export entrants increases significantly from one-year to eight-year horizons. The starter size discounts are very similar to those in Colombia and Chile, but the intensity dynamics are more persistent.

Finally, in the last rows of Tables 1 and 2, we report some moments for U.S. manufacturing plants from other papers. Bernard, Jensen, and Lawrence (1995) report that new exporters export half as intensively as average exporters. Bernard and Jensen (1999) show that new exporters are smaller than incumbent exporters but grow much faster in terms of sales at various horizons. Bernard and Jensen (2004) show that exporting is very persistent and that having exported in the past increases the probability of exporting. They also show that, in the U.S. data, about 50 percent of starters from 1984 to 1992 reentered the export market after spending a year out of the market.9

---

9 Using linked firm-transaction data for the United States (1993–2003), Bernard, Jensen, Redding, and Schott (2009) show that net entry accounts for two to five percent of trade growth annually, about 20 percent over five-year horizons, and 24 percent over 24-year horizons. Expanding net entry to include products and destinations increases the annual and longer-term contribution of the extensive margin.
3 Model

We develop a dynamic general equilibrium model that captures the life cycle of both establishments and exporters. There are two symmetric countries: home and foreign, \{H, F\}. Each country is populated by a unit mass of identical, infinitely-lived consumers that inelastically supply one unit of labor.\footnote{Results with an endogenous labor supply are available. See Section 9 for more details.}

In each country, competitive final-goods producers purchase home and foreign differentiated intermediate inputs. The final good is not traded and is used for consumption, investment, and as a production input.\footnote{Capital accumulation is included to more accurately quantify the gains from trade. In most models, capital accumulation tends to increase the steady-state gains from a cut in trade barriers but makes the steady-state change overstate the welfare gains. Hence, our results are even more surprising.} There exists a one-period nominal bond, denominated in units of the home final good, that pays one unit of the home final good in the next period. Let \(B_t\) denote the home consumer’s holding of bonds purchased in period \(t\), \(B_t^*\) denote the foreign consumer’s holding of this bond, and \(Q_t\) denote the nominal bond price. The home final good is the numeraire, so \(P_t = 1\) in every period. With symmetric economies and symmetric policies, the foreign price level is \(P_t^* = 1\) and bond holdings are \(B_t = 0\). With asymmetric policies, the real exchange rate is \(q_t = P_t^*\), and \(B_t\) will vary.

Intermediate-goods producers in each country are characterized by their productivity, fixed export cost, and iceberg trade cost. Productivity is stochastic. Iceberg costs have an endogenous and stochastic element, while the fixed cost is exogenous. The shocks to productivity and iceberg costs generate movements of establishments into and out of exporting; unproductive establishments exit and new establishments enter.

All intermediate goods producers sell to their own country, but only some export. Exporting requires paying fixed and variable costs. All exporters face the same ad valorem tariff, \(\tau\), but differ in their iceberg transportation cost, \(\xi \geq 1\), and fixed export costs. The tariff is a policy variable, and the revenues collected from the tariff are rebated lump-sum to consumers. The transportation cost is a feature of technology. Fraction \(\xi - 1\) of an export shipment is destroyed in transit. Fixed export costs are paid in units of domestic labor.

We depart from the literature in allowing for three possible iceberg costs \(\xi \in \{\xi_L, \xi_H, \infty\}\)
with $\xi_L \leq \xi_H < \infty$ and two possible fixed export costs $f \in \{f_L, f_H\}$, $f_L \leq f_H$.\footnote{This is the smallest departure from the standard models that allows for new exporter dynamics yet yields rich predictions that differ from standard models.} Fixed export costs and the variable iceberg costs are related. Producers with an iceberg cost of $\xi = \infty$ are nonexporters. A nonexporter can deterministically lower its next-period iceberg cost to $\xi_H$ by paying a cost $f_H$. An exporter with iceberg costs $\xi_t = \{\xi_L, \xi_H\}$ can incur a cost $f_L$ to draw its next-period iceberg cost. We assume that the transition probabilities are Markovian and that the probability of drawing the low iceberg cost, $\xi_L$, is lower for an exporter with a high iceberg cost than for an exporter with a low iceberg cost: $\rho_\xi(\xi_L|\xi_H) \leq \rho_\xi(\xi_L|\xi_L)$. Thus, part of exporting is making an investment that may lead to a lower marginal cost of exporting in the future. If an exporter does not pay $f_L$, it is choosing to exit the export market, and its next-period iceberg cost rises to $\xi = \infty$. To reenter, it will have to pay $f_H$ and go through the growth process to accumulate a better $\xi$.

This formulation of fixed and iceberg costs is quite general and nests the most common approaches to modeling trade. When $f_L < f_H$, there is a sunk cost of exporting, as in Das, Roberts, and Tybout (2007). When $f_L = f_H$ and $\xi_L = \xi_H$, exporting is a static decision. When $f_L = f_H = 0$ and $\xi_L = \xi_H$, there is no export decision, and this is a general version of the Krugman (1980) model of monopolistic competition.

An establishment is created by hiring $f_E$ domestic workers and begins producing in the following period. The measure of country $j \in \{H, F\}$ establishments with technology $z$, iceberg costs $\xi$, and fixed costs $f$ is $\varphi_{j,t}(z, \xi, f)$.\footnote{Here, $f$ is the fixed cost that the producer has to pay if it decides to export: $f = f_H$ if $\xi = \infty$ and $f = f_L$ otherwise. Note that the producer-specific state is given by $(z, \xi)$. However, we describe producers with $(z, \xi, f)$ to explicitly denote the producer’s fixed cost.} Establishment exit (“death”) is exogenous and depends on the current productivity level.\footnote{Introducing endogenous exit from a fixed production cost is straightforward and yields similar results to our benchmark model.} The state variables of the economy include the measure of establishments, $\varphi_{j,t}(z, \xi, f)$, from each country and the capital stock in each country. For notational ease, economy-wide state variables are subsumed in the time subscript.
### 3.1 Consumers

Home consumers choose consumption, investment, and bonds to maximize utility subject to the sequence of budget constraints

\[
V_{C,0} = \max E_0 \sum_{t=0}^{\infty} \beta^t U(C_t),
\]

(4) \( C_t + K_t + Q_t B_t \leq W_t L_t + R_t K_{t-1} + (1 - \delta) K_{t-1} + B_{t-1} + \Pi_t + T_t, \)

where \( \beta \in (0, 1) \) is the subjective time discount factor; \( C_t \) is final consumption; \( K_{t-1} \) is the capital available in period \( t \); \( W_t \) and \( R_t \) denote the real wage rate and the rental rate of capital; \( \delta \) is the depreciation rate of capital; \( \Pi_t \) is real dividends from home producers; and \( T_t \) is the real lump-sum transfer of local tariff revenue. Investment is \( I_t = K_t - (1 - \delta) K_{t-1} \).

The foreign consumer’s problem is analogous. Foreign prices and allocations are denoted with an asterisk. The foreign budget constraint is

(5) \( P^*_t C^*_t + P^*_t K^*_t + Q_t P^*_t B^*_t \leq W^*_t P^*_t L_t + R^*_t P^*_t K^*_{t-1} + (1 - \delta) P^*_t K^*_{t-1} + P^*_t B^*_{t-1} + \Pi^*_t + T^*_t, \)

where all prices are quoted in units of the home final good.

The first-order conditions for the consumers’ utility maximization problems are

(6) \( Q_t = \beta E_t \frac{U_{C,t+1}}{U_{C,t}} = \beta E_t \frac{U^*_{C,t+1}}{U^*_{C,t}} \frac{P^*_t}{P^{t+1}_t}, \)

(7) \( 1 = \beta E_t \frac{U_{C,t+1}}{U_{C,t}} (R_{t+1} + 1 - \delta) = \beta E_t \frac{U^*_{C,t+1}}{U^*_{C,t}} \frac{P^*_t}{P^{t+1}_t} (P^{*}_{t+1} R^{*}_{t+1} + 1 - \delta), \)

where \( U_{C,t} \) denotes the derivative of the utility function with respect to its argument. Equation (6) is the no-arbitrage condition for bonds that equates the difference in expected consumption growth across countries to the expected change in the real exchange rate. Equation (7) is the standard Euler equation for capital accumulation in each country.
### 3.2 Final-goods Producers

Final goods are produced by combining home and foreign intermediate goods. The aggregation technology is a CES function

\[ D_t = \left\{ \sum_{j \in \{H,F\}} \sum_{\xi \in \{\xi_L,\xi_H,\infty\}} \int z y_{j,t}^d (z, \xi, f)^{\theta-1} \varphi_{j,t} (z, \xi, f) \, dz \right\}^{\frac{\theta-1}{\theta}}, \]

where \( y_{j,t}^d (z, \xi, f) \) are inputs of intermediate goods purchased from country \( j \)'s intermediate goods producers. The elasticity of substitution between intermediate goods is \( \theta > 1 \).

The final-goods market is competitive. Given the price of inputs, the final-goods producer chooses intermediate inputs, \( y_{j,t}^d \), to solve

\[ \max \Pi_{F,t} = D_t - \sum_{\xi \in \{\xi_L,\xi_H,\infty\}} \int z P_{H,t} (z, \xi, f) y_{H,t}^d (z, \xi, f) \varphi_{H,t} (z, \xi, f) \, dz \]

\[ - (1 + \tau) \sum_{\xi \in \{\xi_L,\xi_H\}} \int z P_{F,t} (z, \xi, f) y_{F,t}^d (z, \xi, f) \varphi_{F,t} (z, \xi, f) \, dz, \]

subject to the production technology in (8). Here, \( P_{j,t} (z, \xi, f) \) are the home-country prices of intermediate goods produced in country \( j \) establishments. Solving the problem in (9) yields the input demand functions,

\[ y_{H,t}^d (z, \xi, f) = [P_{H,t} (z, \xi, f)]^{-\theta} D_t, \]

\[ y_{F,t}^d (z, \xi, f) = [(1 + \tau) P_{F,t} (z, \xi, f)]^{-\theta} D_t, \]

where the final-goods price is defined as

\[ P_t^{1-\theta} = \sum_{\xi \in \{\xi_L,\xi_H,\infty\}} \int z \left[ P_{H,t} (z, \xi, f)^{1-\theta} \varphi_{H,t} (z, \xi, f) + [(1 + \tau) P_{F,t} (z, \xi, f)]^{1-\theta} \varphi_{F,t} (z, \xi, f) \right] \, dz. \]

### 3.3 Intermediate-goods Producers

An intermediate-goods producer is described by its technology, iceberg cost, and fixed cost, \((z, \xi, f)\). It produces using capital, \( k \), labor, \( l \), and materials, \( x \), according to

\[ y_t (z, \xi, f) = e^z [k_t (z, \xi, f)^{\alpha} l_t (z, \xi, f)^{1-\alpha}]^{1-\alpha_x} x (z, \xi, f)^{\alpha_x}. \]
The markets that the producer serves in the current period are predetermined, so the producer maximizes current-period gross profits by choosing prices for each market, \( P_{H,t}(z, \xi, f) \) and \( P_{H,t}^*(z, \xi, f) \), labor \( l_t(z, \xi, f) \), capital \( k_t(z, \xi, f) \), and materials \( x_t(z, \xi, f) \) to solve

\[
\Pi_t(z, \xi, f) = \max P_{H,t}(z, \xi, f) y_{H,t}(z, \xi, f) + P_{H,t}^*(z, \xi, f) y_{H,t}^*(z, \xi, f) \\
- W_t l_t(z, \xi, f) - R_t k_t(z, \xi, f) - P_t x_t(z, \xi, f),
\]

subject to the production technology (13), a constraint that supplies to home- and foreign-goods markets, \( y_{H,t}(z, \xi, f) \) and \( y_{H,t}^*(z, \xi, f) \), are feasible,

\[
y_t(z, \xi, f) = y_{H,t}(z, \xi, f) + \xi y_{H,t}^*(z, \xi, f),
\]

and the constraints that supplies to home- and foreign-goods markets are equal to the demands from final-goods producers from (10) and their foreign analogue,

\[
y_{H,t}(z, \xi, f) = y_{dH,t}(z, \xi, f),
\]

\[
y_{H,t}^*(z, \xi, f) = y_{dH,t}^*(z, \xi, f).
\]

Given its downward-sloping demand curve, the monopolistic producer charges a constant markup over marginal cost in each market,

\[
P_{H,t}(z, \xi, f) = \frac{\theta}{\theta - 1} MC_t e^{-z},
\]

\[
P_{H,t}^*(z, \xi, f) = \frac{\theta}{\theta - 1} \xi MC_t e^{-z},
\]

where

\[
MC_t = \alpha_x^{-\alpha_x} (1 - \alpha_x)^{-1-\alpha_x} \left[ \left( \frac{R_t}{\alpha} \right)^\alpha \left( \frac{W_t}{1 - \alpha} \right)^{1-\alpha} \right]^{1-\alpha_x}.
\]

Note that when \( \xi = \infty \), the producer is a nonexporter.

The value of the producer with \((z, \xi, f)\), if it decides to export in period \( t + 1 \), is

\[
V_t^1(z, \xi, f) = -W_t f + n_s(z) Q_t \sum_{\xi' \in \{\xi_L, \xi_H\}} \int_{z'} V_{t+1}(z', \xi', f_L) \phi(z'|z) \rho_\xi(\xi'|\xi) dz',
\]

and the value of the producer, if it does not export in period \( t + 1 \), is

\[
V_t^0(z, \xi, f) = n_s(z) Q_t \int_{z'} V_{t+1}(z', \infty, f_H) \phi(z'|z) dz',
\]
where \( n_s(z) \) is the probability that the producer will survive until the next period. Note that this probability varies with the producer's productivity. In the calibrated model, the survival probability is increasing in productivity, \( z \). The value of the producer is

\[
V_t(z, \xi, f) = \Pi_t(z, \xi, f) + \max \left\{ V^1_t(z, \xi, f), V^0_t(z, \xi, f) \right\}.
\]

Clearly, the value of a producer depends on its fixed cost, iceberg cost, and productivity. Given that there are three possible levels of iceberg costs, there are now three possible cutoffs, \( z_{m,t} \), with \( m \in \{L, H, \infty\} \). The critical level of productivity for exporting, \( z_{m,t} \), satisfies

\[
V^1_t(z_{m,t}, \xi_m, f) = V^0_t(z_{m,t}, \xi_m, f).
\]

It is straightforward to show that the threshold for exporting is largest for nonexporters and smallest for exporters with the low iceberg cost (\( z_{\infty,t} > z_{H,t} \geq z_{L,t} \)).

### 3.4 Entry

New establishments are created by hiring \( f_E \) workers in the period prior to production. Entrants draw their productivity from the distribution \( \phi_E(z') \). Entrants cannot export in their first productive period. The free-entry condition is

\[
V^E_t = -W_t f_E + Q_t \int_{z'} V_{t+1} (z', \infty, f_H) \phi_E (z') \, dz' \leq 0.
\]

The mass of entrants in period \( t \) is \( N_{E,t} \), while the mass of incumbents, \( N_t \), consists of the two types of exporters and the nonexporters,

\[
N_{L,t} = \int_z \phi_{H,t} (z, \xi_L, f_L) \, dz,
\]

\[
N_{H,t} = \int_z \phi_{H,t} (z, \xi_H, f_L) \, dz,
\]

\[
N_{\infty,t} = \int_z \phi_{H,t} (z, \infty, f_H) \, dz.
\]

The mass of exporters equals \( N_{1,t} = N_{L,t} + N_{H,t} \); the mass of nonexporters equals \( N_{0,t} = N_{\infty,t} \); and the mass of establishments equals \( N_t = N_{1,t} + N_{0,t} \). The fixed costs of exporting imply that only a fraction, \( n_x,t = N_{1,t}/N_t \), of home intermediates are available in the foreign country in period \( t \).
Given the critical level of productivity for exporters and nonexporters, \( z_{m,t} \), the starter ratio (the fraction of establishments among nonexporters that start exporting) and the stopper ratio (the fraction of exporters among surviving establishments who stop exporting) are, respectively, 

\[
\begin{align*}
  n_{0,t+1} &= \frac{\int_{-\infty}^{\infty} n_s(z) \varphi_{H,t}(z, f_H) \, dz}{\int_{-\infty}^{\infty} n_s(z) \varphi_{H,t}(z, f_H) \, dz}, \\
  n_{1,t+1} &= \frac{\sum_{m \in \{L,H\}} \int_{-\infty}^{\infty} n_s(z) \varphi_{H,t}(z, f_L) \, dz}{\sum_{m \in \{L,H\}} \int_{-\infty}^{\infty} n_s(z) \varphi_{H,t}(z, f_L) \, dz}.
\end{align*}
\]

The mass of establishments evolves according to 

\[
\begin{align*}
  \varphi_{t+1}(z', f_H) &= \sum_{m \in \{L,H,\infty\}} \int_{-\infty}^{z_{m,t}} \rho_{\xi}(\xi|\xi_m) \int_{-\infty}^{\infty} n_s(z) \varphi_{H,t}(z, f) \phi(z'|z) \, dz, \\
  \varphi_{t+1}(z', \xi_H, f_L) &= \sum_{m \in \{L,H\}} \rho_{\xi}(\xi|\xi_m) \int_{-\infty}^{z_{m,t}} \rho_{\xi}(-\infty, \xi_m) \int_{-\infty}^{\infty} n_s(z) \varphi_{H,t}(z, f) \phi(z'|z) \, dz, \\
  \varphi_{t+1}(z', \xi_L, f_L) &= \sum_{m \in \{L,H\}} \rho_{\xi}(\xi|\xi_m) \int_{-\infty}^{z_{m,t}} \rho_{\xi}(\xi, \infty) \int_{-\infty}^{\infty} n_s(z) \varphi_{H,t}(z, f) \phi(z'|z) \, dz.
\end{align*}
\]

