Spending Multiplier during Sudden Stop Crises

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February 10, 2018

Abstract

This paper studies the effect of government spending policy during sudden stop crises. Using a quarterly data-set of 30 small open economies, I find that government spending is more effective in stimulating consumption and appreciating real exchange rate during sudden stops than during normal times. To rationalize this, I build a two-sector model with the collateral constraint on external debt. During recession, an adverse international shock reduces consumption and undermines the value of collateral. The collapsing asset price in turn tightens the financial constraint, deteriorates the real absorption, and sets-in a fully-blown debt-deflation mechanism in spirit of E. G. Mendoza (2010). In this context, an increase in government purchase exerts a counteracting force by raising asset prices and stimulating real activities. More importantly, if the government can commit certain paths of spending in the future, the expected real appreciations further relax the financial constraint today. Lastly, I use a calibrated model to explore the multiplier effect under different exchange rate regimes, the asymmetric multipliers, and the multipliers for different shock persistence.

Keywords: Spending Multiplier, Sudden Stop Crisis, Fisher’s Debt-Deflation, Collateral Constraint, Downward Nominal Wage Rigidity.

JEL Classification: E62, F34, F41, F44, H50

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1 Introduction

Are government spending multipliers higher during sudden stop crisis? While the stimulus packages were hastily put together upon sudden stops in many countries, the governments of others took the route of austerity. At the center of the debate is the uncertainty regarding the size of fiscal multipliers during small open economy (SOE) financial crises.

This paper provides an empirical and theoretical investigation into the effect of government spending shock on real activities and asset prices. First of all, I provide the cross-country evidence that the spending multiplier depends on financial status of the economy. Secondly, I build a model with a novel channel through which Fisher’s debt-deflation mechanism can account for the state-dependence of fiscal multipliers.

The empirical part uses quarterly data for a panel of 30 small open economies from 1980 to 2016. The sample covers a wide range of countries with different level of incomes, financial institutions and exchange rate regimes, and it contains most of the sudden stop events in the last two decades that are found in the existing papers (see Calvo et al., 2004, for example). Following Ramey & Zubairy (2014) (RZ), I estimate the government spending multipliers using local projection method. The shocks are identified assuming that it takes at least one quarter for the government spending to respond to innovations in economic fundamentals. The regression result shows that a positive government spending shock is more effective in increasing consumption and appreciating real exchange rate during sudden stop crisis than during normal times. And the increased multiplier effect during sudden stop is more pronounced for the countries under predetermined exchange rate system. Moreover, due to the occurrence of sudden stop crisis, I find asymmetric effect of government spending shocks: the multiplier of the fiscal contraction is higher than that of the fiscal expansion, especially during sudden stop episodes. Lastly, the multiplier is higher for the greater shock persistence.

I show that a simple small open economy model with financial constraint and downward nominal wage rigidity (DNWR) can partly account for these observations. I start from investigating the optimal policy in a three-period model to illustrate the key mechanisms of this paper. Then, I use an infinite-horizon model to quantify the business cycle-dependent multipliers. Lastly, I show the results are robust to a variety of model specifications.

The model I use is a two-sector small open economy with limited access to international financial markets. The form of the financial constraint is in spirit of Kiyotaki & Moore (1997) and E. G. Mendoza (2010). Non-tradable goods are produced using labor and capital while the tradable endowment follows a stochastic process. In order to smooth consumption, the private agents are indebted in the international financial market, but the borrowing is restricted by the current value of capital. In addition to constraint on debt, non-tradable good producers face a working-capital constraint in the sense that each firm need to pledge a fraction of their wage-bill before production takes place, and this also requires capital as collateral. In this paper, I assume the government spending is exogenous and lump-sum taxes are always available.

Under an adverse international shock, the decrease of tradable good consumption reduces the
non-tradable good price. Since capital is productive in non-tradable sector while external debts are denominated in tradable goods, a (dynamic) currency mismatch problem arises. The currency depreciation reduces the tradable value of firms’ dividends, depressing the capital price. In turn, the collapsing capital price lowers the borrowing limit, and as long as the financial constraint binds, the financial accelerator akin Fisher’s debt-deflation mechanism is set in motion: this causes a vicious loop of falling consumption demand, drops of capital prices, and the sudden stop of capital inflows. Moreover, since the representative agents are atomic, they don’t incorporate the effect of their decisions on asset prices. That gives rise to the well-known pecuniary externality in the literature (e.g., Bianchi, 2011; Bianchi & Mendoza, 2013), which calls for policy interventions in the financial markets.

I show that in this environment, the increase in government purchase helps in counteracting the financial deflation and increasing real activities. First of all, the higher government spending appreciates real exchange rate, creating a positive wealth effect on households. If this effect dominates the traditional negative wealth effect associated with fiscal expansion, private consumption increases. The higher level of consumption reduces the rate at which households discount future returns, driving up capital price and relaxing the financial constraint. Moreover, due to the currency mismatch, the current capital price is determined by the present value of future dividends. If the government is able to commit certain paths of fiscal policies in the future, the expected real appreciations increase the current capital price further. When there is working-capital constraint on producing firms, the smaller financing cost encourages labor-hiring. This shrinks the crowding-out effect on non-tradable good consumption as well. Overall, an increase in government purchase is more effective in stimulating consumption and production when the financial constraint binds, and turns the vicious debt-deflation spiral into a virtuous one.

DNWR is well-documented in the literature. Schmitt-Grohé & Uribe (2016) shows that DNWR is important to capture the characteristics of SOE crisis. In my model, I include DNWR to understand how the above mechanism applies to economies under different exchange rate regimes. For a country lacking the ability to depreciate the nominal exchange rate, DNWR prevents real wage falling beyond some floor during recession, and the economy is subject to the possibility of involuntary unemployment. Because the wage rigidity makes the economy more demand-driven, an increase in government purchase has larger stimulative effect on production. That results in higher multiplier on output and consumption volumes. However, the demand channel also interacts with the financial channel. After a positive government spending shock, the overproduction of non-tradable goods dampens the increase in non-tradable good price and lessens the favorable effect on the price of collateral and international borrowings. I will show that in the full model this offsetting effect is strong enough so that the increase in value-multipliers (the multipliers measured in tradable values) during sudden stop becomes less significant than otherwise without DNWR.

Solving the model non-linearly allows me to analyze the state-dependent effect of government spending. Under pegged exchange rate policy, the model predicts consumption multiplier on impact can be greater than 0 (at 0.37) during the identified sudden stop periods and is always negative.
(at -0.18) in normal times. On the other hand, with flexible exchange rate, the multiplier increases from -0.31 in normal times to -0.12 in sudden stop periods. Overall, the model is able to reproduce several salient features that are generally consistent with the data: Even without DNWR, spending multipliers of output and consumption are higher during sudden stops than that during normal times, and the real exchange rate also appreciates more; DNWR makes the government spending more effective in stimulating consumption and output during crisis; The multiplier associated with a negative shock is higher than that associated with a positive shock; Multipliers in financial crisis get higher when shocks are more persistent.

In the baseline model, I make the assumption that the aggregate amount of capital is fixed. Due to the controversial issue of investment multiplier in the literature, I extend the model to incorporate investment technology. A positive government spending shock increases domestic interest rate, and at the same time, relaxes the financial constraint on international borrowings. So, the investment multiplier depends on the relative strength of the traditional channel and the financial channel. In the calibrated version of my model, I show that the investment is crowded-in by government spending and the effect is even stronger during financial crisis.

Sensitivity analysis examine other model specifications such as: the GHH preference (Greenwood et al., 1988), tradable good production, and intermediate exchange rate policy coefficients. The GHH utility eliminates the wealth effect on labor supply, but it also weakens the government spending effect on the financial constraint. My simulation shows the main result that financial constraint leads to a higher spending multiplier in recessions than in expansions remains, although the difference becomes smaller under GHH utility. Similarly, the financial channel of government spending is also weaker when there is tradable good production and labors are flexible to move between sectors.

Taking together, the findings of this paper highlight the necessity to analyze the state-dependent effect of government spending. More importantly, the understanding of financial market friction is nontrivial and quantitatively relevant for the conduct of fiscal policy.

1.1 Contact the Literature

This paper contributes to the literature studying government spending multipliers in financial recessions. Burgeoning empirical studies provide evidence on the state-dependent multipliers and generally find larger multipliers in recessions than in expansions based on U.S. data (e.g., Auerbach & Gorodnichenko, 2012b; Ramey & Zubairy, 2014). With a closed economy model, Carrillo & Poilly (2013) argues the credit market friction increases spending multipliers during liquidity trap through the capital accumulation channel. However, the additional multiplier effect in their paper relies on the unresponsive monetary policy at ZLB. Their prediction that inflation has to react more to a government spending shock in recessions than in expansions contradicts with the empirical finding of Dupor & Li (2015).

Fiscal multiplier research for small open economies are relatively rare\(^1\). Based on a data-set\(^1\) empirical evidence of fiscal multipliers in small open economies are also controversial. The early papers are
of 44 countries, Ilzetzki et al. (2013) (IMV) finds the output effect of government consumption depends on a list of country characteristics: the level of incomes, exchange rate regimes, the level of government indebtedness. Similarly, Miyamoto et al. (2016) finds an increase in government purchase causes real exchange rate to appreciate and increases consumption in developing countries, but the opposite for advanced countries. And they emphasize the importance of real exchange rate in explaining the divergent macro-effect of government purchases. Despite the empirical relevance, the theoretical channels about government spending effect in SOE are uncertain. One view is that public debt sustainability calls for pro-cyclical fiscal policy (e.g., Cuadra et al., 2010). On the other hand, Sin (2016) proposes that fiscal multipliers are large when the bond-financed government spending alleviates liquidity frictions in the private financial market. Within my knowledge, this is the first paper using a theoretical model to study the effect of government spending in sudden stop crisis. And I also emphasize the role of real exchange rate to account for the mechanism.

The method in this paper employs the real business cycle model featuring occasionally binding collateral constraint and pecuniary externality. Examples are Bianchi (2011), Bianchi & Mendoza (2013), Benigno et al. (2013), Schmitt-Grohé & Uribe (2017). During the financial crisis, the collapsing asset prices restrict borrowings and reduce real absorptions, which makes the financial constraint even tighter. But private agents fails to internalize this effect on the price of collateral, leading to overborrowing in competitive equilibrium relative to constrained efficient outcome (as in Bianchi, 2011; Bianchi & Mendoza, 2013, for example). Because the pecuniary externality creates a role of financial intervention, most of these papers investigate the macro-prudential policy and consider the welfare benefit of capital controls. While they conduct normative analysis, the discussions below are mainly positive, and I focus on macro-effect of government spending.

In a recent paper, Fornaro (2015) evaluates the welfare ranking of different monetary policy rules in a sudden stop prone economy, considering the trade-off between price rigidity and financial stability. Similarly, Ottonello (2012) draws implications on the optimal exchange rate policy in a model where collateral constraint is based on current income. In contrast, Devereux et al. (2015) explores the combination of capital control and monetary policy in an open economy where both financial friction and domestic price rigidities are present. Given all these papers focus on either monetary policy or capital control policy, the novelty here lies in that I study the financial aspect on government spending, although I also consider its interaction with different exchange rate regimes in an environment where both nominal and financial frictions exist.

The rest of the paper is organized as follows. Section 2 provides the empirical estimation of government spending multipliers using the cross-country data and motivates the theoretical framework. Section 3 studies the optimal government spending in a three-period model and explores the key mechanisms of this paper. Section 4 sets-up the full model and section 5 calibrates it to Mexican economy. Section 6 presents simulation results and shows how the quantitative features of the model are consistent with the data. Section 6 provides sensitivity analysis. Section 7 concludes.

Monacelli & Perotti (2010), Kim & Roubini (2008). They find an increase in government purchase raises private consumption and depreciates real exchange rate, which creates a challege for theoretical models. Ravn et al. (2012) provides a framework based on deep habit formation to reconcile these facts.
2 Empirical Evidence: Higher Multipliers during Sudden Stop Recessions

To explore the channel though which the sudden stop event affects fiscal multipliers, I collect a quarterly data-set on GDP components, real exchange rate and current account ratio for a group of small open economies going earliest to 1981\(^2\). The real variables are nominal variables deflated by GDP deflators if they exist, or CPIs if they do not (represented in local currency units)\(^3\). The real effective exchange rates are indexed numbers based on CPI-adjusted nominal exchange rates.

Consistent with the literature, I define a sudden stop event as the big current account reversal happening amid of deep economic recession\(^4\). To compare the fiscal policy effect near and away from a sudden stop event, for each identified sudden stop period, I create a 8-quarter event window with 2 periods before and 6 periods after it. This provides a clear-cut way to define sudden stop episode and generally capture the spirit of “big and sudden” capital flow stop as in Calvo et al. (2004). The definition of exchange rate regime follows Ilzetzki et al. (2013) and Ilzetzki et al. (2017)\(^5\). These definitions result in a sample data-set of 30 countries with each country having at least one sudden stop event in the history. 15 countries are classified as pegging exchange rate.

2.1 Econometric Method

The estimation strategy follows Ramey & Zubairy (2014) (RZ), who modifies the local projection method advanced in Jordà et al. (2005)\(^6\). Compared to SVAR/threshold SVAR, this method has two advantages: First, it is more robust to misspecification because it does not impose implicit dynamic restrictions on the shape of the impulse responses; Second, it is easy to accommodate state-dependence and to study non-linear issues. This flexibility allows me to extend my base-line regression to a variety of dimensions. The identification scheme follows Blanchard & Perotti (2002)/(BP)’s assumption that fiscal policy requires at least one period to respond to innovations in economic fundamentals\(^7\). IMV claims that the validity of this assumption crucially relies on the availability of quarterly data. With lower frequency data, it is much less compelling to assume within period government spending cannot respond to contemporaneous innovations in aggregate states. Given this argument and the data availability, it is reasonable to adopt this empirical strategy. In appendix C, I test the instrument relevance using Kleinpergan-Paap rk F statistics in the

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\(^2\)The data coverage is shown in appendix E

\(^3\)I also estimate the model using variables measured in U.S. dollars. The results are reported in appendix C.

\(^4\)Specifically, I denote those periods with GDP 1.5 standard deviations below trend and the reversal of current account-GDP ratio from the last year larger than two standard deviations as sudden stop type of crisis.

\(^5\)More specifically, I include the country with no legal tender, hard pegs, crawling pegs, and de facto or pre-announced bands or crawling bands with margins no larger than +(−)2% as pegged exchange rate regime. While all other countries are classified as flexible.

\(^6\)RZ extends the “three-steps method” of first creating impulse response and then simulating fiscal multiplier into a “one-step method” where multipliers can be estimated directly, and they argue that these two methods give the same result if there is no data loss. I mainly use the “one-step method” here but in appendix C, I confirm my baseline result is robust to the previous one.

\(^7\)IMV also uses the identification scheme of BP in a panel-data regression. But they only estimate the fiscal multiplier in linear models.
baseline estimation.

First of all, in a linear regression, I estimate the effect of government spending on key macro-variables:

$$\sum_{j=0}^{h} x_{i,t+j} = c + \alpha_i + \sum_{j=0}^{h} g_{i,t+j} + \Phi_h(L) z_{i,t-1} + u_{i,t+h}$$

where $i$ is the country index and $t$ is the time indicator. $h = 1, ..., H$ represents the time horizons when I study cumulative effect of government spending. $x$ is the variable of our interest: GDP, consumption, investment and log-difference in real exchange rate. The control vector includes lagged economic fundamentals and fiscal variables,

$$z_{i,t-1} = [g_{i,t-1}, GDP_{i,t-1}, drex_{i,t-1}, CA_{i,t-1}/GDP_{i,t-1}]'$$

which are lagged values of government consumption, GDP, log-difference in real exchange rate, current account-GDP ratio. $\Phi(L)$ is the polynomial of lag operator of degree 4. The endogenous regressor of integral of government spending is instrumented by the extracted residuals from a SVAR regression for each country: $shock_{i,t} = u^9(g_{i,t})$, which is the orthogonalized part of contemporary government spending observations. The major advantage of this regression is that the estimation of $m^{x,h}$ is directly the cumulative multiplier of $x$ at horizon $h$. The details of my econometric specifications are in appendix C.