### 3.5 Government and Aggregate Variables

The government collects tariffs and redistributes the revenue lump-sum to domestic consumers. The government’s budget constraint is 

\[
T_t = \tau \sum_{\xi \in \{\xi_L, \xi_H\}} \int P_{F,t}(z, \xi, f_L) y_{F,t}(z, \xi, f_L) \varphi_{F,t}(z, \xi, f_L) \, dz.
\]

Nominal exports and imports are, respectively, 

\[
\begin{align*}
  EX_t^N &= \sum_{\xi \in \{\xi_L, \xi_H\}} \int P_{H,t}(z, \xi, f_L) y_{H,t}(z, \xi, f_L) \varphi_{H,t}(z, \xi, f_L) \, dz, \\
  IM_t^N &= \sum_{\xi \in \{\xi_L, \xi_H\}} \int P_{F,t}(z, \xi, f_L) y_{F,t}(z, \xi, f_L) \varphi_{F,t}(z, \xi, f_L) \, dz.
\end{align*}
\]

Home nominal GDP is the sum of value added from intermediate- and final-goods producers, 

\[
Y_t^N = C_t + I_t + EX_t^N - IM_t^N.
\]

The trade-to-GDP ratio is \( TR_t = \frac{EX_t^N + IM_t^N}{2Y_t^N} \), and \( IMD_t \) is the expenditure on imported goods relative to home goods, 

\[
IMD_t = \frac{(1 + \tau) \sum_{\xi \in \{\xi_L, \xi_H\}} \int P_{F,t}(z, \xi, f_L) y_{F,t}(z, \xi, f_L) \varphi_{F,t}(z, \xi, f_L) \, dz}{\sum_{\xi \in \{\xi_L, \xi_H, \infty\}} \int P_{H,t}(z, \xi, f) y_{H,t}(z, \xi, f) \varphi_{H,t}(z, \xi, f) \, dz}.
\]
so the share of expenditures on domestic goods is

\begin{equation}
\lambda_t = \frac{1}{1 + IMD_t},
\end{equation}

and the trade elasticity is

\begin{equation}
\varepsilon_t = -\frac{\ln \left(\frac{IMD_t}{IMD_{t-1}}\right)}{\ln \left(\frac{(1 + \tau_t)}{(1 + \tau_{t-1})}\right)}.
\end{equation}

Labor used in production, rather than to pay fixed costs, \(L_{P,t}\), is

\begin{equation}
L_{P,t} = \sum_{\xi \in \{\xi_L, \xi_H, \infty\}} \int_z l_t(z, \xi, f) \varphi_{H,t}(z, \xi, f) \, dz.
\end{equation}

The domestic labor hired by exporters to cover the fixed costs of exporting, \(L_{X,t}\), equals

\begin{equation}
L_{X,t} = \sum_{m \in \{L, H\}} f_L \int_{z_{m,t}}^{\infty} \varphi_{H,t}(z, \xi_m, f_L) \, dz + f_H \int_{z_{\infty, t}}^{\infty} \varphi_{H,t}(z, \infty, f_H) \, dz.
\end{equation}

From (41), we see that the trade cost, measured in units of domestic labor, depends on the exporter status from the previous period. Aggregate profits are the difference between profits and fixed costs,

\begin{equation}
\Pi_t = \sum_{\xi \in \{\xi_L, \xi_H, \infty\}} \int_z \Pi_t(z, \xi, f) \varphi_{H,t}(z, \xi, f) \, dz - W_t L_{X,t} - W_t f_E N_{E,t}.
\end{equation}

Even though there is free entry in the model, aggregate profits are generally positive. These profits compensate consumers for waiting for their investment in producers to mature. With \(\beta = 1\), these profits will equal zero in steady state.

### 3.6 Equilibrium Definition

In an equilibrium, variables satisfy several resource constraints. The final-goods market-clearing conditions are \(D_t = C_t + I_t + X_t\), and \(D_t^* = C_t^* + I_t^* + X_t^*\), where \(X_t\) is total material inputs in production, given by

\begin{equation}
X_t = \sum_{\xi \in \{\xi_L, \xi_H, \infty\}} \int_z x_t(z, \xi, f) \varphi_{H,t}(z, \xi, f) \, dz.
\end{equation}
Each individual goods market clears; the labor market-clearing conditions are \( L = L_{P,t} + L_{X,t} + \int E N_{E,t} \) and the foreign analogue; the capital market-clearing conditions are
\[
K_{t-1} = \sum_{\xi \in \{\xi_L, \xi_H, \infty\}} \int z k_t(z, \xi, f) \varphi_{H,t}(z, \xi, f) \, dz,
\]
and the foreign analogue. The government budget constraints are given by (34) and its foreign analogue. The profits of each country’s establishments, \( \Pi_t \), are distributed to its consumers. The international bond market-clearing condition is given by \( B_t + B^*_t = 0 \).

An equilibrium is a collection of allocations for home consumers \( C_t, B_t, \) and \( K_t \); allocations for foreign consumers \( C^*_t, B^*_t, \) and \( K^*_t \); allocations for home final-goods producers; allocations for foreign final-goods producers; allocations, prices, and export decisions for home intermediate producers; allocations, prices, and export decisions for foreign intermediate producers; labor used for exporting costs and for entry costs by home and foreign producers; transfers \( T_t, T^*_t \) by home and foreign governments; real wages \( W_t, W^*_t \); real rental rates of capital \( R_t, R^*_t \); and bond prices \( Q_t \) that satisfy the following conditions: (i) the consumer allocations solve the consumer’s problem; (ii) the final-goods producers’ allocations solve their profit-maximization problems; (iii) intermediate-goods producers’ allocations, prices, and export decisions solve their profit-maximization problems; (iv) the entry conditions hold; (v) the market-clearing conditions hold; and (vi) the transfers satisfy the government budget constraint.

4 Calibration

In this section, we discuss the calibration of the model. We also make clear how the nature of entry and trade costs depends on modeling the growth in exporters’ export intensity. We calibrate the model to match features of the U.S. economy and first describe the functional forms and parameter values of our benchmark economy. The parameter values are summarized in Table 3.

The instantaneous utility function is \( U(C) = \frac{C^{1-\sigma}}{1-\sigma} \), where \( 1/\sigma \) is the intertemporal elasticity of substitution. The discount factor, \( \beta \), depreciation rate, \( \delta \), and risk aversion, \( \sigma \), are standard: \( \beta = 0.96, \delta = 0.10, \) and \( \sigma = 2 \).
The distribution of establishments is determined by the structure of shocks. To eliminate the role of the elasticity of substitution, $\theta$, in establishment dispersion, we assume that producer productivity $z = \frac{1}{\theta - 1} \ln a$. An incumbent’s productivity has an autoregressive component ($\rho < 1$) of $\ln a' = \rho \ln a + \varepsilon$, $\varepsilon \sim N(0, \sigma_\varepsilon^2)$. With an AR(1) shock process, the conditional distribution is normal, $\phi(\ln a'| \ln a) = N(\rho \ln a, \sigma_\varepsilon^2)$, and the unconditional distribution is $N\left(0, \frac{\sigma_\varepsilon^2}{1 - \rho^2}\right)$. Entrants draw productivity based on the unconditional distribution $\ln a' = \mu_E + \varepsilon_E$, $\varepsilon_E \sim N\left(0, \frac{\sigma_\varepsilon^2}{1 - \rho^2}\right)$, where $\mu_E < 0$ is chosen to match the observation that entrants are smaller than incumbents. Establishments receive an exogenous death shock that depends on an establishment’s previous-period productivity, $a$; the probability of death is $n_d(a) = 1 - n_s(a) = \max\{0, \min\{e^{-\gamma_0 a + \gamma_1}, 1\}\}$.

The parameter $\theta$ determines both the producer’s markup and the elasticity of substitution across varieties. We set $\theta = 5$ to yield a producer markup of 25 percent. We set the tariff rate to ten percent to include the direct measure of tariff and non-tariff barriers.

Recall that four parameters determine the dynamics of export intensity: the two iceberg costs $\{\xi_H, \xi_L\}$ and the transition probabilities, which we denote $\{\rho_{LL}, \rho_{HH}\}$. For simplicity, we assume that $\rho_{LL} = \rho_{HH} = \rho_\xi$, so that three parameters determine the trade intensity dynamics.

The labor share parameter in production, $\alpha$, is set to match the ratio of labor income to GDP in the United States (66 percent). In the model, $\alpha_x$ determines the ratio of value added to gross output in manufacturing. In the United States, this ratio averaged 2.8 from 1987 to 1992 and implies that $\alpha_x = 0.81$. The entry cost, $f_E$, is set to normalize the total mass of establishments, $N$, to one in the initial steady state. The mean establishment size is normalized to the mean establishment size in the United States in 1992.

The ten parameters, $\{\gamma_0, \gamma_1, \rho_\varepsilon, \sigma_\varepsilon^2, \mu_E, f_L, f_H, \xi_L, \xi_H, \rho_\xi\}$, are chosen to match the following observations:

2. An initial export intensity of half the mean export intensity (Ruhl and Willis 2017).
3. A five-year export intensity twice the initial export intensity (Ruhl and Willis 2017).
5. An export participation rate of 22.3 percent (1992 CM).


7. Entrants’ labor share of 1.5 percent (Davis, Haltiwanger, and Schuh 1998).

8. Shut-down establishments’ labor share of 2.3 percent (Davis, Haltiwanger, and Schuh 1998).

9. Establishment employment size distribution as in the 1992 CM.

The first three targets summarize the dynamics of export intensity and determine the technology for shipping ($\xi_L$, $\xi_H$, $\rho_\xi$). The next two targets relate exporters to the population of establishments and largely determine the fixed costs ($f_L$, $f_H$). The next three targets help pin down the establishment creation, destruction, and growth process ($\rho_\varepsilon$, $\sigma_\varepsilon$, $\gamma_0$, $\gamma_1$, $\mu_E$): Newborn establishments and dying establishments tend to have few employees, and newborn establishments have high failure rates. Finally, we minimize the distance between the model’s producer size distribution and the size distribution of U.S. establishments.

4.1 The Benchmark Model

The calibration provides an estimate of the establishment creation and exporting technologies. The cost of starting to export is relatively small, only 3.7 percent of the cost of creating an establishment, but it is about 40-percent larger than the cost of continuing to export (0.246 versus 0.176). The high iceberg cost, $\xi_H$, is estimated to be 63-percent larger than the low cost, $\xi_L$ (1.72 versus 1.07), and the idiosyncratic iceberg cost is persistent, $\rho_\xi = 0.916$. Most active exporters have the high iceberg cost and are investing in improving their export ability. In the aggregate, fixed export costs account for 58.1 percent of gross export profits.

Based on the ergodic distribution, Figure 2A shows how the average export intensity, measured as the ratio of export revenue to total revenue, rises with years of exporting experience. Export intensity grows gradually beyond the five-year period being targeted. This reflects a rising probability that a long-term exporter has accumulated the low iceberg cost. Figure 2B shows that the probability of continuing in the export market rises over time after the second year in the market, consistent with the Colombian data in Figure 1B. This
reflects mainly the fact that older exporters are more likely to have become efficient exporters and are less willing to give this up by exiting. This model outcome was not targeted and provides independent validation of the model. These two figures are consistent with the evidence from Ruhl and Willis (2017).

The low export intensity and continuation probabilities suggest that export profits are quite low initially and rise over time. Figure 2C shows how the net profits of a marginal starter (i.e., a producer with productivity $z_\infty$ in period zero) evolve over time when it is subject to shocks that lead it to continue exporting ($\prod_{j=1}^t \pi_t > 0$). In this figure, we plot

$$\mu_t = 100 \times \frac{E(\pi_t - f_t | \pi_j > 0, j = 1, ..., t)}{f_H}.$$ 

In the year prior to exporting, $\mu_0 = -100$ since the producer pays $f_H$ and earns $\pi_0 = 0$. This measure of net profits to entry costs is rising with time in the market, primarily because older exporters tend to be more efficient exporters. Given this profile of gross profits, a new marginal exporter expects to have negative net profits over its first three years in the market, in addition to the loss incurred in the year prior to entry. Over this period, the new exporter is willing to take a loss in order to have the chance to become an efficient exporter in the future. This investment is risky, as many new exporters exit right away.

4.2 The Sunk-Cost Model

Eliminating the variance in iceberg costs, $\xi_L = \xi_H = \xi$, yields the traditional sunk-cost model of Das, Roberts, and Tybout (2007), studied in general equilibrium in Alessandria and Choi (2014). We report the parameter estimates from this version of the model in the “sunk-cost” column in Table 3. We estimate the single iceberg cost, $\xi$, to be 1.43.

Compared to the baseline model, in which the export entry cost is 1.4 times the continuation cost, the sunk component of the entry cost is much larger in this model: The estimated export entry cost is 3.8 times the cost of continuing to export. In the sunk-cost model, an important reason that exporters stay in the market is to avoid paying the large upfront cost of reentering—sunk costs generate persistent exporting. In the benchmark model, this effect is much smaller since the gap between the startup and continuation costs is small. Rather, in
the benchmark model, exporters stay in the market to maintain access to the good exporting technology, $\xi_L$, and to avoid going through the growth process again.

To show how the timing of profits depends on the structure of trade costs, we take the marginal new exporter from the benchmark model (Figure 2C) and subject it to the trade cost structure estimated in the sunk-cost model. The new exporter faces the same productivity shocks and uses the same exit rule as in the benchmark model. In this way, we can see how the path of expected profits varies with exporting tenure for a particular path of productivity and participation decisions. To make profits comparable across models, net profits are measured relative to the export entry cost from the benchmark model. Figure 2C shows that, with the sunk-cost export technology, the upfront investment to enter is about twice as large, and the producer starts earning a net profit from the first period in the market. This reflects, in part, a much higher initial export intensity and a smaller continuation cost (about half that of our benchmark model). Over time, the profit rate does not change much. Comparing the models, the sunk-cost model front-loads the costs and benefits from exporting relative to the benchmark model. The rising net profits in the benchmark model make it clear that the continuation cost in that model is primarily an investment in lowering the future marginal cost of exporting. Figure 2D reports the cumulative profits in both models. It takes almost seven years for the cumulative net profits in the benchmark model to exceed the net profits in the sunk-cost specification.

In the benchmark model with new exporter dynamics, we find the producer-level cost of entering the export market to be relatively low but the aggregate cost of maintaining international trade to be relatively high. In the stationary steady state, payments of fixed export costs are 58.1 percent of total profits in the benchmark model and only 47.6 percent in the sunk-cost model. If we recalibrate the sunk-cost model so that the aggregate share of profits paid to fixed export costs is the same as in the benchmark model, the export entry cost needs to be 11.1 times the continuation cost.\footnote{Obviously, this producer would make different exit decisions in this sunk-cost environment.} We refer to this model as the sunk-cost-high model and summarize its parameters in Table 3. In the sunk-cost-high model, exporting...\footnote{The sunk-cost-high model is calibrated to match observations 5 to 9 on page 20. Additionally, the iceberg cost and the two fixed costs are set to generate the export participation rate (22.3 percent), the ratio of exports to GDP (9.7 percent), and the ratio of fixed export costs to export profits (58.1 percent), as in the benchmark model.}
becomes a very persistent activity: The exporter exit rate in the data and the benchmark model is 17 percent; it falls to 3.95 percent in the sunk-cost-high model. We discuss the sunk-cost-high model in more detail in Section 6.1.

5 Global Trade Liberalization

In this section, we consider the impact of a global change in tariffs on welfare and the dynamics of trade. In particular, we assume an unanticipated worldwide elimination of the ten-percent tariff. We focus on an unanticipated change in tariffs to clarify the aggregate effect of tariffs in the baseline model and a range of simpler models. It is straightforward to consider the more empirically relevant case of anticipated changes in trade policy, such as a gradual liberalization.

Table 4 reports the changes in welfare and trade, and Figure 3 plots the dynamics of some key variables. Even though trade grows gradually, consumption booms during the transition, so the welfare gain is about 15 times larger than the change in steady-state consumption (6.30 versus 0.42). Thus, the conventional view that slow trade adjustment should lower the gain from trade liberalization does not hold in the model with endogenous export participation and exporter growth.

With lower tariffs, trade expands substantially, rising from 9.7 percent of manufacturing shipments to 29.2 percent. Figure 3A shows that this expansion takes time, as the trade elasticity grows slowly. In the first year, only the intensive margin operates, so the trade elasticity is $\theta - 1$. With time, as more exporters enter, continue, and mature, export shipments expand. Ten years after the policy change, the endogenous part of the trade elasticity has increased by only 69 percent of its long-run change: Trade is quite sluggish.\footnote{The endogenous part of the trade elasticity is the part due to entry, expansion, and exit rather than to the static intensive margin.}

A simple way to compare aggregate trade dynamics across models is to measure the discounted average trade elasticity:

\[
(46) \quad \bar{\varepsilon}_t = (1 - \beta) \sum_{t=0}^{\infty} \beta^t \varepsilon_t.
\]
This measure weights the early periods of trade adjustment more than the later periods and provides a relevant measure of the speed of trade adjustment. In our model, the short-run elasticity is four; the discounted trade elasticity is 10.15; and the long-run elasticity is 11.55.

Slow trade elasticity growth, however, does not lead to slow growth in consumption or output (see Figure 3B); consumption and output jump initially. Consumption has a hump shape, peaking seven years after the policy change at 9.75 percentage points above its long-run change of 0.42 percent. Figure 3C shows how different forms of investment change during the transition to the new steady state. Investment in capital initially falls and then recovers strongly as the economy uses capital to smooth out the benefits of the policy change. Capital dynamics imply that output expands a bit more strongly than consumption. Investment in establishment creation falls in the first few years and then recovers to a level of establishment creation that is lower than that in the initial steady state. The stock of establishments falls gradually to its new steady-state level.

The desire to reduce the number of establishments following the policy change is key to the overshooting behavior in the model since it implies that more resources are initially available for production along the transition (see Figure 3C) and that there is a large stock of establishments that can be converted to exporting. The decline in establishments is gradual because the overshooting in aggregate economic activity increases profits enough to offset the negative effect of increased trade on entry. This mechanism is similar to that discussed by Alessandria and Choi (2014) in a model with only a sunk cost. Burstein and Melitz (2013) also argue for overshooting in consumption, but, in their framework with no dynamic exporting decision or capital accumulation, the overshooting arises because of a sharp drop in entry.

The effect of the decline in establishment creation on the aggregate dynamics of the economy can be seen most clearly in a counterfactual experiment in which the mass of entrants does not change. For this experiment, we impose a subsidy on establishment creation, which is financed by a lump-sum tax. We choose the subsidy so that $N_t = 1$ in every period. Figure 4 plots the dynamics of the trade elasticity and consumption in this counterfactual and in the benchmark model. With no change in establishment creation, trade expands by less, as exporters are discouraged from entering in the face of greater local competition.
Consumption declines slightly in the first period, owing to the investments in expanding export participation. It then grows monotonically to the new steady-state level, which is seven percentage points above the level in the benchmark model (7.41 versus 0.42). It takes 20 years for this alternative model to reach the same level of consumption as in the benchmark model.

6 Comparison to other Models

To evaluate the role of producer-level export dynamics in the aggregate effect of tariffs, we consider two variations of the benchmark model. In the first variation, we eliminate the sluggishness in producer-level export growth. This version of the sunk-cost model of Das, Roberts, and Tybout (2007) clarifies the role of export intensity growth in the aggregate economy. In the second variation, we examine a model without a producer-level export decision, but the model is calibrated to generate the same aggregate export growth in the transition and in the new steady state as in our benchmark model. This allows us to explore whether the idea from ACR—that the welfare gain from trade will be identical across models that generate the same trade elasticity—extends to a dynamic environment.

6.1 No Exporter Growth (Sunk-Cost Model)

The slow export growth of producers is important for the response of welfare and trade volumes to a change in trade barriers. To show this, we set $\xi_L = \xi_H = \xi$ so that there is no difference in export intensity between new and old exporters—a variation of the model studied by Alessandria and Choi (2014). It shares the qualitative features of our benchmark model, and we recalibrate this model to match the kinds of features of the U.S. economy that we used to calibrate the benchmark model. Table 3 summarizes the parameters, while Table 4 summarizes the effect of abstracting from export intensity dynamics on aggregate outcomes. Figures 5–7 plot the transition to the new steady state.

Compared to the benchmark model, the sunk-cost model generates a smaller long-run expansion of trade. The trade elasticity is about 63 percent of the benchmark model (7.2 versus 11.5). The transition, though, is relatively faster, as the discounted trade elasticity
is about 68 percent of the benchmark model’s (6.9 versus 10.15). By year three, 90 percent of trade growth has been realized, while in the benchmark model, only 54 percent of trade growth has been realized. Compared to the benchmark model, the sunk-cost model also generates a larger change in steady-state consumption (1.98 versus 0.42) but a smaller welfare gain (4.75 versus 6.30).

The benchmark model generates a larger welfare gain than the sunk-cost model because, even though trade grows more slowly, overshooting is stronger. In terms of consumption, the models generate similar dynamics in the first two to three years. The sunk-cost model, however, peaks four years earlier and at a level below the benchmark model. The gap that opens between the models closes slowly. The more-delayed and more-variable dynamics of consumption in the benchmark model reflect the dynamics of new exporter growth. In the benchmark model, since exporters need time to increase export efficiency, more time and resources are initially used to increase the stock of exporters, so it takes longer to benefit from this entry. The long-run effect on consumption in the sunk-cost model is stronger because there is less substitution between trade and establishment creation than in our benchmark model. Indeed, in the long run, the stock of domestic producers falls by only 4.8 percent in the sunk-cost model versus 13.1 percent in the benchmark model.

Relative to the benchmark model, the sunk-cost model generates a smaller gain from trade reform because new exporters contribute too much to aggregate trade. While both models are calibrated to have the same number of export entrants each period, new exporters in the benchmark model face much higher iceberg trade costs. Consequently, a new exporter in the sunk-cost model will export twice as much as the otherwise-identical new exporter in the benchmark model.

We can make the aggregate variables in the sunk-cost model behave more like those in the benchmark model by increasing the export entry cost. While this may seem counterintuitive—entry costs are already larger in the sunk-cost model—the slow expansion of new exporters in the benchmark model implies that the export continuation costs \( f_L \) are a form of entry cost as well. We report this parameterization in the column labeled “sunk-cost high” in Table 3. We calibrate the sunk-cost-high model to match the same moments as in the sunk-cost model, except that we can no longer target the export exit rate. In its place, we set
the share of aggregate export profits paid to export fixed costs at 0.58, as in the benchmark model. This implies that the sunk-cost-high model and the benchmark model dedicate the same share of aggregate resources to trade and generate the same aggregate trade share.

In the sunk-cost-high model, the much larger entry cost \( f_H / f_L = 11.1 \) increases the selection on productivity into exporting and decreases the export exit rate from 17 percent to 3.95 percent. Since we have calibrated both models to match the export participation rate in the data, the lower exit rate means that the entry rate falls. Cohorts of new exporters in the sunk-cost-high model are smaller than in the original sunk-cost model.

Reducing the importance of the new cohort of exporters leads the sunk-cost-high model to behave more like the benchmark model. Trade grows more and more gradually; the number of establishments shrinks more; and there is more overshooting than in the model with the smaller sunk cost. Thus, our benchmark model with a small export entry cost and new exporter dynamics yields aggregate properties that are more consistent with a traditional sunk-cost model with a very large export entry cost. The sunk-cost-high model comes closer to approximating our benchmark model because it requires relatively similar investments in exporting (measured as fixed costs relative to export profits) to generate a given stream of export revenue.

It is evident from Table 4 that the sunk-cost-high model better approximates the benchmark model than the calibrated sunk-cost model does. Even so, compared to our benchmark model, the welfare gain in the sunk-cost-high model is still lower (5.67 percent versus 6.3 percent), and the long-run change in consumption is still higher (1.65 percent versus 0.42 percent).

6.2 No Export Decision (No-Cost Model)

To further explore how the producer-level details of exporting matter for welfare, we now consider a version of the model in which all establishments export from birth (i.e., there are no fixed export costs, \( f_H = f_L = 0 \)) and face the same iceberg cost (i.e., \( \xi_L = \xi_H \)). This is a variation of the Krugman (1980) model. Without some modification, the trade elasticity is constant in this model. To generate the gradual increase in the trade elasticity that we observe in the benchmark model, it is necessary to introduce an adjustment friction...
to either the final-goods aggregator or the trade cost. We introduce an adjustment cost into the aggregation of intermediates by final-goods producers. Specifically, we modify the aggregator in (8) to include a time-varying weight on imported goods, $g_t$, such that

$$D_t = \left[ \int_z y_{H,t}^d (z) \phi_t (z) \, dz + g_t \int_z y_{E,t}^d (z) \phi_t (z) \, dz \right]^{\theta_t},$$

$$g_t = g_{t-1} \left[ \left( \frac{\lambda_t}{\lambda_{t-1}} \right)^{\rho_g} \right]^{1-\rho_g}, \quad g_{-1} = 1,$$

where $\lambda_t$ is the home intermediate goods’ expenditure share. With $\nu > 0$, the term $g_t$ implies that an increase in the import share will lower the weight on imports in the aggregator. This demand shifter is assumed to depend on aggregate imports and is external to the final-goods producer. It can be interpreted as a cost of adjusting inputs. It affects only the transition and not the steady state.

The parameters of the final goods aggregator, $\nu$ and $\rho_g$, are set to minimize the gap between the trade elasticity in the benchmark model and in this no-cost model,

$$\left\{ \nu^*, \rho_g^* \right\} = \arg \min_{\{\nu, \rho_g\}} \left\{ \sum_{t=0}^{\infty} [\beta^t (\varepsilon_{\text{benchmark},t} - \varepsilon_{\text{nocost},t})]^2 \right\},$$

which yields $\nu^* = 1.89$ and $\rho_g^* = 0.25$. In Figure 5, we plot the trade elasticity in the no-cost model, as well as, in the benchmark and sunk-cost models.

For the no-cost model to match the long-run trade elasticity in the benchmark model, we increase the elasticity of substitution, $\theta$, from 5 to 12.54. This lowers markups from 25 percent to about eight percent, which has the effect of changing the labor share of income, the ratio of gross-output to value added, and the ratio of trade to value added. To maintain the same macro targets for the ratio of trade to shipments, labor share, and materials usage,

---

18 One can think of this specification as representing the challenges that producers face in adjusting their inputs in the short run. This adjustment cost shares some similarities with that in Engel and Wang (2011). Alternatively, we could have generated slow trade growth by making the tariff fall gradually or allowing the iceberg cost to depend on the change in the import share (i.e., $\xi_t = \xi e^{-v \ln \lambda_t/\lambda_{t-1}}$). Both of these approaches yield similar findings in that they reduce consumption along the transition, but the trade elasticity would be constant in these cases.

19 The term $g_t$ can be thought of as a wedge that accounts for the changes in trade that cannot be explained by relative prices. Recent work by Levchenko, Lewis, and Tesar (2010) and Alessandria, Kaboski, and Midrigan (2013) show that there are substantial cyclical fluctuations in this wedge.
we must adjust $\alpha$, $\alpha_x$, and $\xi$. The capital share is doubled from 14 percent to 28 percent; the material usage is lowered from 80 percent to 70 percent; and the iceberg cost is lowered to 1.11. The parameters are reported in the column labeled “no-cost” in Table 3. The column labeled “no-cost” in Table 4 summarizes the aggregate effects of the cut in tariffs in this alternative model, and Figures 5–7 plot some aspects of the transition.

The key focus is on the change in welfare. In the benchmark model, the welfare gain is almost four percentage points larger (6.3 versus 2.3), even though the steady-state change in consumption is about 3.5 percentage points smaller (0.42 versus 3.93). This large gap in welfare occurs because consumption in the benchmark model overshoots the new steady state, while in the no-cost model, consumption grows gradually. The gap in consumption between the models is as large as 7.8 percentage points five years after the policy change. The gradual consumption growth in the no-cost model occurs because the economy decumulates establishments only temporarily, with much smaller magnitudes, and capital and trade grow gradually due to the adjustment cost in the use of inputs in the production of final goods.\textsuperscript{20} This suggests that focusing on the relationship between the trade elasticity and welfare is not sufficient to estimate the gain from trade liberalization. Instead, one must also consider how the scale of the economy is changing.

7 Firm Creation and Trade: The Evidence

We now present some evidence supporting the substantial substitution between firm creation and trade predicted by the benchmark model. This substitution is key to understanding the different aggregate effects between dynamic and static trade models. When exporting is a static decision, there is no substitution between these two margins, but, in the dynamic models, this substitution is increasing in the gap between the short-run and the long-run trade response. For the United States, there is ample evidence of this substitution: As the number of manufacturing establishments per worker has fallen, trade has increased.

In Figure 8, we plot the number of establishments per working-age person against open-

\textsuperscript{20}Eliminating the adjustment cost in inputs would speed up the transition and increase the welfare gain in the no-cost model to 3.5 percent. The path of aggregate dynamics, however, would remain quite different from that in our benchmark model.
ness in the data and the model. Openness is measured as the ratio of trade to gross output from 1972 to 2015.\textsuperscript{21} Establishments are measured relative to the working-age population (16–64) with the level normalized to one in 1980. Over this 43-year period, openness increases from 7.6 percent to 31.9 percent, and the number of establishments per working-age person falls by 40 percent. The number of establishments falls nonlinearly with openness. The increase in trade from 7.5 percent to 22.5 percent is associated with a 12-percent decrease in the number of establishments, while the increase in trade from 22.5 to 32.5 percent coincides with the remaining 17.5 percent decrease in establishments.

The first 43 years of the transition in the benchmark model captures a similar negative and non-linear relationship between the number of establishments and openness over the range in which the model and data overlap. Recall that the model’s trade share increases from about 8.8 percent to 22.6 percent.

We also plot the same relationship in the model targeted to match new exporters’ export share (the \textit{starter model}, which we discuss in Section 9), which generates a larger increase in trade (26 percent versus 22.6 percent in the benchmark model). This model generates a larger drop in the number of establishments, particularly in the late stages of the transition, as the economy converges to a higher level of integration.

In our model, the decrease in trade barriers drives both a decline in the number of establishments and an increase in trade volumes. It seems plausible that the United States’ greater integration into the world economy has generated the same effects. A growing literature has studied the declining business startup rate (e.g., Decker, Haltiwanger, Jarmin, and Miranda 2014), and several potential explanations for the phenomenon have been explored. Notably, though, this literature has not carefully considered decreasing trade costs and, thus, this is an area that warrants further research.

\textsuperscript{21}Specifically, openness refers to exports of goods and services plus imports of good and services, divided by gross output of private goods-producing industries. Imports and exports are measured inclusive of tariffs.
8 Sources of the Gains from Trade

Why does the benchmark model generate a welfare gain that differs from the Krugman model, even though the trade elasticity in the two models is the same at every point in the transition? The short answer is that the scales of the two economies differ substantially, owing to the different incentives to invest in establishment creation.

In the static models considered in ACR, for example, decreasing trade barriers do not generate changes in the number of establishments created: There is no meaningful substitution between creating new establishments and creating exporters. In our benchmark model, however, a decrease in tariffs leads to fewer establishments but many more exporters. This is the source of the additional gains from trade. In Section 8.1, we discuss the analytic solution to a simple version of our benchmark model. The closed-form solutions allow us to observe the ways that exporter dynamics influence the number of firms in the economy, consumption, and trade.

Motivated by our analytic results, in Sections 8.2 and 8.3, we derive a decomposition of the gains from trade that generalizes the formula in ACR, making it easy to see how the benchmark model (and a simpler version) differs from those in the literature.

8.1 An Analytic Solution

To quantify the dynamic aggregate effects of a change in tariffs in the presence of realistic exporter dynamics, we have eschewed the analytical approach favored in the literature for a purely computational approach. Nonetheless, in the appendix, for a special case of our model that eliminates capital accumulation, the input-output structure, and persistent firm productivity dynamics, but maintains the assumption that new exporters face a relatively high marginal trade cost $\xi_H > \xi_L$ with symmetric transition probability, $\rho_\xi \geq 0.5$, we can solve for the long-run impact of a change in tariffs.

Compared to a model without exporter dynamics ($\xi_H = \xi_L$), we find that in the model with new exporter dynamics:

1. The stock of establishments increases more with tariffs, and more so as we increase the discount factor and disadvantage of new exporters ($\rho_\xi$ or $\xi_H/\xi_L$).
2. The trade elasticity is larger.

3. The steady-state change in consumption is smaller for a given level of trade growth.

Our numerical results show that the long-run decline in the stock of establishments is key to the overshooting observed along the transition. It primarily reflects entrants strongly discounting future foreign market access relative to increased foreign competition at home. We show this effect arises even when there is no discounting ($\beta = 1$). In our simple model, the steady state number of establishments equals

$$N = \left( \frac{\beta}{1 - n_s \beta} \right) \left( \frac{1}{\eta \theta} \right) \left( \frac{1}{f_E} \right) \left( \frac{S_0}{b_0 - b_1 S_0} \right),$$  \hspace{1cm} (50)

where the first term is related to discounting and the expected lifespan of an establishment, the second and third terms are related to the profit share and fixed cost, and the last term is related to the new exporter’s disadvantage. In particular, $b_0 = 1 - \left( \frac{1-\beta}{\eta \theta} \right) > 0$, and $b_1 = \frac{n_s \beta (1-\beta)}{(1-n_s \beta) \eta \theta}$, where $\eta$ is the slope parameter in a Pareto productivity distribution. Note that $b_1 > 0$ if $\beta \in (0, 1)$ and $b_1 = 0$ if $\beta = 1$. $S_0$ is a measure of the importance of new exporters in overall expenditures. When there are no new exporter dynamics ($\xi_H = \xi_L$) we find that $S_0 = 1$, while with new exporter dynamics ($\xi_H > \xi_L$) we find that $S_0 < 1$. The entrant’s disadvantage, $S_0$, is increasing in the tariff rate, so cutting tariffs reduces the number of establishments in the long-run, but frees resources for consumption along the transition. This effect is stronger the smaller is $\beta$.