To study the regime-dependent effect of government spending during and away from sudden stop episodes, I estimate the following non-linear regression:

$$\sum_{j=0}^{h} x_{i,t+j} = I_{i,t-1} \times \left[ c_A + \alpha_{A,i} + \sum_{j=0}^{h} g_{i,t+j} + \Phi_{A,h}(L) z_{i,t-1} \right]$$

$$+ (1 - I_{i,t-1}) \times \left[ c_B + \alpha_{B,i} + \sum_{j=0}^{h} g_{i,t+j} + \Phi_{B,h}(L) z_{i,t-1} \right] + u_{i,t+h}$$

Let $A$ denote sudden stop and $B$ denote normal time. $I_{i,t-1}$ is a dummy variable that indicates whether the shock hits the economy at sudden stop or not. Given everything else the same, now the integrals of government spending are instrumented by $I_{i,t-1} \times shock_{i,t}$ and $(1 - I_{i,t-1}) \times shock_{i,t}$. I concentrate on the estimation of coefficients $m^h_A$ and $m^h_B$ which are the multipliers in corresponding financial regimes. I report the results of both linear and non-linear regressions in table 1.

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8I focus on the response of real exchange rate because that underlies the key mechanism studied in this paper. Following IMV, I use the difference in log real exchange rate in the regression instead of the levels.

9Before running the regression, to identify the government spending shocks, I estimate a four-variable recursively identified SVAR for each country (the same as IMV) and extract the residuals from the first line equation. Those are the estimated government spending innovations (BP shocks) and are used as instruments in my regression model.
Table 1: Spending Multipliers during Sudden Stops and Normal Times

<table>
<thead>
<tr>
<th></th>
<th>Linear Model</th>
<th>Sudden Stops</th>
<th>Normal Times</th>
<th>HAC p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>GDP</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Impact</td>
<td>0.61 (0.15)</td>
<td>0.56 (0.50)</td>
<td>0.58 (0.15)</td>
<td>0.97</td>
</tr>
<tr>
<td>1 year</td>
<td>1.40 (0.21)</td>
<td>1.51 (0.42)</td>
<td>1.29 (0.24)</td>
<td>0.64</td>
</tr>
<tr>
<td>2 year</td>
<td>0.83 (0.18)</td>
<td>1.13 (0.29)</td>
<td>0.70 (0.21)</td>
<td>0.22</td>
</tr>
<tr>
<td><strong>Consumption</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Impact</td>
<td>0.15 (0.13)</td>
<td>0.70 (0.40)</td>
<td>0.02 (0.12)</td>
<td>0.11 *</td>
</tr>
<tr>
<td>1 year</td>
<td>0.83 (0.18)</td>
<td>1.60 (0.48)</td>
<td>0.58 (0.17)</td>
<td>0.05 ***</td>
</tr>
<tr>
<td>2 year</td>
<td>0.67 (0.18)</td>
<td>1.52 (0.42)</td>
<td>0.40 (0.20)</td>
<td>0.02 ***</td>
</tr>
<tr>
<td><strong>Real Exchange</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Impact</td>
<td>3.99 (0.73)</td>
<td>8.67 (2.62)</td>
<td>2.95 (0.57)</td>
<td>0.03 ***</td>
</tr>
<tr>
<td>1 year</td>
<td>1.12 (0.27)</td>
<td>3.20 (0.88)</td>
<td>0.52 (0.24)</td>
<td>0.01 ***</td>
</tr>
<tr>
<td>2 year</td>
<td>-0.21 (0.18)</td>
<td>0.59 (0.49)</td>
<td>-0.48 (0.21)</td>
<td>0.06 **</td>
</tr>
</tbody>
</table>

a. ** *** **** indicates the null hypothesis is rejected at significance level of 20%, 10% and 5%, respectively: $H_0 : m^{A,h} = m^{B,h}$.

b. Before running the regression, I transform the data by taking ratios of each quantity variable over trend real GDP. Therefore, the numbers in the table are directly cumulative multipliers in terms of local currency units.

c. To study the effect on real exchange rate, I use the log difference of real exchange rate index as dependent variable in the regression. The numbers in the table represents the cumulative percentage change in real exchange rate after a government spending shock.

2.2 Estimation Result

The main goal of this paper is to understand the mechanisms behind the spending multiplier effect when sudden stop crisis happens. Accordingly, in table 1 and figure 1, I report the short-run cumulative multipliers in sudden stop times versus in normal times. Special attention is devoted to the response of real exchange rate because it is a main driver behind the supposedly large effect of government spending during sudden stops. In the last column of table 1, the p-values are used to test whether there are significant differences between multipliers in different financial states. Given the possibility of correlated residuals across times and countries, I use the heteroscedasticity and autocorrelation-consistent (HAC) standard error. I get the following key finding of this paper:

RESULT 2.1: Government spending is more effective in stimulating consumption and appreciating real exchange rate when the shock happens during sudden stops than that during normal times.

First of all, a positive government spending shock is shown to crowd-in consumption and output on average. The multiplier effect peaks 4 quarters after the shocks. The real exchange rate also appreciates (by 4% on average) on impact, but the effect decays very fast and turns negative after two years. My finding that an increase in government spending causes real appreciation in developing countries is consistent
government spending is more effective in stimulating consumption when the shock hits the economy during a sudden stop episode. On average, the consumption multipliers are more than doubled in the financially distressful periods than that in normal times, but due to the large error bands, the state-dependence is only 5% significant after the first year. The real exchange rate also appreciates more during sudden stops and the error bands of different financial regimes do not overlap.

However, there is no significant difference in GDP multipliers between financial regimes in all time horizons, even though the average values in sudden stops become slightly higher after the shock hits. This result becomes natural considering the theoretical framework I will develop below. I argue that key sudden stop mechanism is closely linked to the behavior of consumption and real exchange rate. In contrast, production activities are more involved and related to other factors, such as domestic nominal and real rigidities, or monetary policy. Accommodating these elements would complicate the model and blur the main result. So, they are left outside the scope of this paper.

Next, to see how the above result depends on exchange rate policy, I estimate the baseline regression separately for countries classified as pegging/flexible exchange rate\textsuperscript{12}. The result is in figure 2 which makes me draw the following conclusion:

**Result 2.2:** The stronger effect of government spending shock during sudden stop is more pronounced for the countries with pegged exchange rate.

One can see from the middle panel of figure 2 that, on average, the consumption multipliers rise from a value around 0 (0) in normal times to 1 (less than 0.5) in sudden stop times for the pegged (flexible) exchange rate countries. Additionally, the real appreciation effect of government spending during sudden stops is also stronger for peggers than for non-peggers. For output, again, there is no significant difference between financial states in all cases, except for peggers, where the

\textsuperscript{12}The details in running the extended regression are shown in appendix C

\textsuperscript{9}with Miyamoto et al. (2016), since most of my countries belong to their sample.
average impact multiplier of sudden stops is slightly higher than that of normal times.

The comparison of normal time multipliers between pegging and flexible exchange rate countries contradicts the Mundell-Flemming’s intuition. For example, one can see consumption multipliers during normal times in upper and lower panels are almost the same. In contrast, Mundell-Flemming predicts a higher fiscal multiplier under pegging exchange rate policy because that is when the positive shock is accompanied by a more accommodating monetary policy. I abstract from that consideration in this paper. Pegging exchange rate policy indeed increases the efficiency of government spending during sudden stops, but not on average.

Figure 3: Spending Multipliers of Positive and Negative Shocks: Consumption

Note: See the note of figure 1 for details. Red (black) curves are multipliers associated with positive (negative) shocks. The multipliers of positive and negative shocks are defined in the same way. For example, a positive consumption multiplier means that the private consumption declines after a negative spending shock.

Given that I have a non-linear model, it is interesting to analyze the fiscal multiplier effect
in other dimensions, one of which is the asymmetric effect of government spending shock. The motivation is that many countries have been adopting fiscal austerity/consolidation over the past decades, and both the media and policy-makers are curious to know their consequences on domestic market\(^\text{13}\). Therefore, I further extend my baseline regression to account for the sign of government spending shock. Specifically, I divide my sample into periods of “fiscal expansion” and “fiscal contraction” based on whether the extracted SVAR residuals are greater than 0 or not. That results in a regression of two regimes. Then, to examine where the asymmetry comes from, I further augment it with two financial regimes, resulting in a four-regime regression. The result is reported in figure 3 and I conclude:

**Result 2.3:** *The consumption multiplier associated with a negative spending shock is higher that that associated with a positive spending shock, and the difference mainly comes from sudden stop episodes.*

Figure 3 shows the asymmetric effect of government spending on private consumption. In the left panel, the multiplier effect on consumption is larger when it is triggered by a negative shock, although the average difference on impact is small. This later result is natural since we know that only sudden stop periods are the times of great market turbulence where asymmetric effects show up, but sudden stop events are rare in the data. Including financial regimes allows me to prove this point. The middle panel shows when sudden stop happens, a negative shock generates quite a large and persistent multiplier effect on consumption, while the multiplier effect of a positive shock is rather muted. However, during normal times, the asymmetry disappears, as in the right panel. As in Pröbsting et al. (2017), fiscal austerity is usually negative, large, persistent government spending shocks that happen during financially distressful periods. The results shown here lend support to their view that the long-lasting austerity measures are sufficiently contractionary to private activities so that their benefit can be easily overturned. So, in order to evaluate the effect of austerity measures comprehensively, the state-dependence of fiscal multiplier should be taken into consideration.

My last empirical exercise comes from the conjecture that the expectation on future policy conduct might be important for the efficiency of fiscal policy today. I divide my sample into two groups of countries based on the level of spending shock persistence. “High (low) persistence” country is the one with the estimated autocorrelation greater (lower) than a threshold (0.7)\(^\text{14}\). Then I run the baseline regression separately for these two country groups. From figure 4, I get the following result:

**Result 2.4:** *The spending multiplier is higher when the government spending shock is more persistent.*

\(^{13}\)Pröbsting et al. (2017), Castro et al. (2013), and Anzoategui (2016) study fiscal austerity measures for periphery European countries. Pragidis et al. (2017) uses a threshold-VAR model to estimate the asymmetric effect of government spending and its state-contingency across U.S. financial cycle.

\(^{14}\)I choose the threshold of 0.7 to make the two groups of countries roughly having the same sample size.
Note: In the left panel, the red (blue) curve is multipliers for the “high (low) persistence” countries. In the middle and right panels, red (black) curves are the multipliers during sudden stop periods (normal times) for each group of countries respectively.

It is well-documented in the literature (e.g., Auerbach & Gorodnichenko, 2012a; Ramey, 2011) that part of the effect of government spending comes from private agents’ anticipation of future fiscal policy. This may create bias in estimating a fiscal regression without using expectational variables. Here I notice that there is another way expectational effect matters, through financial channel. The intuition is that since the collateral price is determined by the present value of future dividends, a financially constrained economy is more forward-looking and sensitive to future fiscal shocks. Given that a higher government spending increases real exchange rate, a more persistent fiscal expansion creates a larger and more desirable impact on current capital price and allows the agents to better front-load consumption. One can see from figure 3 that the more persistent government spending shock has the higher multiplier effect on consumption on average (left panel). Moreover, the dependence of fiscal multiplier on financial states is also stronger in the countries with the more persistent fiscal policy (middle and right panels).

I check the robustness of my empirical results in appendix C. First, I estimate the model again with all variables denominated in U.S. dollars rather than in local currency units. I find the key result becomes more significant. Second, most authors studying fiscal multipliers by local projection method use the “three-step method” of first estimating impulse response, then constructing fiscal multipliers through Monte Carlo simulation. While I rely on the “one-step regression” throughout the paper, I also repeat my baseline exercise based on the “three-step method” and find the same result. Lastly, because in the regression, I control for the past GDP and government consumption, theoretically the BP shocks $\text{shock}_{i,t}$ could be replaced by the current observations of government consumption $g_{i,t}$. I confirm my result still holds when I directly use $g_{i,t}$ as the instruments for future government consumption.

To sum up, this section displays several non-linear features of fiscal multiplier using the cross-country data. To understand the subdued effect of government spending in the financially distressed economy, in the next section, I start with developing a three-period model and try to explore implications from the optimal spending policy.
3 A Three-Period Model

In this section, I analyze the optimal government spending policy in a three-period model and the implied state-dependence of fiscal multipliers. I use a two sector economy with a collateral constraint on external debt. The domestic capital is considered as the collateral of borrowing, but its total amount is fixed at unity. The only uncertainty is a tradable endowment shock in the second period, which makes the financial constraint occasionally bind. From the second period, the government chooses the optimal level of spending and I assume away availability of capital controls\(^{15}\). However, in order to evaluate the welfare benefit of government spending, I assume macro-prudential tax is available in the first period.

3.1 Competitive Equilibrium

There are two sectors: tradable and non-tradable good sector. The non-tradable goods are produced by labor and capital; the tradable goods are endowed and subject to shocks. To simplify, I assume the labor is only elastically supplied in the second period, and the labor supply is fixed at 1 in the last period. There is a continuum of representative agents/households, with each one of them having the life-time utility of the form

\[
U = \log(c^T_t) + E_1 \left[ \beta \left( \log(c_2) - \frac{\chi}{1 + \nu} + \eta G \log(g_2) \right) + \beta^2 \left( \log(c_3) + \eta G \log(g_3) \right) \right],
\]

where \(1/\nu\) is the Frisch elasticity on labor supply. \(\chi\) and \(\eta G\) are respectively labor disutility and utility weight on public consumption. Both the private and public consumption are composite goods from two sectors\(^{16}\)

\[
c_t = \left( \omega c_t \frac{\theta - 1}{\sigma} + (1 - \omega) c_t \frac{\theta - 1}{\sigma} \right)^{\frac{\theta}{\theta - 1}},
\]

\[
g_t = \left( \omega^G g_t \frac{\theta - 1}{\sigma} + (1 - \omega^G) g_t \frac{\theta - 1}{\sigma} \right)^{\frac{\theta}{\theta - 1}},
\]

for \(t = 2, 3\). \(\omega\) and \(\omega^G\) are the weights on tradable goods in private and public consumption. \(\theta\) is the elasticity of substitution. Notice that the government spending enters utility function in a separable form, which means the spending policy doesn’t affect the private equilibrium directly. As will be shown below, the major role of government spending mainly comes from its general equilibrium effect on asset prices. This section is meant to illustrate how this effect helps in alleviating the severity of financial crisis.

In each period, the households choose consumption, labor supply (only \(t = 2\)) and capital

\(^{15}\)To consider the marginal effect of government spending policy, I assume capital controls are not available from the second period onward. The welfare implication of capital control is well-studied in the literature(e.g. Bianchi & Mendoza (2013)). However, I analyze macro-prudential tax in the first period because that represents the average severity of financial crisis or pecuniary externality.

\(^{16}\)I include government spending into utility function only to study the optimal policy. When solving the model, the government spending under competitive equilibrium is set at its steady state.
acquisition (only \( t = 2 \)). To smooth consumption, they need to borrow from foreign investors. The international credit market is incomplete in the second period in the sense that the total amount of borrowing is restricted by the value of collateral. More specifically, in period 1, the households borrow and make tradable consumption. In period 2, the households pay-off the debt, provide labor for non-tradable good production, and borrow from international markets to finance the consumption and capital investment. Capitals are used for production in the next period and are also taken as collateral. In period 3, the households collect the return on production, clear-off their debt balance, and make consumption. Assume tradable good is the numeriare. Let \( p^N_t \) denote the relative price of non-tradable good. \( q^K_t \) is the capital price and \( R^* \) is the gross interest rate. The households' sequential budget constraints are:

\[
\begin{align*}
    c_T^1 + b_1 &= y_T^1 + \frac{1}{R^*}b_2(1 - \tau_1^{MP}) - T_1, \\
    c_T^2 + p^N_T c_T^N + b_2 + q^K_T (k_3 - 1) &= y_T^2 + p^N_T F(1, l_2) + \frac{1}{R^*}b_3 - T_2, \\
    c_T^3 + p^N_T c_T^N + b_3 &= y_T^3 + p^N_T F(k_3, 1) + q^c k_3 - T_3.
\end{align*}
\]

\{b_{t+1}\}_{t=1}^2 \) denotes the households’ borrowing amount in tradable units and \( k_3 \) is the capital share bought by each individual agent. \( F(k, l) = k^{o_K} l^{o_L} \) is a non-increasing-return-to-scale production function in non-tradable sector. Notice that since \( t = 3 \) is the terminal period, and there is no new borrowing and capital investment anymore. The continuing value of capital \( q^c \) is set at 0 in the numerical exercise below\(^{17}\). I assume in the first period there is a debt tax \( \tau_1^{MP} \) for macro-prudential regulation, as will be discussed later. \( \{T_t\}_{t=1}^3 \) are the lump-sum taxes collected by the government to finance its deficit. Throughout the paper, I assume lump-sum tax/subsidy are always available\(^ {18}\).