The trade elasticity, $\varepsilon$, is complicated when there are new exporter dynamics. The elasticity is

$$\varepsilon = \frac{\hat{\Psi}_L - \hat{\Psi}_H}{\hat{\tau}},$$  \hspace{1cm} (51)

$$= \eta \theta - 1 + (\eta - 1) \left( 1 - \frac{\zeta_0}{\zeta_1} \right) \frac{\hat{a}_H}{\hat{\tau}}$$

$$- \left( 1 - \frac{\xi_H^{-\theta} \hat{\Psi}_H}{\Psi_X} \right) \left\{ \left( \frac{\hat{N}_L}{\hat{N}_{n_L}} \right) \frac{\hat{\tau}}{\tau} + \left[ \left( \frac{\xi_L}{\xi_H} \right)^{1-\theta} \left( \frac{\Psi_L}{\Psi_H} \right) - 1 \right] \left( \hat{\Psi}_L - \hat{\Psi}_H \right) \right\}.$$
Note that when \( \xi_H = \xi_L \), the elasticity becomes \( \varepsilon = \eta^\theta - 1 \). Since \( 0 < \zeta_0 < 1, \zeta_1 > 1 \), \( \hat{a}_H/\hat{\tau} > 0, \left( \frac{N_L}{N_L^H} \right)/\hat{\tau} < 0 \), and \( \left( \hat{\Psi}_L - \hat{\Psi}_H \right)/\hat{\tau} \leq 0 \), we have \( \varepsilon > \eta^\theta - 1 \). Thus, the model with new exporter dynamics has a greater trade elasticity compared to the model without them.

Turning to steady state consumption, note that consumption is proportional to wage growth scaled by labor income’s share of GDP,\(^{22}\)

\[
C \propto \frac{W}{\left( \frac{W}{C} \right)},
\]

and that each of these terms can be derived analytically. From the labor market clearing condition, labor’s share of income is

\[
\frac{W}{C} = \frac{b_0 - b_1 S_0}{S},
\]

where \( S \) is a measure of the tariff distortion: \( S > 1 \) for any \( \tau > 1 \), and \( S = 1 \) for \( \tau = 1 \) or \( \tau \to \infty \) (autarky). Therefore, trade liberalization (moving from \( \tau > 1 \) to \( \tau = 1 \)) raises labor’s share of GDP, \( W/C \). It also raises the entrant’s disadvantage, \( S_0^{-1} \), which further increases the labor share of GDP when \( \beta < 1 \).

We are also able to show that the wage is proportional to the usual domestic expenditure share, \( \lambda \), plus additional terms related to the tariff distortions, \( S \), and the relative size of entrants to incumbents, \( S_0 \),

\[
W \propto (b_0 - b_1 S_0)^{-\frac{1}{\theta - 1}} S^{\frac{\eta - 1}{\theta (\theta - 1)}} \left( \frac{S_0}{\lambda} \right)^{\frac{1}{\eta (\theta - 1)}}.
\]

Notice that the exponent on the domestic expenditure share is not the trade elasticity. This is because we are considering tariffs rather than trade costs. If we considered trade costs, the exponent would be the trade elasticity. Combining (53) and (54) yields

\[
C \propto (b_0 - b_1 S_0)^{-\frac{\theta}{\theta - 1}} S^{\frac{\eta - 1}{\theta (\theta - 1)}} \left( \frac{S_0}{\lambda} \right)^{\frac{1}{\eta (\theta - 1)}}.
\]

\(^{22}\)In this simple model there is no physical capital investment and trade is balanced, so expenditure-side GDP there is only final-goods consumption.
Now, given a common change in $\lambda$ across models, there will be some offsetting effects on $S_0$ in the new exporter model so that it will generate smaller long run gains.

### 8.2 A Model without Capital and Materials

We begin by deriving a decomposition of the gains from trade in a variation of the benchmark model that abstracts from capital and material inputs. In the next section, we add capital and material inputs.

The labor market-clearing condition implies that

$$\frac{P_t C_t}{S_t} = \frac{\theta}{\theta - 1} W_t L_{p,t}, \quad (56)$$

where $S_t = \frac{1 + \tau_t \zeta_t^{-1}}{1 + \zeta_t}$ is the distortion from a tariff ($\zeta_t = \tau_t \xi_t^{\theta - 1} \leq 1$). Notice that, if there is only iceberg trade costs (as is usually considered in the literature), $S = 1$. The change in consumption from a change in tariff policy is

$$\hat{C}_t = \hat{W}_t - \hat{P}_t + \hat{S}_t + \hat{L}_{p,t}, \quad (57)$$

where a "hat" variable is the change relative to the previous steady state. Normalizing the wage to one, the change in the price level is

$$\hat{P}_t = \frac{\hat{\lambda}_t - \hat{N}_t - \hat{\Psi}_{d,t}}{\theta - 1}, \quad (58)$$

where $\hat{\Psi}_d = \int_{a_d}^{\infty} a \phi_t(a) \, da$ is the average ability of domestic producers ($a$ is the elasticity-adjusted productivity of a producer, $\ln a = \rho(\theta - 1)$). The price level is increasing in the domestic expenditure share ($\lambda$) and decreasing in the number of establishments ($N$) and the average productivity of establishments ($\Psi_d$). These effects are stronger when the markup is larger. Combining (57) and (58) yields the change in consumption,

$$\hat{C}_t = \frac{\hat{N}_t + \hat{\Psi}_{d,t} - \hat{\lambda}_t}{\theta - 1} + \hat{S}_t + \hat{L}_{p,t}. \quad (59)$$

We begin by analyzing (59) for two of the standard trade models in the literature, deriving
the main result in ACR. Consider a static trade model, such as the Krugman (1980) model, without tariffs. A change in the iceberg cost, $\xi$, implies that $\hat{L}_p = \hat{S} = \hat{N} = \hat{\Psi}_d = 0$, so that

$$
(60) \quad \hat{C}^K_t = -\hat{\lambda}_t \frac{1}{\theta - 1} = -\frac{\hat{\lambda}_t}{\varepsilon^K},
$$

where $(\theta - 1)^{-1}$ is the markup and the inverse of the trade elasticity, $\varepsilon^K$.

Next, consider the typical presentation of the Melitz (2003) model with a fixed operating cost and a Pareto distribution of productivity with slope $\eta$, but without tariffs. In this model, a change in the iceberg cost, $\xi$, also implies that $\hat{L}_p = \hat{S} = \hat{N} = 0$, but $\hat{\Psi}_d = \frac{\eta - 1}{\eta} \hat{\lambda}$,

$$
(61) \quad \hat{C}^M_t = \left(\frac{\eta - 1}{\eta}\right) \frac{\hat{\lambda}_t - \hat{\lambda}_t}{\theta - 1} = -\frac{\hat{\lambda}_t}{\eta (\theta - 1)} = -\frac{\hat{\lambda}_t}{\varepsilon^M},
$$

where the trade elasticity is now $\varepsilon^M = \eta (\theta - 1)$. When the Krugman and Melitz models are disciplined to have the same trade elasticity $(\varepsilon^M = \varepsilon^K)$, a change in trade costs will yield the same gain from trade, as shown in ACR.

In our dynamic exporting model, however, $\hat{L}_p$, $\hat{S}$, and $\hat{N}$ generally do change in the new steady state. Since accumulating varieties is costly, when $\hat{N} \neq 0$, the transition will matter for steady-state consumption. Along the transition, the number of establishments affects the productivity distribution of establishments through the producer-level growth process, so $\hat{\Psi}_d$ is complicated. With risk-neutral consumers, we can express the welfare gain from a change in tariffs as

$$
(62) \quad \hat{\bar{C}} = \frac{\hat{N} + \hat{\Psi}_d - \hat{\lambda}}{\theta - 1} + \hat{L}_p + \hat{S},
$$

where the bars denote the discounted average of a variable,

$$
(63) \quad \hat{X}_t = (1 - \beta) \sum_{t=0}^{\infty} \beta^t \hat{X}_t.
$$

In Figure 9, we plot the dynamics of consumption and the contribution of each of the components in (59) and (62), following the elimination of a ten-percent tariff in the benchmark
model without capital and material inputs. Table 5 reports the discounted and steady-state change in each variable. Consumption overshoots and declines slightly in the long run. The overshooting of consumption is a result of several competing forces. Labor used in production and average productivity both increase temporarily. The variety effect \((N)\) gradually reduces consumption, while the growth in trade \((-\lambda)\) gradually boosts consumption. The lost tariff revenue \((S)\) reduces welfare.

In total, the decline in steady-state consumption of 0.17 percent is attributed to a 4.09-percent increase from trade that is offset by the reduction in varieties \((-2.89\text{ percent})\), labor used in production \((-0.56\text{ percent})\), and tariff revenue \((-0.81\text{ percent})\). The average productivity of producers is unchanged across the steady states.

Compared to the change in steady-state consumption, the welfare gain of 0.66 percent arises because, even though trade contributes less (3.50 percent versus 4.09 percent), along the transition, it grows faster than the stock of establishments shrinks. In addition, the labor used in production declines less and there is an increase in average productivity.

In Figure 9B, we plot the dynamics of utility in the simple model and the predicted change in utility from applying the ACR formula, adjusted for the change in the tariff, to the data generated by the model.\(^{23}\) We adjust for the change in the tariff because the ACR formula is derived for a change in iceberg costs and not tariffs. The ACR formula understates the gain over the first 16 years and then overstates the gain, as utility in the model quickly declines below the previous steady-state level. Over the first 20 years, the ACR formula predicts an average utility gain of 0.33 percent, and the mean absolute deviation of the predicted utility from the model utility is 0.54 percent.

### 8.3 A Model with Capital and Materials

Using the benchmark model, we can construct a similar decomposition that includes physical capital and material inputs in production. It is straightforward to show that output can be

\(^{23}\)To focus on utility, the change in consumption is scaled by the discount factor \(\beta^t\).
decomposed as

\begin{equation}
\hat{Y} = \hat{TVE} + \alpha \left( \hat{K} - \hat{L}_p \right) + \hat{L}_p + \left[ S - \alpha_x \left( \frac{\theta - 1}{\theta} \right) \right],
\end{equation}

where $TVE$ is the trade-variety-efficiency effect; $K$ is capital; $\alpha$ is capital’s share of value added; and $\alpha_x$ is the share of material inputs in gross output.

We report this decomposition in Table 6, in which we further decompose the trade-variety-efficiency effect into

\begin{equation}
\hat{TVE} = -\frac{\hat{\lambda}}{(\theta - 1)(1 - \alpha_x)} + \frac{\hat{N}}{(\theta - 1)(1 - \alpha_x)} + \frac{\hat{\Psi}_d}{(\theta - 1)(1 - \alpha_x)}.
\end{equation}

The change in steady-state output (0.52 percent) is driven by an increase in the trade-variety-efficiency effect (4.25 percent) and a decrease in production labor (–2.11 percent) and adjusted tariff revenue (–2.27 percent). The cut in tariffs generates capital deepening, which adds 0.65 percent to the new steady-state output level. In discounted terms, the increase in the trade-variety-efficiency effect dominates (8.64 percent) a smaller change in production labor (–1.08 percent) and greater capital deepening (1.31 percent). The tariff revenue term (–2.27) is the same as the steady-state change.

The bottom panel of Table 6 provides further detail on the trade-variety-efficiency effect. The general patterns are similar to those in the model without capital and material inputs. The trade-variety-efficiency effect is driven by a decrease in establishments offset by a larger change in trade. As in the previous decomposition, changes in the average productivity of establishments play a small role.

9 Sensitivity

In this section, we modify several features of our benchmark model and we discuss how these modifications change (or do not change) our findings. We show that the substitution between trade and establishments in the aggregate responses to trade liberalization is even stronger when we modify the model to more closely match the importance of new exporters.
in the economy.

### 9.1 Sensitivity to New Exporter Size

Our benchmark model does a good job of accounting for the behavior of new exporters that we observe in the data. The model has trouble, however, accounting for the relative importance of new exporters in aggregate trade: New exporters export too much. In our benchmark model, the average new exporter exports 65.5 percent as much as the average exporter, compared with 25 percent in the data. Even so, the benchmark model does much better than the sunk-cost model (138 percent) or the sunk-cost-high model (301 percent).

We consider three modifications to the export fixed- and variable-cost setup and study their implications for aggregate trade growth and welfare. In doing so, we further draw out the relationship between firm creation and trade. We find that making new exporters smaller generally leads to larger welfare gains and long-run trade elasticities.

Since our baseline calibration closely matches other aspects of producer growth and dynamics, we focus on changing the process for fixed and variable trade costs rather than on reestimating the whole model. In each model extension, we target the same aggregate export intensity, export participation, and exit rate from exporting. The first margin ensures that any comparison of aggregate effects is appropriate. The last two margins ensure that each model has the same share of new exporters in the steady state. We summarize the new parameters in Table 3 and the results in Table 4.

**Exporter Reentry**

Our first extension allows an exporter to exit the export market and reenter at a later date by paying a smaller fixed entry cost. Specifically, we allow an exporter the option to take a one-period break from the export market and return the following period by paying $f_R < f_H$.\(^{24}\) Consistent with the data (Table 2), recent exporters in the model are now more likely to export than a firm that last exported three periods ago. We set $f_R/f_L = 2/3$. This increases the option value in export entry, so, to make this model match the aggregate export data, we must increase the cost of exporting for first-time exporters. We find that $100 \times f_H/f_E = 5.9$, compared with 3.8 in the benchmark.

\(^{24}\)We present the modified model in the appendix.
In the reentry model, the average exports of new exporters relative to incumbent exporters is 63.2 percent—adding reentry does not shrink the aggregate importance of new exporters. Reentry allows for smaller reentrants, but the larger initial entry cost creates more selection and requires larger (more productive) initial entrants. In our parameterization, these two forces roughly cancel each other out. Adding reentry makes establishments and exporters greater substitutes, and the model’s long-run trade elasticity from a ten-percent tariff reduction rises from 11.55 to 12.2 (Table 4). The number of establishments falls by 14.4 percent, compared with 13.1 percent in the benchmark model, and long-run consumption increases by 0.70 percent, compared with 0.42 percent in the benchmark model. The welfare gain rises to 6.93 percent.

Search

In our second extension, we make the variable trade cost upon export entry stochastic. In the benchmark model, a new exporter always faced $\xi = \xi_H$. We now assume that, with probability $\eta$, the variable trade cost is $\xi = \xi_H$, and with probability $1 - \eta$, the variable trade cost is $\xi = \infty$. We interpret this setup as a simple model of searching for, and sometimes failing to find, an export market.

We set $\eta = 0.33$. To keep the exit decision unaffected, we assume that $f_R = f_H/\eta$. To maintain the same aggregate export intensity, we scale down the variable trade costs. We find that the average exports per new exporter are 53.2 percent of those of the average incumbent exporter, compared with 25 percent in the data and 65.5 percent in the benchmark model. It is possible to further reduce the probability of a match ($\eta$), which will reduce the importance of new exporters, but doing so makes it difficult for the model’s size distributions to match those in the data. Establishments and exporters are also better substitutes in this model. The trade elasticity rises to 12.8 and the number of establishments falls by 15.8 percent. Long-run consumption, however, grows by less than it does in the benchmark, rising by only 0.21 percent, but welfare increases by more, 7.00 percent.

New Exporter Intensity

Our last extension is an alternative calibration in which we directly target the exports of new exporters. We choose the gap between the high and low export costs so that the average exports per new exporter are 25.0 percent of the average incumbent exporter’s, as in the
This requires increasing the high variable trade cost to $\xi_H = 2.2$ and decreasing the low variable trade cost $\xi_L = 1.0$. This version of the model makes establishments and exporters the most substitutable. The trade elasticity now rises to 13.7 and the number of establishments falls by 17.7 percent. Long-run consumption rises 0.11 percent, but welfare rises the most, by 7.28 percent.

9.2 Sensitivity to Preference Parameters

Compared to static models, our dynamic model of the exporter life cycle generates non-trivial transitions and different long-run effects from trade reforms. In this section, we consider two other factors that influence post-liberalization transitions and the gains from trade: the intertemporal elasticity of substitution and the interest rate.

The intertemporal elasticity of substitution, $1/\sigma$, determines how agents value fluctuations in consumption over time. We have set this to $1/\sigma = 1/2$, but there is much disagreement about its value. When we raise this elasticity to $1/\sigma = 1$, we make consumption more volatile and the interest rate less volatile (Figure 11). Changing the intertemporal elasticity has no impact on the long-run effect of a change in tariffs but leads to a faster transition. This has almost no impact on the trade elasticity, but leads to a faster reduction in establishments and more overshooting in consumption and capital. The faster transition arises because fluctuations in consumption are less costly than in our benchmark case.

Next, we vary the real interest rate ($r = 1/\beta$) by considering two alternative values for $\beta = 0.95, 0.98$. To keep the capital-output ratio constant, we maintain the same capital share parameter, $\alpha$, and adjust the capital depreciation rate by the change in the discount factor.\(^{26}\) The export response, measured by the trade elasticity, is again largely unaffected by changing the interest rate. Raising the interest rate, however, leads firms to discount profits faster so that the benefits of a cut in tariffs are discounted more for new firms. This leads to a larger drop in firm entry in the new steady state. This large drop in entry makes the long-run change in consumption negative, with consumption falling 0.6 percent. The larger

\(^{25}\) An alternate approach is to make the low variable trade cost an absorbing state, which generates similar results.