In equilibrium, the aggregate amount of capital is fixed at unity \( k_3 = 1 \) and the individual borrowing coincides with the aggregate, \( b_2 = B_2 \) and \( b_3 = B_3 \). Both tradable and non-tradable

\[
\frac{1}{R^*}b_3 \leq \kappa q^K_2 k_3, \tag{7}
\]

where \( \kappa \) is the collateral rate. This constraint says foreign creditors restrict loans so that the total value of borrowings at time 2 does not exceed a fraction of the foreign currency value of households’ end of period capital holdings. A larger value of \( \kappa \) allows the agents to borrow more, thus making the private agents more indebted. Taking initial condition \( b_1 \) and equilibrium prices \( \{p^N_2, p^N_3, q^K_2\} \) as given, the households’ problem is to maximize utility 1 subject to budget constraint 4-6 and the financial constraint 7.

In equilibrium, the aggregate amount of capital is fixed at unity \( k_3 = 1 \) and the individual borrowing coincides with the aggregate, \( b_2 = B_2 \) and \( b_3 = B_3 \). Both tradable and non-tradable

\(^{17}\)This assumption is crucial here. In the three-period model, the continuing value of capital \( q^c \) determines the shape of government spending policies. However, the welfare implication that optimal government spending can improve asset prices and real absorption always holds. I choose \( q^c = 0 \) here because that makes the result of the three-period model consistent with that of the full model.

\(^{18}\)Like in the optimal policy literature, all distortionary taxes considered here are only for corrective purpose.
Here, I intentionally make simplification assumption on the first and third period problem in order to focus on discussion of the financial constraint in the second period. Under these simplifying assumptions, the competitive equilibrium can be defined as:

**Definition 3.1.** Taking initial condition $b_1$ and government’s policy $\{g_2^N, g_2^T, g_3^N, g_3^T\}$ as given, A competitive equilibrium consists of a set of prices $\{p_2^N, p_3^N, q_2^K\}$ and a set of allocation choices $\{c_t^N\}_{t=2}, \{c_t^T\}_{t=1}, \{k_3, B_2, B_3, l_2\}$ such that:

1. Given prices, the consumption, labor, capital and borrowing choices solve the representative agents’ problem.

2. The capital market, bond market, and both the tradable and non-tradable goods market clear.

In order to solve the problem, I take the first order conditions. Since most of the discussions below focus on fiscal policy during financial crisis, first I write the Euler equations in period 2 and 3 as:

\[
\frac{u_{L}(2)}{u_{CT}(2)} = p_2^N F_L(1, l_2), \quad (13)
\]

\[
[u_{CT}(2) - \mu_2] = \beta R^* [u_{CT}(3)], \quad (14)
\]

\[
[u_{CT}(2) - \kappa \mu_2] q_2^K = \beta \left\{ u_{CT}(3) \left[ q^* + p_3^N F_K(1, 1) \right] \right\}, \quad (15)
\]

\[
p_t^N = \frac{1 - \omega}{\omega} \left( \frac{c_t^N}{c_t^T} \right)^{\frac{1}{\theta}}, \quad \text{for} \quad t = 2, 3. \quad (16)
\]

$\mu_2 \geq 0$ is the Lagrangian multiplier on the collateral constraint. The first equation determines the equilibrium labor at time 2. Notice that the labor demand is proportional to $p_2^N$, which means the current real appreciation has a stimulating effect on production. The second one is the inter-temporal borrowing condition. When the financial constraint binds, the increase in $\mu_2$ raises the marginal cost of borrowing. The next two are asset pricing equations that lie in the heart of the key mechanisms discussed in this paper. From the third one, capital price is determined by the future
foreign currency value of dividends discounted by a rate that is proportional to inter-temporal substitution of tradable consumption. \( \mu_2 \) increases the value of capital serving as collateral. The last equation determines consumption allocation between sectors. In equilibrium, non-tradable good price is proportional to the ratio of tradable and non-tradable consumption. The financial amplification effect underlies the interaction between these two equations. An adverse shock in tradable sector decreases the non-tradable good price through (16) and at the same time depresses the capital price through (15). We also have the complementary slackness condition:

\[
\mu_2 \geq 0, \quad \mu_2 \left( \frac{1}{R^*} b_3 - \kappa q^{K}_2 \right) = 0.
\] (17)

At \( t = 1 \), the households only make borrowing decisions, taking future problems as given. The Euler equation is

\[
(1 - \tau^M_1)u_{CT}(1) = \beta R^* E [u_{CT}(2)].
\] (18)

The households choose an optimal level of borrowing so that the marginal benefit equals to the marginal cost. How does the second period problem affect the first period borrowing? In choosing an optimal level of debt, the households are forward-looking and anticipate the probability and severity of debt crisis in the future. So, they are facing a trade-off between front-loading consumption and precautionary saving motive. At the same time, we will show below that, compared to the government’s problem, the private agents fails to internalize the effect of their borrowing decisions on asset prices and are subject to constrained inefficiency.

The solution of competitive equilibrium is characterized by equation 7 to equation 18.

### 3.2 Optimal Government Spending: Discretionary and Commitment

It is well-documented in the literature that a financially constrained economy is subject to pecuniary externalities both before and during crisis. In the above model, a negative \( y^T_2 \) shock at time 2 creates a downward pressure on capital price, making the benefit of borrowing decrease. Since asset prices are aggregate variables, an atomic agent fails to internalize the effect of its borrowing decision on asset prices, and thus the borrowing capacity of others. As a result, allocations of the competitive equilibrium are inefficient. The government’s problem, however, can incorporate this effect. By choosing the right policy instruments (e.g. capital control tax, spending, etc.), the government can correct these externalities and move the equilibrium closer to the constrained-efficient outcome.

In this subsection, I first define the optimal policy, then restrict the problem to the case of only government spending policy available. The problem also depends on whether we allow the period 2 government to commit fiscal policy in period 3. I first consider the discretionary government’s policy and then go to the optimal policy under commitment.

Generally, the discretionary optimal policy can be defined recursively. In each period, the government chooses all current allocations taking private equilibrium conditions and future government’s problem as given. In the example above, let \( V_t(S_t) \) be the value function at beginning of
period \( t \), defined over the state-space \( S_t = (B_t, y_T^t) \). Using letters with \( \sim \) to denote functionals on the state-space, then we have the following definition:

**Definition 3.2.** The **discretionary optimal policy** of the model is defined as value functions \( \{ \tilde{V}_t^r \}_{t=1}^2 \), policy functions of allocations \( \{ \tilde{C}_t^N \}_{t=1}^3 \), \( \{ \tilde{G}_t^N \}_{t=1}^3 \), \( \{ \tilde{G}_t^T \}_{t=1}^3 \), \( \tilde{L}_2 \), \( \{ B_{t+1} \}_{t=1}^2 \) and prices \( \tilde{P}_2^N \), \( \tilde{P}_3^N \), \( \tilde{Q}_2^K \), \( \tilde{\mu}_2^{D, SP} \) such that: for each \( t = 1, 2 \), given future policy and value functions, the current policies solve the following problem\(^{19}\)

\[
\tilde{V}_t(B_t, y_T^t) = \max_{\{c_t^T, c_t^N, y_T^t, B_{t+1}, q_t^K, \tilde{\mu}_t^{D, SP} \}} \left( c_t^{1-\sigma} - \chi \frac{l_t^{1+\nu}}{1+\nu} + \eta_G \frac{g_t^{1-\sigma}}{1-\sigma} \right) + \beta \tilde{E} \tilde{V}_{t+1}(B_{t+1}, y_T^t),
\]

subject to resource constraint of equation 8-12 and implementability constraint of equation 7, 13-18. \( \tilde{\mu}_2^{D, SP} \) is the Lagrangian multiplier on collateral constraint for the discretionary government’s problem.