\(^{26}\) In the neoclassical growth model $K/Y = \frac{\alpha}{(1-\beta)+\sigma}$. In our model, the profit share is affected by discounting, too.
discount factor implies that the welfare gain is less than in our benchmark (6.1 percent versus 6.3 percent). Lowering the interest rate increases the benefits in the long-run: steady state consumption now rises by 3.2 percent. It also yields larger consumption growth in the early transition, with consumption growth peaking at 12 percent in year seven compared with 10.5 percent in the benchmark model. The 6.8 percent increase in welfare, however, is not much larger than that in our benchmark model. The small difference in welfare across these alternative calibrations with vastly different long-run effects makes clear that understanding the early periods of reforms—rather than the long-run effects—is much more important for judging the benefits of reform.²⁷

### 9.3 Further Sensitivity

Our findings are robust to introducing a per-period operating cost as found in traditional firm-dynamics models in Hopenhayn (1992) or Melitz (2003). With a fixed operating cost, the extra substitution between active and inactive firms actually increases the incentive to create a plant and, thus, the firm creation rate falls even more than in our benchmark model. Finally, our findings are also robust to introducing an endogenous labor supply decision. With endogenous labor supply, the long-run effects of trade liberalization will depend on whether income or substitution effects dominate and how the lost tariff revenue is replaced. For standard balanced-growth preferences and lump sum taxes, we continue to find substantial overshooting.

### 10 Unilateral Trade Liberalization

We next consider the effect of an unanticipated unilateral cut in the home tariff. The model is well-suited to this experiment since it allows for international borrowing and lending, capital accumulation, and different short-run and long-run trade elasticities. As in the global tariff reduction, the steady-state change in consumption does not yield a good approximation.

²⁷It may seem odd that the low interest rate economy generates a welfare gain similar to that in our benchmark economy even though there is more overshooting and substantially larger long-run consumption growth. Recall, however, that periods are not valued equally when we change β, so the overshooting period is a much smaller share of lifetime utility when β is high.
of the welfare gain. In particular, for the reforming country, the steady-state change in consumption predicts a sizeable loss, even though welfare actually increases.

In Figure 12, we plot the dynamics of some key variables, and Table 7 reports the welfare gain and change in steady-state consumption. We solve the model under three alternative financial market assumptions: trade in a non-contingent bond, financial autarky, and complete markets. The latter two cases illustrate the role of financial markets and the effects of the wealth transfer from the loss of tariff revenue. We also include the results for the no-cost model with a non-contingent bond to clarify how the structure of trade costs and the source of slow trade adjustment influence the welfare gain from liberalization and the pattern of international borrowing and lending.

When countries trade only a non-contingent bond, the home country’s welfare rises by 0.51 percent and the foreign country’s rises by 5.7 percent. Home steady-state consumption falls by 2.43 percent and foreign steady-state consumption rises by 2.82 percent. As in the global reform, there is substantial overshooting of consumption, so the change in steady-state consumption does not provide a good approximation of the welfare gain. This overshooting arises, in large part, from a strong reduction in the investment in new establishments. In the new steady state, the number of home establishments falls by 6.5 percent and foreign establishments falls by 5.91 percent. The strong decline in establishments in both countries arises because, as the thresholds for exporting decline substantially in both countries, local producers face more competition from importers.

In Figure 12, we can see that, along the transition, the home country runs a trade surplus in the first 11 years following the liberalization. The surplus peaks in year two at 0.72 percent of GDP. The home country accumulates net external assets equal to 6.9 percent of GDP. Its real exchange rate depreciates by 5.4 percent initially and then appreciates slightly for a total depreciation of 4.5 percent. This depreciation, and the large increase in foreign income, leads to a stronger expansion of exporting among the home producers.

In the model without trade in financial assets, the home country’s welfare increases a bit more than in the bond economy (0.55 versus 0.51), and the foreign country’s welfare increases a bit less (5.66 versus 5.70). While there is a minor effect on welfare, the differences in steady-state consumption growth are larger, as home steady-state consumption now drops
more (−2.85 versus −2.43) and foreign consumption rises more (3.22 versus 2.82). These long-run differences largely reflect the accumulation of assets by the home country in the bond economy.

When we model countries that trade a complete set of contingent claims, the wealth effect from the loss in tariff revenue is eliminated. In contrast to the bond economy and financial autarky, the home country is now the main beneficiary of the reform, as its welfare increases by 4.34 percent, while the foreign country’s welfare increases by only 1.91 percent (Table 7). The trade balance is also significantly different with complete markets. The home country runs a trade deficit of 2.3 percent of GDP in year one, which gradually expands to 2.8 percent of GDP in the steady state.

Lastly, we consider unilateral tariff reform in the no-cost model with slow trade adjustment, from Section 6.2. As with the global reform, we find that welfare and consumption paths in the no-cost model are quite different from those in the benchmark model. Most striking is the implication for welfare: In the no-cost one-bond model, home country welfare decreases by 0.62 percent, compared to the 0.51 percent increase in the benchmark one-bond model (Table 7). In the steady state, home consumption falls only 0.06 percent, compared to a decline of 2.43 percent in the benchmark model. The last panel of Figure 12 shows that borrowing and lending are qualitatively similar to the benchmark model, in that the home country initially runs trade surpluses, but the fluctuations in the trade balance are muted. The home country accumulates assets of 6.9 percent of GDP in the benchmark model, compared to 2.6 percent in the no-cost model. Without slow trade adjustment, net foreign assets would remain equal to zero. Thus, the source of gradual trade adjustment matters in determining the pattern of international borrowing.

11 Conclusions

What conclusions emerge from our analysis?

First, the relationship between trade and the benefits of trade, measured by welfare, depends strongly on how exporting substitutes for firm creation. Somewhat paradoxically, this substitution and, hence, the welfare gain, is stronger when marginal exporters are less
important. When marginal exporters are small, more investment is undertaken in increasing the exports of incumbent exporters, unlike in the canonical sunk-cost model. For the United States, we find evidence that this substitution is even stronger than predicted by our model.

Second, the long-run effects of a change in trade policy depend on the dynamic incentives to export. They cannot be recovered from a static trade model or from the formula proposed by ACR. Moreover, because aggregate trade grows slowly, these long-run effects are strongly discounted and, thus, are not the key determinants of the welfare gains from a change in trade policy.

Third, the latter stages of trade integration, as trade converges to its new steady state, can be characterized by falling incomes. Whether this has contributed to the trade backlash and slow growth in the post Great Recession (and Great Trade Liberalization) world remains an open question. Indeed, it would be useful to apply our theory to understand the impact of the massive trade liberalization that has expanded U.S. manufacturing openness from seven percent to 35 percent over the last 40 years. There are, of course, challenges to this approach—in particular, specifying the timing of and expectations regarding changes in trade policy, a challenge that does not arise in static trade models (Alessandria and Mix 2017).

Our theoretical and empirical analysis suggests several challenges to the existing empirical approach and directions for future research.

First, the non-linear, and non-monotonic, relationship between trade and growth suggests revisiting empirical work on the effects of trade and growth. Second, given that the benefits of trade are front-loaded, while trade is back-loaded, it would be useful to consider how different generations gain or lose from reform, as well as the incentives to initiate and maintain these trade reforms. Third, our analysis suggests that a better understanding of the substitution between firm creation and export capacity is paramount. To highlight this mechanism, we have assumed a simple process by which firms become better exporters. Continuing to integrate recent work on the growth patterns of exporters into general equilibrium models may yield further insights. Fourth, our quantitative theory can be used to formulate and evaluate alternative formulas to approximate the gains from policy reform in a world with different short-run and long-run trade elasticities. It can also be used to clarify how forward-looking variables, such as asset prices, can be useful in identifying the impact of changes in
trade policy. Preliminary work along these lines is promising (Alessandria, Choi, and Ruhl 2018).

References


## Table 1: New Exporter Importance and Growth

<table>
<thead>
<tr>
<th></th>
<th>Participation</th>
<th>Export Share</th>
<th>Starter Size Discount</th>
<th>Intensity Dynamics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>8-year</td>
<td>1-year</td>
<td>8-year</td>
<td>1-year</td>
</tr>
<tr>
<td>Chile (98–06)</td>
<td>56.7</td>
<td>11.8</td>
<td>39.2</td>
<td>3.5</td>
</tr>
<tr>
<td>Colombia (81–89)</td>
<td>57.2</td>
<td>17.6</td>
<td>38.4</td>
<td>4.2</td>
</tr>
<tr>
<td>Balanced Panels</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chile (98–06)</td>
<td>27.4</td>
<td>11.8</td>
<td>9.2</td>
<td>4.4</td>
</tr>
<tr>
<td>Colombia (81–89)</td>
<td>34.7</td>
<td>13.8</td>
<td>14.5</td>
<td>3.1</td>
</tr>
<tr>
<td>U.S. Compustat (84–92)</td>
<td>28.2</td>
<td>4.1</td>
<td>11.0</td>
<td>1.1</td>
</tr>
<tr>
<td>United States (84–92)</td>
<td>42.0</td>
<td>14.0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: Columns 1 and 2 report the fraction of exporting producers that entered during either the eight-year sample period or in an average year. Columns 3 and 4 report the share of total exports accounted for by producers that entered during either the eight-year sample period or in an average year. Columns 5 and 6 report the average sales of a new exporter relative to the average sales of incumbent exporters. Column 7 reports the export-intensity autocorrelation from (1). T, in column 8, is the number of years an average exporter needs stay in the market to reach the average export intensity in the sample.
<table>
<thead>
<tr>
<th></th>
<th>Incumbent (1)</th>
<th>Starter (2)</th>
<th>Re-entrant (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chile (98–06)</td>
<td>0.81</td>
<td>0.65</td>
<td>0.26</td>
</tr>
<tr>
<td>Colombia (81–89)</td>
<td>0.90</td>
<td>0.66</td>
<td>0.30</td>
</tr>
</tbody>
</table>

**Balanced Panels**

<table>
<thead>
<tr>
<th></th>
<th>Incumbent (1)</th>
<th>Starter (2)</th>
<th>Re-entrant (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chile (98–06)</td>
<td>0.83</td>
<td>0.66</td>
<td>0.29</td>
</tr>
<tr>
<td>Colombia (81–89)</td>
<td>0.90</td>
<td>0.68</td>
<td>0.35</td>
</tr>
<tr>
<td>U.S. Compustat (84–92)</td>
<td>0.93</td>
<td>0.83</td>
<td>0.03</td>
</tr>
<tr>
<td>United States (84–92)</td>
<td>0.66</td>
<td></td>
<td>0.27</td>
</tr>
</tbody>
</table>
Table 3: Model Parameters

<table>
<thead>
<tr>
<th>Common Parameters</th>
<th>( \beta )</th>
<th>( \sigma )</th>
<th>( \delta )</th>
<th>( \tau )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>0.96</td>
<td>2.0</td>
<td>0.10</td>
<td>0.10</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Model-specific Parameters</th>
<th>Benchmark Cost</th>
<th>Sunk Cost</th>
<th>Sunk-cost High</th>
<th>No Cost</th>
<th>Reentry Cost</th>
<th>Search Cost</th>
<th>Starters</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta )</td>
<td>5.0</td>
<td>5.0</td>
<td>5.0</td>
<td>12.54</td>
<td>5.0</td>
<td>5.0</td>
<td>5.0</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.132</td>
<td>0.132</td>
<td>0.132</td>
<td>0.276</td>
<td>0.132</td>
<td>0.132</td>
<td>0.132</td>
</tr>
<tr>
<td>( \alpha_x )</td>
<td>0.810</td>
<td>0.810</td>
<td>0.810</td>
<td>0.704</td>
<td>0.810</td>
<td>0.810</td>
<td>0.810</td>
</tr>
<tr>
<td>( 100 \times \gamma_1 )</td>
<td>2.26</td>
<td>2.26</td>
<td>2.26</td>
<td>2.26</td>
<td>2.26</td>
<td>2.26</td>
<td>2.26</td>
</tr>
<tr>
<td>( \rho_z )</td>
<td>0.654</td>
<td>0.654</td>
<td>0.654</td>
<td>0.654</td>
<td>0.654</td>
<td>0.654</td>
<td>0.654</td>
</tr>
<tr>
<td>( \sigma_z )</td>
<td>1.32</td>
<td>1.32</td>
<td>1.32</td>
<td>1.32</td>
<td>1.32</td>
<td>1.32</td>
<td>1.32</td>
</tr>
<tr>
<td>( \mu_E )</td>
<td>-1.34</td>
<td>-1.34</td>
<td>-1.34</td>
<td>-1.34</td>
<td>-1.34</td>
<td>-1.34</td>
<td>-1.34</td>
</tr>
<tr>
<td>( \theta f_E )</td>
<td>32.7</td>
<td>33.2</td>
<td>32.7</td>
<td>34.7</td>
<td>32.6</td>
<td>30.7</td>
<td>32.5</td>
</tr>
<tr>
<td>( 100 \times f_H/f_E )</td>
<td>3.76</td>
<td>5.79</td>
<td>18.3</td>
<td>0.0</td>
<td>5.9</td>
<td>1.12</td>
<td>2.76</td>
</tr>
<tr>
<td>( f_H/f_L )</td>
<td>1.40</td>
<td>3.81</td>
<td>11.1</td>
<td>—</td>
<td>2.29</td>
<td>0.38</td>
<td>0.89</td>
</tr>
<tr>
<td>( \xi_H )</td>
<td>1.72</td>
<td>1.43</td>
<td>1.34</td>
<td>1.11</td>
<td>1.71</td>
<td>1.67</td>
<td>2.20</td>
</tr>
<tr>
<td>( \xi_L )</td>
<td>1.07</td>
<td>1.43</td>
<td>1.34</td>
<td>1.11</td>
<td>1.06</td>
<td>1.04</td>
<td>1.00</td>
</tr>
<tr>
<td>( \rho_\xi )</td>
<td>0.916</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>0.916</td>
<td>0.916</td>
<td>0.916</td>
</tr>
<tr>
<td>( \eta )</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>0.333</td>
<td>1.0</td>
</tr>
<tr>
<td>( f_R/f_L )</td>
<td>1.40</td>
<td>3.81</td>
<td>11.1</td>
<td>—</td>
<td>0.667</td>
<td>1.14</td>
<td>0.89</td>
</tr>
</tbody>
</table>

Overall Fit (RMSE): Size distributions

| Estab. + Empl. | 0.70 | 0.70 | 0.78 | 1.29 | 0.70 | 0.70 | 0.70 |
| Export | 14.6 | 15.7 | 3.8 | 49.6 | 11.9 | 4.5 | 14.4 |

Fixed Trade Costs Relative to

| Plant Creation Cost | 10.8 | 8.7 | 10.8 | 0.0 | 12.2 | 11.1 | 11.5 |
| Export Profits | 58.1 | 47.6 | 58.1 | 0.0 | 71.5 | 65.1 | 68.7 |

Selected Moments (Data)

| Exit rate (17.0) | 17.0 | 17.0 | 3.90 | — | 17.0 | 17.0 | 17.0 |
| Starter ratio (25.1) | 65.5 | 138.0 | 301.0 | — | 63.2 | 52.7 | 25.0 |
| Starter export share (4.9) | 12.8 | 19.2 | 23.1 | — | 12.4 | 9.7 | 4.9 |
| 5-yr incumbent share | 49.0 | 30.0 | 43.0 | — | 52.7 | 52.9 | 58.7 |
Table 4: Effect of Eliminating a ten-percent Tariff

<table>
<thead>
<tr>
<th></th>
<th>Bench</th>
<th>Sunk-cost</th>
<th>Sunk-cost High</th>
<th>No-cost</th>
<th>Reentry</th>
<th>Search</th>
<th>Starters</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Change in</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Welfare</td>
<td>6.35</td>
<td>4.75</td>
<td>5.67</td>
<td>2.34</td>
<td>6.93</td>
<td>7.00</td>
<td>7.28</td>
</tr>
<tr>
<td>SS Consumption</td>
<td>0.42</td>
<td>1.98</td>
<td>1.65</td>
<td>3.93</td>
<td>0.70</td>
<td>0.21</td>
<td>0.11</td>
</tr>
<tr>
<td>SS Establishments</td>
<td>−13.1</td>
<td>−4.80</td>
<td>−7.20</td>
<td>0.0</td>
<td>−14.4</td>
<td>−15.8</td>
<td>−17.7</td>
</tr>
<tr>
<td><strong>Trade elasticity</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Discounted</td>
<td>10.15</td>
<td>6.90</td>
<td>8.68</td>
<td>10.15</td>
<td>10.75</td>
<td>10.96</td>
<td>11.87</td>
</tr>
<tr>
<td>Steady state</td>
<td>11.55</td>
<td>7.19</td>
<td>9.44</td>
<td>11.55</td>
<td>12.23</td>
<td>12.77</td>
<td>13.7</td>
</tr>
</tbody>
</table>

Notes: Welfare change is a value of $x$ that satisfies $\sum_{t=0}^{\infty} \beta^t U (C_{-1}e^x) = \sum_{t=0}^{\infty} \beta^t U (C_t)$, where $C_{-1}$ is the consumption level in the initial steady state. The discounted trade elasticity is $\bar{\varepsilon} = (1 - \beta) \sum_{t=0}^{\infty} \beta^t \varepsilon_t$, where $\varepsilon_t$ is the trade elasticity based on the difference in trade between period $t$ and the initial steady state. The long-run trade elasticity is $\lim_{t \to \infty} \varepsilon_t$.