This formulation of optimal policy follows the “primal approach” with the government choosing allocations subject to implementability constraints and leaving asset prices market-determined. In the numerical solution below, I use the method of a time-consistent social planner’s problem in Bianchi & Mendoza (2013). Bianchi & Mendoza (2013) shows that the complete formulation of optimal policy defined by the “primal approach” is equivalent to the social planner’s “recursive constrained-efficient equilibrium” where redundant constraints are dropped\(^{20}\).

The important implementability constraints here are asset pricing equations 15 and 16. In choosing consumption and borrowing, the government internalizes the effect on \( p^N \) and \( q^K \), thus the financial constraint, making the social marginal value of wealth different from private one. However, whether the optimal allocations can be decentralized depends on policy instruments. Bianchi & Mendoza (2013) shows with debt taxes, the constrained-efficient outcome can be decentralized. However, since I emphasize the net effect of government spending in this paper, I assume the state-contingent tax is not available from the second period. But, from the government’s perspective, the marginal value of wealth is still evaluated at its social level.

One thing to notice is that the capital price (equation 15) is defined in the recursive way: the capital price today depends on future policies and prices. In this simple economy, once the financial constraint is binding in period 2, the government wishes the period 3 government to increase real exchange rate, because this helps the period 2 government increase capital price and relax financial constraint as well. However, when period 3 arrives, the past benefit is sunk and the government does not consider the effect of its decisions on the period 2 problem. Therefore, a time-inconsistency problem arises. The optimal policy of the discretionary government is sub-optimal relative to the solution under commitment. The government is willing to make commitment on future policies to

\(^{19}\)Since there is no financial constraint in period 1 and period 3, the problem could be reduced. But to save space, I set out the problem in general form here.

\(^{20}\)Similarly, Biljanovska (2017) also shows the more and less restricted Ramsey plans are possibly equivalent depending on the set of policy instruments.
alleviate the severity of financial crisis, only it cannot credibly do so since its incentives change once next period arrives. Now, I define the optimal policy under commitment as the following:

**Definition 3.3.** The **optimal policy under commitment** is to choose all allocations \( \{c_i^N\}_{i=2}^3, \{c_i^T\}_{i=1}^3, \{g_i^K\}_{i=1}^3, l_2, \{b_{t+1}\}_{t=1}^2 \) and prices \( \{p_{21}^N, p_{23}^N, q_{21}^K, \mu_{C,SP}^2\} \) at time 1 to maximize the life-time utility 1 subject to resource constraint of 8-12 and implementability constraints 7, 13-18. \( \mu_i^{C,SP} \) is the Lagrangian multiplier on collateral constraint for the commitment problem.

### 3.3 Analysis

#### 3.3.1 Illustration of Period 2

The model described in the previous sections is rich enough to display non-linear feature of fiscal policy. Before going to the solution of optimal policies, I analyze the effect of government spending \( \{g_2^N, g_2^T, g_3^N, g_3^T\} \) on period 2 problem. To provide some intuition, consider the non-tradable good price

\[
p_2^N \uparrow = \frac{w c N(2)}{w c T(2)} = \frac{1 - \omega}{\omega} \left( \frac{y_2^T - y_2^T - b_2 + \frac{1}{\pi} b_3 \uparrow}{y_2^N - y_2^N \uparrow} \right)^{\frac{1}{2}},
\]

where I plug-in the market clearing conditions. The government spending has a direct effect on non-tradable good price, but this effect depends on its composition (or home-bias \( \omega G \)). Giving everything else equal, a higher non-tradable spending (or lower tradable spending) increases \( p_2^N \). The higher \( p_2^N \) pushes-up labor demand and increases non-tradable good production.

In addition to the direct effect, the collateral constraint creates an induced effect through the capital price. It works through two channels. First, because a higher \( p_2^N \) makes the tradable good relatively cheaper, the agent becomes wealthier and the level of final consumption increases, especially when the elasticity of substitution \( \theta \) is low. The higher consumption lowers the rate at which households discount dividends, increasing the capital price (**Wealth Channel**). Second, because the debt is denominated in foreign currencies while the collateral is a domestic asset, the higher \( p_3^N \) increases the value of dividends at time 3 and also boosts-up the period 2 capital price (**Currency Mismatch Channel**). Through these two channels, an increase (decrease) in non-tradable (tradable) government purchase reverses the debt-deflation effect and relaxes the financial constraint when it binds.

Figure 5 illustrates the conditional effect of an increase in \( g_2^N \) or \( g_3^N \) on the period 2 borrowing decisions. If the financial constraint does not bind (left panel), borrowing level is determined by Euler equation 14. The increase in \( g_3^N \) crowds-out \( c_2^N \). Because tradable and non-tradable goods are complements, the marginal benefit of borrowing decreases. As a result, equilibrium borrowing \( (b_3^*) \) is also reduced. However, if the constraint binds (right panel), the level of borrowing is determined by capital price, which in turn depends on borrowing level itself. The increase of \( g_2^N \) and \( g_3^N \) props-up capital price through the channels discussed above, enlarging agents’ borrowing capacity. The endogenous feedback loop is also present. The higher borrowing and the associated real absorption
in turn raise $p^N_2$ and $q^K_2$, which further relaxes the financial constraint. In the end, this turns the vicious cycle into a virtuous one.

### 3.3.2 Result: Period 2 and 3

What does the optimal government spending look like? Firstly, in the problem of discretionary policy, the government chooses the current spending level, only considering its contemporary effect on capital price. The related first order conditions are:

$$\lambda_{2,T}^D = u_{CT}(2) + \frac{\kappa \mu_{SP}^D}{1 - \kappa \mu_{SP}^D} \frac{\partial \beta \Lambda_{2,3}}{\partial c_T^T} \left[p^N_3 F_k(3)\right] = u_{GT}(2)$$

$$\lambda_{2,N}^D = u_{CN}(2) + \frac{\kappa \mu_{SP}^D}{1 - \kappa \mu_{SP}^D} \frac{\partial \beta \Lambda_{2,3}}{\partial c_N^N} \left[p^N_3 F_k(3)\right] = u_{GN}(2)$$

where $\Lambda_{2,3} = \frac{u_{CT}(3)}{u_{CT}(2)}$ is the consumption pricing kernel. Notice that the marginal value of wealth $\lambda_{2,T}^D$, $\lambda_{2,N}^D$ are evaluated from the social perspective, incorporating the externalities on financial constraint. This is different from the marginal utility if $\mu_{SP}^D > 0$. When two goods are complementary enough, $\sigma < \frac{1}{b}$, we have $\frac{\partial \beta \Lambda_{2,3}}{\partial c_T^T} > 0$ and $\frac{\partial \beta \Lambda_{2,3}}{\partial c_N^N} < 0$. This gives the period 2 government an incentive to increase (decrease) non-tradable (tradable) spending when financial constraint is more binding.

Now, consider the optimal policy under commitment. Given that there is no financial constraint in period 1, the period 2 problem under commitment is similar to the discretionary one. However, the period 3 government also internalizes the effect of its decisions on past financial constraint. The
related first order conditions are,

\[ \lambda_{3}^{C,T} = u_{CT}(3) + \frac{\kappa \mu_{2}^{C,SP}}{1 - \kappa \mu_{2}^{C,SP}} \frac{\partial \Lambda_{2,3}}{\partial c_{3}} \left[ p_{3}^{N} F_{k}(3) \right] = u_{GT}(3) \quad (21) \]

\[ \lambda_{3}^{C,N} = u_{CN}(3) + \frac{\kappa \mu_{2}^{C,SP}}{1 - \kappa \mu_{2}^{C,SP}} \frac{\partial \Lambda_{2,3}}{\partial c_{3}} \left[ p_{3}^{N} F_{k}(3) \right] = u_{GN}(3) \quad (22) \]

\( \lambda_{3}^{C,T}, \lambda_{3}^{C,N} \) are the social marginal value of wealth under commitment. The sign of these two externalities is ambiguous. However, with proper parameter values, I show that \( \frac{\partial \Lambda_{2,3}}{\partial c_{3}} < 0 \), which means it is beneficial to commit a higher level of non-tradable spending in period 3 whenever constraint binds in period 2.

Figure 6: The Optimal Government Spending and Financial Constraint

Note: The upper panel shows the equilibrium borrowing and consumption in the second period for the case of competitive equilibrium (black), contemporary optimal spending (blue), and the optimal spending under commitment (red). The lower panel shows the shape of optimal government spending policy across debt space.

The policy functions are displayed in figure 6. First of all, consistent with our intuition, the optimal government spending displays non-linear features both during and after financial crisis. In period 2, the government increases \( g_{2}^{N} \) (decreases \( g_{2}^{T} \)) at high debt states where the financial
constraint is binding. The optimal policies enables the government to relax the financial constraint and increase borrowings (upper left panel). The upper right panel of figure 6 shows that even in this three-period model, the financial channel is strong enough so that the final consumption is crowded-in. Secondly, the government under commitment also increases $g_3^N$ at high debt states, which further relaxes the financial constraint. The commitment effect of $g_T^3$ is quite small.

One thing to notice is that government spending in the model is relatively wasteful. The major role of fiscal policy comes from its impact on asset prices. However, whether an increase in asset price can stimulate real activities depends on whether financial constraint binds or not, which implies the state-dependence of fiscal stimulating effect.

### 3.3.3 Result: Period 1

How does the presence of optimal spending during crisis affect borrowing in the first period and the welfare? In the optimal policy problem, the period 1 borrowing is determined by the following Euler equation

$$\frac{1}{R}u_{CT}(1) = \beta\mathbb{E}\left[u_{CT}(2) + u_{CT}(2)\frac{\kappa \mu_2^{SP}}{1 - \kappa \mu_2^{SP}} \frac{\partial \beta \Lambda_{2,3} \left[p_3^N F_k(3)\right]}{\partial c_2^T} \right].$$

First, private agents fail to consider the social cost of borrowing in inviting greater likelihood of financial crisis tomorrow, which is the emphasized over-borrowing phenomenon in the literature (e.g., Bianchi, 2011). Second, with the proper fiscal policy managing the crisis *ex post*, the financial crisis becomes less costly and the precautionary motive in the first period is even smaller compared to the case with no *ex post* policy. Moreover, with different policy instruments available *ex post*, the borrowing externality in period 1 is also different. These incentives are illustrated by a comparison analysis of the marginal benefits and marginal costs of borrowing, as shown in appendix E.3.

Table 2 shows the equilibrium borrowing and welfare. Consistent with the intuition, the optimal government spending mitigates the pain of financial crisis, reducing the precautionary motives. As a result, the agents borrow more under optimal policies, followed with the higher probability of crisis. The macro-prudential tax is a measure to represent the pecuniary externality and so that depends on both probability and severity of crisis. From the table, even when the economy is more indebted under the optimal spending policies, the macro-prudential tax rate is still smaller. The optimal spending policies generally increase private welfare. The welfare benefit mainly comes from two reasons: the smaller precautionary saving motives let agents more able to front-load consumption; at the same time, the cost of crisis is alleviated by *ex post* policy.

---

21Benigno et al. (2016) explains how the presence of different *ex post* policy can affect the role of *ex ante* policy.
Table 2: Three-Period Model

<table>
<thead>
<tr>
<th></th>
<th>C.E. with $\tau MP$</th>
<th>Optimal $G_2$</th>
<th>Optimal $G_2, G_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^*_2$</td>
<td>2.2713</td>
<td>2.2739</td>
<td>2.2744</td>
</tr>
<tr>
<td>Prob. of Crisis ($P(\mu_2 &gt; 0)$)</td>
<td>0.52</td>
<td>0.54</td>
<td>0.54</td>
</tr>
<tr>
<td>Sev. of Crisis ($E\mu_2$)</td>
<td>0.0519</td>
<td>0.0488</td>
<td>0.0484</td>
</tr>
<tr>
<td>$\tau MP$</td>
<td>0.0237</td>
<td>0.0221</td>
<td>0.0216</td>
</tr>
<tr>
<td>$V_1$</td>
<td>-1.2396</td>
<td>-1.2366</td>
<td>-1.2375</td>
</tr>
</tbody>
</table>

a. C.E. means the competitive equilibrium with government spending fixed at its steady state. But, in order to explore the net welfare benefit of government spending policy, I assume there is always an optimal capital control (state-contingent tax) on debt in the first period. So, the increase in equilibrium borrowing indicates the cost of crisis becomes smaller when government spending is used to manage crisis after it happens.

b. To be consistent with the full model where government spending is wasteful, here I consider the welfare implication of government spending on private agents. $V_1$ means the life-time welfare from private utilities, which is

$$V_1 = \log(c^T_1) + E_1 \left[ \beta \left( \log(c_2) - \chi \frac{\tau^1 + \nu}{1 + \nu} \right) + \beta^2 \log(c_3) \right].$$

That explains why the welfare under commitment is smaller than the one under discretionary policy.

### 3.4 Quantifying the Static Multipliers

Given the channels discussed above, it is natural to explore how the spending multiplier depends on the states of the economy. In this section, I treat the government spending as exogenous and in fixed proportion between the two sectors. I define the static multiplier as: $\frac{\Delta x_2}{\Delta g_2}$, where $x_2$ is the variable of our interests. Figure 7 shows the multiplier on tradable consumption $c^T_2$ across parameter spaces. The multipliers of other endogenous variables are shown in appendix E.3.

First, we know from the optimal policy analysis that a promised fiscal expansion in the future relaxes the current financial constraint and enhances real activities. So, the higher shock persistence (larger $\rho_G$) implies the higher multiplier on $c^T_2$. Second, since the financial effect of government spending works through non-tradable good price, so parameter values making $p_{2N}^N, p_{3N}^N$ more sensitive to government spending shock can potentially generate higher multipliers. The lower elasticity of substitution (smaller $\theta$) makes the wealth effect stronger under a positive shock, thus increasing fiscal multipliers during crisis. Greater home-bias of government consumption (smaller $\omega_G$) creates a larger increase in $p_{2N}^N$ for a positive shock of the given size, increasing the multipliers. Lastly, the smaller size of tradable sector (smaller $\omega$) also implies the higher multipliers when the constraint binds.

In summary, this section studies the state-dependent effect of government spending in an optimal policy environment. First, with enough home-bias and large degree of complementarity, the government spending is more stimulative during financial crisis than that during normal times. Second, under policy commitment, the promise in future fiscal policy has an expansionary effect on current activities.
Figure 7: The Spending Multiplier and Financial Constraint

Note: The simulated multipliers here $\frac{\Delta c_t}{\Delta g_t}$ depend on the starting point in period 2: $(b_2, y_T^g, g_2)$. Suppose the non-binding (binding) state is $y_T^g = 1.01$ ($y_T^g = 0.99$) while $b_2$ and $g_2$ are kept at their middle values. The benchmark parameter values are $\rho_G = 0.95$, $\theta = 0.45$, $\omega^G = 0$, $\omega = 0.35$ respectively, and I change one parameter at a time. I assume government purchases are allocated to tradable and non-tradable sectors in fixed proportion: $g_t = \max\{\frac{\omega}{1-\omega}g_{Tt}^N, \omega g_{Nt}^G\}$ ($t = 2, 3$), and they follow the process: $g_3 = \bar{g}^{1-\rho_G}g_2^{\rho_G}$ where $\bar{g}$ is the steady state.

4 Full Model

This section extends the three-period model to an infinite horizon environment. There are two differences: 1.) I impose a working-capital constraint on the firms’s problem as in E. G. Mendoza (2010) so that the financial condition has a direct impact on domestic production; 2.) The firms’ labor demand is subject to the DNWR in the spirit of Schmitt-Grohé & Uribe (2016). This way to model the labor market friction is meant to capture the observation of cyclical unemployment in SOE financial crisis and I use that to consider my government spending channel under different exchange rate systems.\(^{22}\) In the quantitative analysis below, demand channel and financial friction

\(^{22}\)Many papers use this form of one-side rigidity to account for the asymmetry over the business cycle (e.g., Félix et al., 2013; Ottonello, 2012). It is motivated by the fact that crisis scenario usually embodies a temporary increase in both the financial frictions and nominal frictions.
are two important factors to explain the business cycle-dependent multipliers, and I study the interaction of these two channels and their relative importance. The wasteful government spending follows a fiscal rule with a part of an exogenous shock. There are three agents in the economy: an infinitely lived firm-household makes consumption and production decisions; the government conducts fiscal policy; the central bank makes exchange rate policy. The same as before, the international debt market is incomplete.