Table 5: Consumption Decomposition after Eliminating a 10-percent Tariff

<table>
<thead>
<tr>
<th></th>
<th>$\hat{C}$</th>
<th>$\hat{N}$</th>
<th>$\hat{\Psi}_d$</th>
<th>$\hat{\lambda}$</th>
<th>$\hat{L}_p$</th>
<th>$\hat{S}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discounted</td>
<td>0.66</td>
<td>−2.13</td>
<td>0.19</td>
<td>3.50</td>
<td>−0.09</td>
<td>−0.81</td>
</tr>
<tr>
<td>Steady-state</td>
<td>−0.17</td>
<td>−2.89</td>
<td>0</td>
<td>4.09</td>
<td>−0.56</td>
<td>−0.81</td>
</tr>
</tbody>
</table>

Notes: The discounted change in a variable is the difference between $\bar{x} = (1 - \beta) \sum_{t=0}^{\infty} \beta^t x_t$ and the initial steady state.

Table 6: Output Decomposition after Eliminating a 10-percent Tariff

<table>
<thead>
<tr>
<th></th>
<th>$(\hat{C})$</th>
<th>$\hat{Y}$</th>
<th>$\hat{TVE}$</th>
<th>$\alpha(\hat{K} - \hat{L})$</th>
<th>$\hat{L}_p$</th>
<th>$S - \frac{\alpha_x(\theta-1)}{\theta}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discounted</td>
<td>(6.35)</td>
<td>6.59</td>
<td>8.64</td>
<td>1.31</td>
<td>−1.08</td>
<td>−2.27</td>
</tr>
<tr>
<td>Steady-state</td>
<td>(0.42)</td>
<td>0.52</td>
<td>4.25</td>
<td>0.65</td>
<td>−2.11</td>
<td>−2.27</td>
</tr>
</tbody>
</table>

Decomposing the trade variety effect

<table>
<thead>
<tr>
<th></th>
<th>$\hat{TVE}$</th>
<th>$\hat{N}$</th>
<th>$\hat{\Psi}_d$</th>
<th>$\hat{\lambda}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discounted</td>
<td>8.64</td>
<td>−10.40</td>
<td>0.92</td>
<td>18.12</td>
</tr>
<tr>
<td>Steady-state</td>
<td>4.25</td>
<td>−17.26</td>
<td>0.00</td>
<td>21.51</td>
</tr>
</tbody>
</table>

Notes: The discounted change in a variable is the difference between $\bar{x} = (1 - \beta) \sum_{t=0}^{\infty} \beta^t x_t$ and the initial steady state.
Table 7: Effect of Unilaterally Eliminating a 10-percent Tariff

<table>
<thead>
<tr>
<th>Change</th>
<th>Benchmark</th>
<th>No-cost</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bond</td>
<td>Fin. Autarky</td>
</tr>
<tr>
<td>Welfare</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Home</td>
<td>0.51</td>
<td>0.55</td>
</tr>
<tr>
<td>Foreign</td>
<td>5.70</td>
<td>5.66</td>
</tr>
<tr>
<td>SS Consumption</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Home</td>
<td>-2.43</td>
<td>-2.85</td>
</tr>
<tr>
<td>Foreign</td>
<td>2.82</td>
<td>3.22</td>
</tr>
<tr>
<td>SS Establishments</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Home</td>
<td>-6.60</td>
<td>-6.70</td>
</tr>
<tr>
<td>Foreign</td>
<td>-5.9</td>
<td>-5.9</td>
</tr>
</tbody>
</table>

Notes: Welfare gain is a value of $x$ that satisfies $\sum_{t=0}^{\infty} \beta^t U (C_{t-1} e^x) = \sum_{t=0}^{\infty} \beta^t U (C_t)$, where $C_{t-1}$ is the consumption level in the initial steady state. The discounted trade elasticity is $\bar{\varepsilon} = (1 - \beta) \sum_{t=0}^{\infty} \beta^t \bar{\varepsilon}_t$, where $\bar{\varepsilon}_t$ is the trade elasticity based on the difference in trade between period $t$ and the initial steady state. The long-run trade elasticity is $\lim_{t \to \infty} \bar{\varepsilon}_t$. 
13 Figures

Figure 1: New exporter dynamics in Colombia

A. Trade Intensity

B. Survival Rates
Figure 2: New exporter dynamics in stationary steady state

A. Export Intensity

B. Survival Rate (Annual)

C. Marginal Starter Net Profits

D. Cumulative Net Profits
Figure 3: Elimination of 10-percent tariff in benchmark model

A. Trade Elasticity

B. Consumption, Wage, Output, and Production Labor

C. Investment (I), Entry (Ne), and Establishments (N)
Figure 4: Effect of entry adjustment on trade and consumption

A. Trade Elasticity

B. Consumption
Figure 5: Trade elasticity, several models
Figure 6: Comparing consumption, wage, and output dynamics

A. Consumption

B. Wage

C. Output
Figure 7: Comparing investment, entry, and labor dynamics

A. Investment

B. Entry

C. Labor in Production

- Benchmark
- Sunk
- No Cost
Figure 8: Establishment creation and trade
Figure 9: Decomposition in simple model

A. Consumption Decomposition

B. Utility ($\beta^c_t$)
Figure 10: Decomposition in the benchmark model

A. Output Decomposition

B. Trade-Variety-Efficiency
Figure 11: Sensitivity to interest rate and intertemporal elasticity

A. Consumption (C)

- Benchmark
- Log preferences
- 5% interest rate
- 2% interest rate

B. Capital (K)

C. Establishments (N)

D. Trade Elasticity
Figure 12: Transition following unilateral liberalization

- C
- C^*
- N_c
- N_c^*
- Real Exchange Rate
- Net Exports/GDP

Legend: Blue (Bond), Black (Complete), Dotted (Fin. Autarky), Red (No-cost)
The General Model of Exporting

The value of a producer that has never exported (as opposed to a firm that has exported and is idle) is

\begin{equation}
V_t^N(z, \infty, f_H) = \Pi_t(z, \infty, f_H) + \max \left\{ V_t^{N,0}(z, \xi, f), V_t^{N,1}(z, \xi, f) \right\},
\end{equation}

where \( V_t^{N,0} \) is the value of continuing as a nonexporter,

\begin{equation}
V_t^{N,0}(z, \infty, f_H) = n_s(z) Q_{t+1} \int_{z'} V_{t+1}^{N}(z', \infty, f_H) \varphi(z'|z) dz',
\end{equation}

and \( V_t^{N,1} \) is the value of entering the export market in the next period,

\begin{equation}
V_t^{N,1}(z, \infty, f_H) = -W_t f_H + \eta n_s(z) Q_{t+1} \int_{z'} \sum_{\xi' \in \{\xi_L, \xi_H\}} V_{t+1}^{X}(z', \xi', f_L) \rho(\xi'|\xi) \varphi(z'|z) + (1 - \eta) n_s(z) Q_{t+1} \int_{z'} V_{t+1}^{N}(z', \infty, f_H) \varphi(z'|z).
\end{equation}

Notice that, compared to the benchmark case, there is now a possibility that the producer is unsuccessful after paying \( f_H \). With probability \( 1 - \eta \), the producer remains a nonexporter, with variable trade cost \( \infty \) and export entry cost \( f_H \). The value of an exporting producer is

\begin{equation}
V_t^X(z, \xi, f) = \Pi_t(z, \xi, f) + \max \left\{ V_t^{X,1}(z, \xi, f_L), V_t^{X,0}(z, \xi, f_R) \right\},
\end{equation}

where the value of the firm, if it pays \( f \), and continues to export is

\begin{equation}
V_t^{X,1}(z, \xi, f) = -W_t f + n_s (z, \xi, f) Q_{t+1} \int_{z'} \sum_{\xi' \in \{\xi_L, \xi_H\}} V_{t+1}^{X}(z', \xi', f_L) \rho(\xi'|\xi) \varphi(z'|z).
\end{equation}

The decision of an idle exporter is

\begin{equation}
V_t^{X,0}(z, \infty, f_R) = \Pi_t(z, \infty, f_H) + \max \left\{ -W_t f_R + EV_{t+1}^{X}(z', \xi_H, f_L), EV_{t+1}^{N}(z', \infty, f_H) \right\}.
\end{equation}
Analytical Solutions to a Model with New-Exporter Dynamics [Not for publication]

In this appendix, we analytically examine the role of new-exporter dynamics in the long-run distortions from tariffs. We show that with new-exporter dynamics that the following are true:

1. The trade elasticity is higher.

2. The steady-state change in consumption is smaller for a given trade growth.

3. The stock of establishments increases with tariffs, and more so as we increase the discount factor and disadvantage of new-exporters.

To make these points most clearly, we keep the main elements related to new-exporter dynamics and eliminate other elements. Specifically, as in the benchmark model, new exporters have a disadvantage in that they face a relatively high marginal trade cost $\xi_H > \xi_L$ when they start exporting. We eliminate capital accumulation, input-output structure, and general transition probability for iceberg costs. Notes with these extensions are available from the authors.

We make the following modification from the benchmark model to obtain the analytical solutions. First, each period producers draw their elasticity-adjusted productivity $a = e^{(\theta-1)z}$ from a Pareto distribution with $a > 1$ and the slope parameter of $\eta > 1$.

The pdf of the distribution is given by $\phi(a) = \eta a^{-(\eta+1)}$. Second, the exogenous shutdown probability is constant with $n_d = 1 - n_s$. Third, producers face the fixed costs in exporting $f_X = f_L = f_H$ measured in labor units. See Alessandria and Choi (2014) for results when there is a sunk export cost ($f_H \geq f_L$). The payment should be made when they export. Fourth, there are fixed costs in production $f_P$ measured in labor units. If a producer does not pay the fixed cost in production, it becomes dormant in the current period. For notational simplicity, we replace $1 + \xi$ with $\xi$, and $1 + \tau$ with $\tau$. As in the benchmark model, we assume $\rho = \rho_\xi (\xi_H | \xi_H) =$

---

28With the elasticity adjusted productivity $a$, the relative size of a producer is proportional to $a$. 

68
For notational convenience, we focus on the symmetric steady-state. We will skip the agent’s problems as they are identical to the benchmark model.

### B.1 Consumers

The first order conditions from the consumer’s problem in the steady-state give the budget constraint,

\[(70) \quad C = WL + \Pi + T\]

### B.2 Final Good Producers

The final good producers’ problem yields the demand for goods,

\[(71) \quad y_H^d (a, \xi) = \left[ P_{H,t} (a, \xi) \right]^{-\theta} C, \]

\[(72) \quad y_F^d (a, \xi) = \left[ \tau P_F (a, \xi) \right]^{-\theta} C, \]

and the normalized final good’s price index, \( P = 1, \)

\[(73) \quad 1 = \sum_{\xi \in \{\xi_L, \xi_H, \infty\}} \int \left[ P_{H,t} (a, \xi) \right]^{1-\theta} \varphi_{H,t} (a, \xi) + \left[ \tau P_{F,t} (a, \xi) \right]^{1-\theta} \varphi_{F,t} (a, \xi) \right] da, \]

Note that we replace the productivity \( z \) with its level \( a. \)

### B.3 Intermediate Good Producers

The first order conditions give the pricing rule

\[(74) \quad P_H (a, \xi) = \left( \frac{\theta W}{\theta - 1} \right) a^{\frac{1}{\theta-\sigma}} \]

\[(75) \quad P_H^* (a, \xi) = \xi \left( \frac{\theta W}{\theta - 1} \right) a^{\frac{1}{\theta-\sigma}}, \]

---

\(^{29}\)We assume that \( \rho_\xi (\xi_L|\xi_L) > \rho_\xi (\xi_L|\xi_H) \) by setting \( \rho > 1/2. \)
and the demands for inputs

\[ l(z, \xi) = a^{\frac{1}{1-\theta}} y(a, \xi). \]

The total output of a producer with \((a, \xi)\) is given by

\[ y(a, \xi) = y^d_H(a, \xi) + \xi y^d_H(a, \xi) = \left(\frac{\theta W}{\theta - 1}\right)^{-\theta} (1 + \tau^{-\theta} \xi^{1-\theta}) a^{\frac{\theta}{\theta - 1}} C. \tag{77} \]

Using (77), we can rewrite the demands for inputs as

\[ l(a, \xi) = (1 + \tau^{-\theta} \xi^{1-\theta}) \left(\frac{\theta W}{\theta - 1}\right)^{-\theta} aC. \tag{78} \]

The operating profit of a producer with \((a, \xi)\) is given by

\[ \Pi(a, \xi) = P_H(a, \xi) y_H(a, \xi) + P^*_H(a, \xi) y^*_H(a, \xi) - Wl(a, \xi) \]
\[ = P_H(a, \xi) y(a, \xi) - \left(\frac{\theta - 1}{\theta}\right) (1 + \tau^{-\theta} \xi^{1-\theta}) \left(\frac{\theta W}{\theta - 1}\right)^{1-\theta} aC \]
\[ = \left(\frac{1}{\theta}\right) (1 + \tau^{-\theta} \xi^{1-\theta}) \left(\frac{\theta W}{\theta - 1}\right)^{1-\theta} aC. \tag{79} \]

**Marginal Productivity:** Let \(\Pi_0 = \left(\frac{1}{\theta}\right) \left(\frac{\theta W}{\theta - 1}\right)^{1-\theta} C\), the operating profit of a non-exporter with \(a = 1\). The marginal active producer’s productivity \(a_P\) satisfies

\[ Wf_P = \Pi(a_P, \infty) = \Pi_0 a_P. \tag{80} \]

Here, we assume that \(f_P\) is sufficiently high so that \(a_P > 1\), and \(f_X\) is relatively high so that the marginal active producer with \(a_P\) does not export. The value of a active producer with \((a, \xi)\) is given by

\[ v(a, \xi) = \Pi_0 (1 + \tau^{-\theta} \xi^{1-\theta}) a - Wf_P - Wf + n_s \beta EV(\xi), \tag{81} \]
where \( EV (\xi) \) is the expected value of the producer with \( \xi \) last period, and \( f \) is the optimal choice of the producer with \( \xi \). Specifically,

\[
(82) \quad EV (\infty) = \int_1^{a_H} v(a, \infty) \phi(a) \, da + \int_{a_H}^{\infty} v(a, \xi_H) \phi(a) \, da,
\]
\[
(83) \quad EV (\xi_H) = \rho \left[ \int_1^{a_H} v(a, \infty) \phi(a) \, da + \int_{a_H}^{\infty} v(a, \xi_H) \phi(a) \, da \right]
+ (1 - \rho) \left[ \int_1^{a_L} v(a, \infty) \phi(a) \, da + \int_{a_L}^{\infty} v(a, \xi_L) \phi(a) \, da \right],
\]
\[
(84) \quad EV (\xi_L) = (1 - \rho) \left[ \int_1^{a_H} v(a, \infty) \phi(a) \, da + \int_{a_H}^{\infty} v(a, \xi_H) \phi(a) \, da \right]
+ \rho \left[ \int_1^{a_L} v(a, \infty) \phi(a) \, da + \int_{a_L}^{\infty} v(a, \xi_L) \phi(a) \, da \right],
\]

where the marginal exporters’ productivity \( a_H \) and \( a_L \) satisfy

\[
(85) \quad v(a_H, \xi_H) = v(a_H, \infty),
\]
\[
(86) \quad v(a_L, \xi_L) = v(a_L, \infty).
\]

Using (81) we can rewrite the conditions as

\[
(87) \quad Wf_X = \Pi_0 \tau^{-\theta} \xi_H^{1-\theta} a_H + n_s \beta [EV (\xi_H) - EV (\infty)],
\]
\[
(88) \quad Wf_X = \Pi_0 \tau^{-\theta} \xi_L^{1-\theta} a_L + n_s \beta [EV (\xi_L) - EV (\infty)].
\]

Let

\[
(89) \quad \Psi_j = \int_{a_j}^{\infty} a \phi(a) \, da = \left( \frac{\eta}{\eta - 1} \right) a_j^{1-\eta},
\]
\[
(90) \quad n_j = \int_{a_j}^{\infty} \phi(a) \, da = a_j^{-\eta},
\]
where \( j \in \{P, H, L\} \). Then, we can rewrite the expected value of a non-exporter as

\[
EV(\infty) = \Pi_0 (\Psi_P + \tau^{-\theta} \xi^1_H \Psi_H) - n_P W f_P - n_H W f_X \\
+ n_s \beta [n_H EV(\xi_H) + (1 - n_H) EV(\infty)] \\
= \left( \frac{\Pi_0}{\eta} \right) (\Psi_P + \tau^{-\theta} \xi^1_H \Psi_H) + n_s \beta EV(\infty) \\
= \left( \frac{1}{1 - n_s \beta} \right) \left( \frac{\Pi_0}{\eta} \right) (\Psi_P + \tau^{-\theta} \xi^1_H \Psi_H).
\]

The entry condition is given by

\[
W f_E = \beta EV(\infty) \\
= \left( \frac{\beta}{1 - n_s \beta} \right) \left( \frac{\Pi_0}{\eta} \right) (\Psi_P + \tau^{-\theta} \xi^1_H \Psi_H).
\]

The expected values of an exporter with \( \xi = \xi_H \) can be rewritten as

\[
EV(\xi_H) = \Pi_0 \left[ \Psi_P + \rho \tau^{-\theta} \xi^1_H \Psi_H + (1 - \rho) \tau^{-\theta} \xi^1_L \Psi_L \right] \\
- n_P W f_P - [\rho n_H + (1 - \rho) n_L] W f_X \\
+ n_s \beta \rho [n_H EV(\xi_H) + (1 - n_H) EV(\infty)] \\
+ n_s \beta (1 - \rho) [n_L EV(\xi_L) + (1 - n_L) EV(\infty)] \\
= \left( \frac{\Pi_0}{\eta} \right) [\Psi_P + \rho \tau^{-\theta} \xi^1_H \Psi_H + (1 - \rho) \tau^{-\theta} \xi^1_L \Psi_L] + n_s \beta EV(\infty).
\]

Similarly, the expected values of an exporter with \( \xi = \xi_L \) can be rewritten as

\[
EV(\xi_L) = \left( \frac{\Pi_0}{\eta} \right) [\Psi_P + (1 - \rho) \tau^{-\theta} \xi^1_H \Psi_H + \rho \tau^{-\theta} \xi^1_L \Psi_L] + n_s \beta EV(\infty).
\]

From (91), (93) and (94), we have

\[
EV(\xi_H) - EV(\infty) = \left( \frac{\Pi_0}{\eta} \right) (1 - \rho) \tau^{-\theta} (\xi^1_L \Psi_L - \xi^1_H \Psi_H),
\]

\[
EV(\xi_L) - EV(\infty) = \left( \frac{\Pi_0}{\eta} \right) \rho \tau^{-\theta} (\xi^1_L \Psi_L - \xi^1_H \Psi_H).
\]
From (87), (88), (95) and (96), we have the marginal exporters’ productivity conditions as

\[ Wf_X = \Pi_0 \tau^{-\theta} \xi_H a_H + \left( \frac{n_s \beta}{\eta} \right) \Pi_0 (1 - \rho) \tau^{-\theta} \left( \xi_L^{-\theta} \Psi_L - \xi_H^{-\theta} \Psi_H \right), \]

(97) \[ Wf_X = \Pi_0 \tau^{-\theta} \xi_L a_L + \left( \frac{n_s \beta}{\eta} \right) \Pi_0 \rho \tau^{-\theta} \left( \xi_L^{-\theta} \Psi_L - \xi_H^{-\theta} \Psi_H \right). \]

(98) The masses of non-exporters, \( N_0 \), exporters with \( \xi_H \), \( N_H \), and exporters with \( \xi_L \), \( N_L \), are given by

\[ N_0 = [ (n_s N_0 + N_E) + \rho n_s N_H + (1 - \rho) n_s N_L ] (1 - n_H) + [ (1 - \rho) n_s N_H + \rho n_s N_L ] (1 - n_L). \]

(99) \[ N_H = [ (n_s N_0 + N_E) + \rho n_s N_H + (1 - \rho) n_s N_L ] n_H, \]

(100) \[ N_L = [ (1 - \rho) n_s N_H + \rho n_s N_L ] n_L, \]

(101) \[ N = N_0 + N_H + N_L. \]

where \( N \) is the mass of all producers, and \( N_E \) is the mass of entrants, \( N_E = (1 - n_s) N \).

Using the masses of producers, we can rewrite (73) with (74) and (75) as,

\[ 1 = \left( \frac{\theta W}{\theta - 1} \right)^{1-\theta} N \left( \Psi_P + \tau^{-\theta} \Psi_X \right) \]

(103) \[ \Psi_X = \xi_H^{-\theta} \left[ \left( \frac{n_s N_0 + N_E}{N} \right) + \rho n_s \left( \frac{N_H}{N} \right) + (1 - \rho) n_s \left( \frac{N_L}{N} \right) \right] \Psi_H + \xi_L^{-\theta} \left[ (1 - \rho) n_s \left( \frac{N_H}{N} \right) + \rho n_s \left( \frac{N_L}{N} \right) \right] \Psi_L = \xi_H^{-\theta} \Psi_H \left( \frac{N_H}{N n_H} \right) + \xi_L^{-\theta} \Psi_L \left( \frac{N_L}{N n_L} \right) \]

(104) \[ = \xi_H^{-\theta} \Psi_H \left( \frac{N_H}{N n_H} \right) + \xi_L^{-\theta} \Psi_L \left( \frac{N_L}{N n_L} \right) \]

Labor Market Clearing Condition: The total labor in production is given by

\[ L_P = \sum_{\xi \in \{ \xi_L, \xi_H, \infty \}} \int l(z, \xi, f) \varphi_H(a, \xi) da = (\theta - 1) \left( \frac{\Pi_0}{W} \right) N \left( \Psi_P + \tau^{-\theta} \Psi_X \right). \]
The total labor used for fixed costs in production is given by

\[ N_{n_f} = \frac{\Pi_0}{W} \left( \frac{\eta - 1}{\eta} \right) N\Psi_P. \]  

The total fixed cost in exporting is given by

\[ \begin{align*}
L_X &= [n_s N_0 + N_E + \rho n_s N_H + (1 - \rho) n_s N_L] n_H f_X + [(1 - \rho) n_s N_H + \rho n_s N_L] n_L f_X \\
&= \frac{\Pi_0}{W} \left( \frac{N_H}{n_H} \right) \left( \frac{\eta - 1}{\eta} \right) \tau^{-\theta} \xi^{1-\theta}_H \Psi_H \\
&\quad + \frac{\Pi_0}{W} \left( \frac{N_H}{n_H} \right) \frac{n_s \beta}{\eta} (1 - \rho) \tau^{-\theta} n_H \left( \xi^{1-\theta}_L \Psi_L - \xi^{1-\theta}_H \Psi_H \right) \\
&\quad + \frac{\Pi_0}{W} \left( \frac{N_L}{n_L} \right) \frac{n_s \beta}{\eta} \tau^{-\theta} n_L \left( \xi^{1-\theta}_L \Psi_L - \xi^{1-\theta}_H \Psi_H \right) \\
&= \frac{\Pi_0}{W} \tau^{-\theta} N \left[ \left( \frac{\eta - 1}{\eta} \right) \Psi_X + \left( \frac{\beta}{\eta} \right) \left( \Psi_X - \xi^{1-\theta}_H \Psi_H \right) \right].
\end{align*} \]

The total labor used for entry is given by

\[ N_{f_E} = \frac{\beta (1 - n_s)}{1 - n_s \beta} \left( \frac{1}{\eta} \right) \left( \frac{\Pi_0}{W} \right) N \left( \Psi_P + \tau^{-\theta} \xi^{1-\theta}_H \Psi_H \right) \]

The labor market clearing condition is given by

\[ \begin{align*}
L &= (\theta - 1) \frac{\Pi_0}{W} N \left( \Psi_P + \tau^{-\theta} \Psi_X \right) + \frac{\Pi_0}{W} \left( \frac{\eta - 1}{\eta} \right) N \Psi_P \\
&\quad + \frac{\Pi_0}{W} \tau^{-\theta} N \left[ \left( \frac{\eta - 1}{\eta} \right) \Psi_X + \left( \frac{\beta}{\eta} \right) \left( \Psi_X - \xi^{1-\theta}_H \Psi_H \right) \right] \\
&\quad + \frac{\beta (1 - n_s)}{1 - n_s \beta} \left( \frac{1}{\eta} \right) \left( \frac{\Pi_0}{W} \right) N \left( \Psi_P + \tau^{-\theta} \xi^{1-\theta}_H \Psi_H \right).
\end{align*} \]

Rearranging it, we have

\[ \frac{WLS}{C^*} = 1 - \frac{1 - \beta}{\theta \eta} - \frac{n_s \beta (1 - \beta)}{(1 - n_s \beta) \theta \eta} S_0. \]
where

\begin{align}
(110) \quad S &= \frac{\Psi_P + \tau^{1-\theta}\Psi_X}{\Psi_P + \tau^{-\theta}\Psi_X}, \\
(111) \quad S_0 &= \frac{\Psi_P + \tau^{-\theta}\xi_{1-\theta}\Psi_H}{\Psi_P + \tau^{-\theta}\Psi_X}.
\end{align}

Note that $S$ can be interpreted as a measure of the tariff distortion. For any $\tau > 1$, $S > 1$, and for $\tau = 1$, $S = 1$. Also note that $S_0^{-1}$ can be interpreted as a measure of entrant’s disadvantage. Clearly, $S_0 = 1$ if $\xi_H = \xi_L$, and $S_0 < 1$, if $\xi_H > \xi_L$.

**Aggregates:** The expenditure on imported goods relative to that on home goods

\begin{equation}
(112) \quad IMD = \frac{\tau \sum_{\xi \in \{\xi_L, \xi_H\}} \int_a P_F (a, \xi) y_F (a, \xi) \varphi_F (a, \xi) \, da}{\sum_{\xi \in \{\xi_L, \xi_H, \infty\}} \int_a P_H (a, \xi) y_H (a, \xi) \varphi_H (a, \xi) \, da} = \frac{\tau^{1-\theta}\Psi_X}{\Psi_P}.
\end{equation}

The share of expenditures on domestic goods is given by

\begin{equation}
(113) \quad \lambda = \frac{1}{1 + IMD} = \frac{\Psi_P}{\Psi_P + \tau^{1-\theta}\Psi_X}.
\end{equation}

**B.4 Long-Run Growth**

**Productivity Thresholds, $a_P$, $a_H$, and $a_L$:** The productivity thresholds are determined by 4 equations (80), (92), (97) and (98). With these 4 equations, we have 3 equations that determine the productivity thresholds

\begin{align}
(114) \quad \left( \frac{f_E}{f_P} \right) a_P &= \left( \frac{\beta}{1 - n_s \beta} \right) \left( \frac{1}{\eta} \right) \left( \Psi_P + \tau^{-\theta}\xi_{1-\theta}\Psi_H \right), \\
(115) \quad \left( \frac{f_X}{f_P} \right) a_P &= \tau^{-\theta}\xi_{1-\theta} a_H + \left( \frac{n_s \beta}{\eta} \right) (1 - \rho) \tau^{-\theta} \left( \xi_{1-\theta} \Psi_L - \xi_{1-\theta}\Psi_H \right), \\
(116) \quad \left( \frac{f_X}{f_P} \right) a_P &= \tau^{-\theta}\xi_{1-\theta} a_L + \left( \frac{n_s \beta}{\eta} \right) \rho \tau^{-\theta} \left( \xi_{1-\theta} \Psi_L - \xi_{1-\theta}\Psi_H \right).
\end{align}

From (115) and (116), we have

\begin{equation}
(117) \quad \xi_{1-\theta} a_H - \xi_{1-\theta} a_L = \left( \frac{n_s \beta}{\eta} \right) (2 \rho - 1) \left( \xi_{1-\theta} \Psi_L - \xi_{1-\theta}\Psi_H \right).
\end{equation}
Note that we should have $\xi_{L}^{1-\theta} \Psi_{L} > \xi_{H}^{1-\theta} \Psi_{H}$ by construction. Thus, we have

$$\text{(118) } \xi_{H}^{1-\theta} a_{H} > \xi_{L}^{1-\theta} a_{L},$$

and $a_{H} > a_{L}$ with $\xi_{H} > \xi_{L}$. It follows that $\Psi_{H} < \Psi_{L}$ and $n_{H} < n_{L}$. Taking the log-linearization of (117), we have

$$\text{(119) } \hat{a}_{L} = \left( \frac{\xi_{H}}{\xi_{L}} \right)^{1-\theta} \left( \frac{a_{H}}{a_{L}} \right) \left[ \frac{1 - n_{s}\beta (2\rho - 1) n_{H}}{1 - n_{s}\beta (2\rho - 1) n_{L}} \right] \hat{a}_{H}.$$

Since $\xi_{H}^{1-\theta} a_{H} > \xi_{L}^{1-\theta} a_{L}$ and $n_{H} < n_{L}$, we have $\hat{a}_{L}/\hat{a}_{H} > 1$. Taking the log-linearization of (114) and (115), we have

$$\text{(120) } \hat{a}_{P} = -\frac{\tau^{\theta} \xi_{H}^{1-\theta} \Psi_{H}}{\eta \Psi_{P} + \tau^{\theta} \xi_{H}^{1-\theta} \Psi_{H}} \left[ \theta \hat{\tau} + (\eta - 1) \hat{a}_{H} \right],$$

$$\text{(121) } \hat{a}_{P} = -\theta \hat{\tau} + \left( \frac{f_{P}}{f_{X}} \right) \tau^{\theta} a_{P}^{-1} \left[ \xi_{H}^{1-\theta} a_{H} \hat{a}_{H} \right.$$

$$- \left( \frac{n_{s}\beta}{\eta} \right) (1 - \rho) (\eta - 1) \left( \xi_{L}^{1-\theta} \Psi_{L} \hat{a}_{L} - \xi_{H}^{1-\theta} \Psi_{H} \hat{a}_{H} \right) \left. \right] .$$

Rearranging them with (119), we have

$$\text{(122) } \hat{a}_{H} = \frac{\theta \eta \Psi_{P}}{(\eta - 1) \tau^{\theta} \xi_{H}^{1-\theta} \Psi_{H} + [\eta \Psi_{P} + \tau^{\theta} \xi_{H}^{1-\theta} \Psi_{H}] \left( \frac{\hat{\tau}}{\hat{\xi}} \right) \hat{a}_{H},}$$

$$\text{(123) } \hat{a}_{P} = -\frac{\theta \left[ \zeta_{0} + (\eta - 1) \zeta_{1} \right] \tau^{\theta} \xi_{H}^{1-\theta} \Psi_{P}}{\eta \zeta_{0} \Psi_{P} + [\zeta_{0} + (\eta - 1) \zeta_{1}] \tau^{\theta} \xi_{H}^{1-\theta} \Psi_{P}} \hat{\tau},$$

where

$$\text{(124) } \zeta_{0} = 1 - \frac{n_{s}\beta (1 - \rho) (n_{L} - n_{H})}{1 - n_{s}\beta (2\rho - 1) n_{L}} ,$$

$$\text{(125) } \zeta_{1} = \left( \frac{f_{X}}{f_{P}} \right) \tau^{\theta} \xi_{H}^{1-\theta} \left( \frac{a_{P}}{a_{H}} \right)$$

$$= 1 + n_{s}\beta (1 - \rho) n_{H} \left( \frac{1}{\eta - 1} \right) \left[ \left( \frac{\xi_{L}}{\xi_{H}} \right)^{1-\theta} \left( \frac{\Psi_{L}}{\Psi_{H}} \right) - 1 \right] .$$
Note that $\zeta_1 > 1$ and $0 < \zeta_0 < 1$ since

$$
(126) \quad \zeta_0 = \frac{1 - n_s \beta \rho n_L + n_s \beta (1 - \rho) n_H}{1 - n_s \beta (2\rho - 1) n_L} > 0.
$$

Clearly, $\hat{a}_H/\hat{\tau} > 0$, and $\hat{a}_P/\hat{\tau} < 0$. From (119) we also have $\hat{a}_L/\hat{\tau} > 0$. That is, with a tariff cut $\hat{\tau} < 0$, the exporting thresholds $a_H$ and $a_L$ both fall, but the production threshold $a_P$ rises.

From (123), we can find that following a cut in the tariff rate, $a_P$ falls less in the model with exporter dynamics relative to the model without it at the margin for the same initial trade share and the tariff rate. To see that, we can rewrite (123) as

$$
(127) \quad \frac{\hat{a}_P}{\hat{\tau}} = -\theta \left[ 1 + \left( \frac{\Psi_P}{\tau^{-\theta} \Psi_X} \right) \frac{\Psi_X}{\Psi_H} \right]^{-1}.
$$

From (113), we have

$$
(128) \quad \frac{\Psi_P}{\tau^{-\theta} \Psi_X} = \tau \left( \frac{\lambda}{1 - \lambda} \right).
$$

From (104), we have

$$
(129) \quad \frac{\Psi_X}{\xi_H^{1-\theta} \Psi_H} = 1 + \left( \frac{N_L}{N N_L} \right) \left[ \left( \frac{\xi_L}{\xi_H} \right)^{1-\theta} \left( \frac{\Psi_L}{\Psi_H} \right) - 1 \right].
$$

From (99), (100) and (101), we have the fraction of exporters with $\xi_H$ and $\xi_L$ as

$$
(130) \quad \left( \frac{N_L}{N N_L} \right) + \left( \frac{N_H}{N N_H} \right) = 1,
$$

$$
(131) \quad (1 - \rho n_s n_L) \left( \frac{N_L}{N} \right) = (1 - \rho) n_s \left( \frac{N_H}{N} \right) n_L,
$$

Rearranging them, we have the fraction of producers with $\xi_L$ as

$$
(132) \quad \frac{N_L}{N N_L} = \frac{(1 - \rho) n_s n_H}{1 + (1 - \rho) n_s n_H - \rho n_s n_L}.
$$
So, we have

\[ \frac{\eta \Psi_X}{\xi_H^{1-\theta} \Psi_H} = \eta + \frac{\eta (1 - \rho) n_s n_H}{1 + (1 - \rho) n_s n_H - \rho n_s n_L} \left[ \left( \frac{\xi_L}{\xi_H} \right)^{1-\theta} \left( \frac{\Psi_L}{\Psi_H} \right) - 1 \right]. \]

From (124) and (125), we have

\[ 1 + (\eta - 1) \frac{\zeta_1}{\zeta_0} = 1 + \frac{1 - n_s \beta (2 \rho - 1) n_L}{1 + n_s \beta (1 - \rho) n_H - n_s \beta \rho n_L} \left\{ \eta - 1 + n_s \beta (1 - \rho) n_H \left[ \left( \frac{\xi_L}{\xi_H} \right)^{1-\theta} \left( \frac{\Psi_L}{\Psi_H} \right) - 1 \right] \right\} \\
= \eta + \frac{n_s \beta (1 - \rho) n_H}{1 + n_s \beta (1 - \rho) n_H - n_s \beta \rho n_L} \left[ \left( \frac{\xi_L}{\xi_H} \right)^{1-\theta} \left( \frac{\Psi_L}{\Psi_H} \right) - 1 \right] \left\{ \left( \frac{n_L}{n_H} - 1 \right) \left( \frac{\xi_L}{\xi_H} \right)^{1-\theta} \left( \frac{\Psi_L}{\Psi_H} \right) - 1 + [1 - n_s \beta (2 \rho - 1) n_L] \right\}. \]

From (117), we have

\[ 1 - \left( \frac{\xi_L}{\xi_H} \right)^{1-\theta} \left( \frac{\Psi_L}{\Psi_H} \right) \left( \frac{n_H}{n_L} \right) = n_s \beta (2 \rho - 1) \left( \frac{1}{\eta - 1} \right) n_H \left[ \left( \frac{\xi_L}{\xi_H} \right)^{1-\theta} \left( \frac{\Psi_L}{\Psi_H} \right) - 1 \right]. \]

Rearranging it, we have

\[ n_s \beta (2 \rho - 1) n_L = \frac{(\eta - 1) \left( \frac{n_L}{n_H} - 1 \right)}{\left( \frac{\xi_L}{\xi_H} \right)^{1-\theta} \left( \frac{\Psi_L}{\Psi_H} \right) - 1} - (\eta - 1). \]

Applying this to (134), we have

\[ 1 + (\eta - 1) \frac{\zeta_1}{\zeta_0} = \eta + \frac{\eta n_s \beta (1 - \rho) n_H}{1 + n_s \beta (1 - \rho) n_H - n_s \beta \rho n_L} \left[ \left( \frac{\xi_L}{\xi_H} \right)^{1-\theta} \left( \frac{\Psi_L}{\Psi_H} \right) - 1 \right]. \]

We have

\[ 1 + n_s \beta (1 - \rho) n_H - n_s \beta \rho n_L = 1 + n_s (1 - \rho) n_H - n_s \rho n_L + (1 - \beta) n_s [\rho n_L - (1 - \rho) n_H] \]

\[ \geq 1 + n_s (1 - \rho) n_H - n_s \rho n_L. \]
since $\rho > 1/2$ and $n_L > n_H$. This gives

\[
\frac{n_s \beta (1 - \rho) n_H}{1 + n_s \beta (1 - \rho) n_H - n_s \beta \rho n_L} < \frac{n_s (1 - \rho) n_H}{1 + n_s (1 - \rho) n_H - n_s \rho n_L}.
\]

Thus, we have

\[
\frac{\hat{a}_P}{\hat{\tau}} |_{\xi_H > \xi_L} < \frac{\hat{a}_P}{\hat{\tau}} |_{\xi_H = \xi_L}.
\]

**Exporters with $\xi_L$:** From (132) we have the fraction of producers with $\xi_L$ as

\[
\frac{N_L}{N n_L} = \left[ 1 + \frac{1 - \rho n_s n_L}{(1 - \rho) n_s n_H} \right]^{-1}.
\]

Since $\hat{\alpha}_H/\hat{\tau} > 0$, and $\hat{\alpha}_L/\hat{\tau} > 0$, we have $\hat{n}_H/\hat{\tau} < 0$, and $\hat{n}_L/\hat{\tau} < 0$. Thus, from (140), the fraction of producers with $\xi_L$ falls with a rise in the tariff rate,

\[
\frac{\left( \frac{N_L}{N n_L} \right)}{\hat{\tau}} < 0, \quad \text{and} \quad \frac{\left( \frac{N_L}{N} \right)}{\hat{\tau}} < 0.
\]

**The Trade Elasticity:** Log-linearizing (104) gives

\[
\hat{\Psi}_X = \hat{\Psi}_H + \left( 1 - \frac{\xi_H^{1-\theta} \Psi_H}{\Psi_X} \right) \left\{ \frac{\hat{N}_L}{\hat{N} n_L} + \left[ \frac{\left( \frac{\xi_L}{\xi_H} \right)^{1-\theta} \left( \frac{\Psi_L}{\Psi_H} \right)}{\xi_h} - 1 \right] \left( \hat{\Psi}_L - \hat{\Psi}_H \right) \right\}.
\]

We have

\[
\frac{\hat{\Psi}_H}{\hat{\tau}} = - (\eta - 1) \frac{\hat{a}_H}{\hat{\tau}} < 0,
\]

\[
\frac{\left( \frac{N_L}{N n_L} \right)}{\hat{\tau}} < 0,
\]

\[
\frac{\hat{\Psi}_L - \hat{\Psi}_H}{\hat{\tau}} = - (\eta - 1) \frac{(\hat{\alpha}_L - \hat{\alpha}_H)}{\hat{\tau}}
\]

\[
= - (\eta - 1) \left\{ \left( \frac{\xi_H}{\xi_L} \right)^{1-\theta} \left( \frac{a_H}{a_L} \right) \left[ 1 - n_s \beta (2\rho - 1) n_H \right] - 1 \right\} \frac{\hat{a}_H}{\hat{\tau}} < 0.
\]
Thus, we have $\hat{\Psi}_X < \hat{\Psi}_H < 0$. From (121), we have

(146) $\hat{a}_P = -\theta\hat{\tau} + \left(\frac{\zeta_0}{\zeta_1}\right)\hat{a}_H$.

Using (142) and (146), the trade elasticity $\varepsilon$ is given by

(147) $\varepsilon = \frac{\left(\hat{\Psi}_P/\hat{\tau}\right)}{\hat{\Psi}_X} - \theta - 1 + (\eta - 1) \left(1 - \frac{\zeta_0}{\zeta_1}\right)\hat{a}_H$

$$= \left(1 - \frac{\zeta_1^{1-\theta}\Psi_H}{\Psi_X}\right) \left\{ \left(\frac{N_L}{N_H}\right)\hat{\tau} + \left[\frac{\left(\frac{\zeta_L}{\zeta_H}\right)^{1-\theta}\left(\frac{\Psi_L}{\Psi_H}\right)}{\eta - 1}\right] \left(\frac{\hat{\Psi}_L - \hat{\Psi}_H}{\hat{\tau}}\right) \right\}.$$  

Since $0 < \zeta_0 < 1$, $\zeta_1 > 1$, $\hat{a}_H > 0$, $\left(\frac{N_L}{N_H}\right)\hat{\tau} < 0$, and $\left(\frac{\hat{\Psi}_L - \hat{\Psi}_H}{\hat{\tau}}\right) \leq 0$, we have $\varepsilon > \eta\theta - 1$. Note that when $\xi_H = \xi_L$, the elasticity becomes $\varepsilon|_{\xi_H=\xi_L} = \eta\theta - 1$. Thus, the model with exporter dynamics has a greater trade elasticity compared to the model without it, $\xi_H = \xi_L$.

**Entrant’s disadvantage** ($S_0^{-1}$): The entrant’s disadvantage is measured with $S_0^{-1}$ in (111). Log-linearizing the equation gives

(148) $\hat{S}_0 = \frac{\Psi_P\hat{\Psi}_P + \tau^{-\theta}\xi_H^{-\theta}\Psi_H \left(-\theta\hat{\tau} + \hat{\Psi}_H\right)}{\Psi_P + \tau^{-\theta}\xi_H^{-\theta}\Psi_H} - \frac{\Psi_P\hat{\Psi}_P + \tau^{-\theta}\Psi_X \left(-\theta\hat{\tau} + \hat{\Psi}_X\right)}{\Psi_P + \tau^{-\theta}\Psi_X}$

$$= \left(\Psi_P + \tau^{-\theta}\xi_H^{-\theta}\Psi_H\right)^{-1} \left(\Psi_P + \tau^{-\theta}\Psi_X\right)^{-1}$$

$$\left\{ \tau^{-\theta} \left[\Psi_X - \xi_H^{-\theta}\Psi_H\right] \Psi_P\hat{\Psi}_P + \tau^{-\theta} \left[\Psi_X - \xi_H^{-\theta}\Psi_H\right] \Psi_P\theta\hat{\tau} + \xi_H^{-\theta}\Psi_H \Psi_P \Psi_X \left(1 - \frac{\hat{\Psi}_H}{\hat{\Psi}_X}\right) \right\}.$$  

Since $\hat{\Psi}_P/\hat{\tau} > 0$, $\hat{\Psi}_X/\hat{\tau} < 0$, and $0 < \hat{\Psi}_H/\hat{\Psi}_X < 1$, we have $\hat{S}_0/\hat{\tau} > 0$. That is, the entrant’s disadvantage, $S_0^{-1}$, rises with a tariff cut.
Labor Share of GDP \((W/L/C)\):  From the labor market clearing condition (109), we have

\[
\frac{WL}{C} = \frac{b_0 - b_1 S_0}{S},
\]

where \(b_0 = 1 - \left(\frac{1-\beta}{\theta \eta}\right) > 0\), and \(b_1 = \frac{n_s \beta (1-\beta)}{(1-n_s \beta) \eta \theta}\). Note that \(b_1 > 0\) if \(\beta \in (0,1)\) and \(b_1 = 0\) if \(\beta = 1\). Also note that \(S > 1\) for any \(\tau > 1\), and \(S = 1\) when \(\tau = 1\) or \(\tau \to \infty\) (autarky). Thus the trade liberalization, \(\tau = 1\) from \(\tau > 1\), raises the labor share of GDP, \(W/L/C\), and the rise in the entrant’s disadvantage, \(S_0^{-1}\), further increases the labor income to output ratio when \(\beta < 1\).

Investment on Establishment Capital \((N)\):  From the entry condition (92) and the price index (103), we have

\[
\begin{align*}
W_fE &= \left(\frac{\beta}{1 - n_s \beta}\right) \left(\frac{1}{\eta \theta}\right) \frac{C}{N} \left(\Psi_P + \tau^{-\theta} \xi_H^{1-\theta} \Psi_H\right) \\
&= \left(\frac{\beta}{1 - n_s \beta}\right) \left(\frac{1}{\eta \theta}\right) CS_0 NS.
\end{align*}
\]

Using (149), we have

\[
N = \left(\frac{\beta}{1 - n_s \beta}\right) \left(\frac{1}{\eta \theta}\right) \left(\frac{L}{f_E}\right) \left(\frac{S_0}{b_0 - b_1 S_0}\right).
\]

Since \(S_0\) falls with the trade liberalization, the mass of producers, \(N\), falls, \(\tilde{N}/\tilde{\tau} > 0\).

Wage Rate \((W)\):  From the price index (103), we have

\[
W^{\theta-1} \propto N \left(\Psi_P + \tau^{1-\theta} \Psi_X\right),
\]
where \( \propto \) denotes ‘proportional to’. From the entry condition (92) and the marginal producer condition (80), we have

\[
(153) \quad a_P \propto \Psi_P + \tau^{-\theta} \xi_H^{1-\theta} \Psi_H. 
\]

We can rewrite it as

\[
(154) \quad \Psi_P \propto \left( \Psi_P + \tau^{-\theta} \xi_H^{1-\theta} \Psi_H \right)^{-(\eta-1)}. 
\]

So, we have

\[
(155) \quad \frac{\Psi_P}{\Psi_P + \tau^{-\theta} \xi_H^{1-\theta} \Psi_H} = \frac{\lambda S}{S_0} \propto \left( \Psi_P + \tau^{-\theta} \xi_H^{1-\theta} \Psi_H \right)^{-\eta}. 
\]

This gives

\[
(156) \quad \Psi_P + \tau^{1-\theta} \Psi_X = \lambda^{-1} \Psi_P \propto \left( \frac{S}{S_0} \right)^{\frac{n-1}{n}} \lambda^{-\frac{1}{n}}. 
\]

With (151) and (156), (152) can be rewritten as

\[
(157) \quad W^{\theta-1} \propto (b_0 - b_1 S_0)^{-1} S_{\eta-1} \left( \frac{S_0}{\lambda} \right)^{\frac{1}{\eta}}, 
\]

or

\[
(158) \quad W \propto (b_0 - b_1 S_0)^{-\frac{1}{\eta-1}} S_{\eta-1} \left( \frac{S_0}{\lambda} \right)^{\frac{1}{\eta}}. 
\]

So, the fall in the tariff distortion from \( S > 1 \) to \( S = 1 \) following the trade liberalization reduces the wage growth rate. The increase in the entrant’s disadvantage, \( S_0^{-1} \) also lowers the wage growth. Having positive discount rate, \( \beta < 1 \) and \( b_1 > 0 \), further reduces the growth of the wage rate.
Consumption \((C)\): Using (149) and (158), consumption can be rewritten as

\[
(159) \ C \propto \left( \frac{C}{W} \right) W
\]

\[
\propto \left[ \frac{S}{b_0 - b_1 S_0} \right] (b_0 - b_1 S_0)^{-\frac{1}{\theta - 1}} S_{\eta-1} \left( \frac{S_0}{\lambda} \right)^{\frac{1}{\eta(\theta - 1)}}
\]

\[
\propto (b_0 - b_1 S_0)^{-\frac{\theta}{\theta - 1}} S_{\eta-1} \left( \frac{S_0}{\lambda} \right)^{\frac{1}{\eta(\theta - 1)}}
\]

Following the trade liberalization, a fall in the share of expenditures on domestic goods \(\lambda\) raises the welfare. However, the reduction of the tariff distortion \(S\) reduces the long-run welfare gains. The increase in the entrant’s disadvantage, \(S_0^{-1}\) also reduces the welfare gains. If the discount rate is positive, \(\beta < 1\) and \(b_1 > 0\), the welfare gains are further reduced because a fall in \(S_0\) lowers the gains additionally through a fall in the mass of producers.

**Long-run Welfare Gains with Same Trade Growth:** Equation (159) shows the relationship between the trade growth, \(\lambda^{-1}\), and the long run welfare gains, and how the tariff distortion \(S\) and the entrant’s disadvantage \(S_0^{-1}\) affect the long run consumption growth. However, it is not clear how the model predicts the long run growth once the model is calibrated to match the trade growth with same tariffs \(\tau\) and initial trade. To find it out we can rewrite the consumption equation as follows. From (149), (152) and (153) we have

\[
(160) \ C \propto \left( \frac{C}{W} \right) \left( \frac{N S a_P}{S_0} \right)^{\frac{1}{\theta - 1}}
\]

\[
\propto \left( \frac{S}{b_0 - b_1 S_0} \right) [(b_0 - b_1 S_0)^{-1} S a_P]^{\frac{1}{\theta - 1}}
\]

\[
\propto (b_0 - b_1 S_0)^{-\frac{\theta}{\theta - 1}} S_{\eta-1} a_P^{\frac{1}{\theta - 1}}.
\]

This gives

\[
(161) \ \frac{\tilde{C}}{\tau} = \frac{\theta}{\theta - 1} \left( \frac{b_1 S_0}{b_0 - b_1 S_0} \right) \frac{\tilde{S}_0}{\tau} + \frac{\theta}{\theta - 1} \frac{\tilde{S}}{\tau} + \frac{\theta}{\theta - 1} \left( \frac{\tilde{a}_P}{\tau} \right).
\]

The first term is positive. The increase in the entrant’s disadvantage, \(S_0^{-1}\), with a fall in \(\tau\) reduces the long run gains. The second term is also positive. The a reduction of the tariff
distortion with a cut in $\tau$ reduces the long run gains. The last term is negative. An increase in the production threshold with a cut in $\tau$ raises the welfare.

For the growth rate of the tariff distortion, from (110), we have

$$
(162) \quad \hat{S} = \frac{\tau^{-\theta}\Psi_X}{\Psi_P + \tau^{-\theta}\Psi_X} [1 - \lambda (\tau - 1) \varepsilon] \hat{\tau}.
$$

Now, consider a special case with $\beta = 1$. In this case, we have

$$
(163) \quad \frac{\hat{S}}{\hat{\tau}} + \frac{\hat{a}_p}{\theta \hat{\tau}} = -\frac{\lambda (\tau - 1) \varepsilon \tau^{-\theta} \Psi_X}{\Psi_P + \tau^{-\theta} \Psi_X}.
$$

If we set $\theta$ to match the trade growth across models ($\xi_H > \xi_L$ and $\xi_H = \xi_L$), we need to assign higher $\theta$ for the model without exporter dynamics. Since $\theta / (\theta - 1)$ is decreasing in $\theta$, and the first term is zero with $\beta = 1$, the model predicts that the welfare gains under $\xi_H = \xi_L$ is lower than that under $\xi_H > \xi_L$.

In a general case with $\beta < 1$, the first term is positive, and $-\frac{\hat{a}_p}{\theta \hat{\tau}} |_{\xi_H > \xi_L} < -\frac{\hat{a}_p}{\theta \hat{\tau}} |_{\xi_H = \xi_L}$. Thus, the discount factor plays a crucial role for the long run welfare gains.