4.1 Firm-Household

The representative firm-households have life-time utility:

\[
E_0 \left[ \sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \right],
\]

where \(c_t\) denotes the final consumption basket as in equation 2. \(E_t\) is the expectation operator conditional on information available at time \(t\). I assume the per-period utility function takes the separable form

\[
u(c_t, l_t) = c_t^{1-\sigma} - 1 \frac{1}{1 - \sigma} - \chi \frac{l_t^{1+\nu}}{1 + \nu}.
\]

\(\sigma\) is the relative risk aversion. \(\chi\) is the weight on labor disutility and \(\nu\) denotes the labor elasticity.

The firm-households are consolidated agents with separable decision-making incentives based on their social status. One hand, the households earn wage income, get tradable endowment, and consume tradable and non-tradable goods. They borrow in the international debt market to smooth consumption. On the other hand, the firms hire labor, make production and investment, and issue working-capital loan. The non-tradable good production follows

\[
y_t^N = z_t F(k_t, l^d_t),
\]

where T.F.P. process \(z_t\) is an exogenous AR(1) process

\[
\log(z_t) = \rho_z \log(z_{t-1}) + \epsilon_{z,t},
\]

with i.i.d. shock \(\epsilon_{z,t} \sim N(-\frac{1}{2}\sigma_z^2, \sigma_z^2)\). To account for disequilibrium in the labor market, I use \(l^d_t\) to denote the labor demand of firms, which could be different from the labor supply \(l_t\), as specified below. The production function is non-increasing-return-to-scale: \(F(k_t, l^d_t) = k_t^{\alpha_K} l^d_t^{\alpha_L}\), with \(\alpha_K \geq 0, \alpha_L \geq 0\) and \(\alpha_K + \alpha_L \leq 1\).

The labors and capitals are owned by the households. The households borrow from foreign investors by issuing short-term bonds, denominated in foreign currency units. The gross interest rate is \(R^*\). The budget constraint denominated in local currencies is given by

\[
E_t c_t^T + P_t^N c_t^N + Q_t^K (k_{t+1} - k_t) + E_t b_t+ = E_t y_t^T + W_t l_t + R_t^K k_t + \frac{1}{R_t^*} E_t b_{t+1} - E_t T_t + E_t \pi_t.
\]
$E_t$ denotes the nominal exchange rate. $P_t^N$ is the nominal non-tradable good price. $Q_t^K$ is the nominal price of capital. $W_t$ and $R_t^K$ are nominal wage rate and rent of capital. $T_t$ is the lump-sum tax/subsidy collected by the government. I assume firm’s profit $\pi_t$ is rebated back to households in a lump-sum fashion. The tradable endowment $y_t^T$ follows an AR(1) process

$$\log(y_t^T) = (1 - \rho_y^T)\bar{y} + \rho_y^T \log(y_{t-1}^T) + \epsilon_{y^T,t},$$

(28)

with i.i.d. shock $\epsilon_{y^T,t} \sim N(-\frac{1}{2}\sigma_y^2, \sigma_y^2)$. The domestic average price level is defined as

$$P_t = \left(\omega E_t^{1-\theta} + (1 - \omega) P_t^N(1-\theta)\right)^{\frac{1}{1-\theta}}.$$

(29)

Here, I assume the foreign price level is constant $P_t^* \equiv 1$. Define the real exchange rate as the price of final consumption good in terms of tradables. Then, the real exchange rate equals nominal exchange rate adjusted by the relative price levels

$$rex_t = \frac{1}{E_t} P_t = \left(\omega + (1 - \omega) P_t^N(1-\theta)\right)^{\frac{1}{1-\theta}},$$

(30)

where $p_t^N = \frac{P_t^N}{E_t}$ is the real price of non-tradable good. Note that the real exchange index $rex_t$ is proportional to $p_t^N$. This makes it convenient to use tradable good as numeraire and re-express the budget constraint as

$$c_t^T + c_t^N + q_t^K(k_{t+1} - k_t) + b_{t+1}^+ = y_t^T + w_t l_t + r_t^K k_t + \frac{1}{R_t} b_{t+1} - T_t + \pi_t,$$

(31)

where I use the lower case to denote the relative prices w.r.t. tradable units, i.e. $w_t = \frac{W_t}{E_t}$, $r_t^K = \frac{R_t^K}{E_t}$, $q_t^K = \frac{Q_t^K}{E_t}$.

The financial market is incomplete and the total credit available is restricted by the value of collateral. There are two types of credit: bond issuance and working capital loan. The bond is due after one period; the working-capital loan is an intra-period contract and does not bear any interest. The form of the collateral constraint is

$$\frac{1}{R_t} b_{t+1} + \phi w_t l_t \leq \kappa q_t^K k_{t+1}.$$

(32)

The parameter $\kappa$ governs the maximum leverage of the economy. In the solution part below, I used this parameter to target the country’s indebtedness. Before making production, the firms are supposed to prepay $\phi$ fraction of the wage bill out of the balance sheet.

---

23 This is different from the common practice in the literature that real exchange rate is the relative price of foreign numeraire in terms of local consumption goods. In my definition, the higher value of $rex_t$ means local currency appreciation.

24 In the working capital constraint of Neumeyer & Perri (2005), hiring factor inputs carries a financing cost determined by $R_t^*$ and thus the decision responds to interest rate shocks. I assume away that part to simply the problem and to isolate the direct effect of asset price fluctuation on productions.
Similar constraints have been used by many papers in the small open economy environment, such as E. G. Mendoza (2010) and Fornaro (2015). In this paper, I do not explicitly derive the credit constraint as the outcome of an optimal contract between lenders and borrowers. But, note that this collateral constraint is meant to capture in reduced form an environment in which informational and institutional frictions affect the credit relationship between domestic and foreign agents. In economic downturns, the depreciating capital price restricts international borrowing and makes the private agents reduce their consumption level. The lower aggregate consumption further reduces the capital price and sets-in the Fisher’s debt-deflation channel, the same as before. From the qualitative side, the severe economic recession driven by asset price deflation is a favorable feature of this model which provides a good laboratory to study sudden stop behavior of small open economies. From the theory side, we also note that this environment incorporates two types of pecuniary externalities emphasized by Dávila & Korinek (2017), since the agents take aggregate prices (such as $w_t$, $p_t^N$ and $q_t^K$) in the budget and collateral constraint as given, but ignore the effect of their individual decisions on these prices and how that changes the feasibility set of others.

Given prices and shocks, the household’s problem is to choose $\{c_t^T, c_t^N, l_t, k_{t+1}, b_{t+1}\}$ maximizing the life-time utility $u$ subject to $31$ and $32$. The firm’s problem is to maximize its profits:

$$\pi_t = \max \left\{p_t^N z_t F(k_t, l_t^d) - w_t l_t^d - r^K_t k_t\right\}. \quad (33)$$

The model is solved by taking first order conditions. The relative consumption demand is determined by

$$p_t^N = \frac{1 - \omega}{\omega} \left(\frac{c_t^T}{c_t^N}\right)^{\frac{1}{\theta}}. \quad (34)$$

The bond Euler equation is given by

$$\frac{1}{R_t^u} u_{CT}(t) (1 - \mu_t) = \beta \mathbb{E} [u_{CT}(t+1)], \quad (35)$$

where $\mu_t$ is the Lagrangian multiplier on the collateral constraint. The left-hand side denotes the marginal utility of borrowing one unit of tradable good in increasing current consumption. If the collateral constraint does not bind ($\mu_t = 0$), this is equated to the expected utility of repaying foreign currency bonds and sacrificing consumption in period $t+1$. When the financial constraint binds ($\mu_t > 0$), the household is not free to borrow as much as it would like to and the marginal benefit of borrowing falls short of the marginal cost. Also note that this term can distort the household’ decisions at time $t$ even if the constraint is not binding at time $t$ but there is a positive probability that it binds in $t+1$. That results in a lower tradable consumption (i.e., stronger precautionary saving) than otherwise in economy without borrowing constraint.

The capital Euler equation is

$$u_{CT}(t) (1 - \kappa \mu_t) q_t^K = \beta \mathbb{E} \left\{u_{CT}(t+1) \left[q_{t+1}^K + p_{t+1}^N z_{t+1} F_k(k_{t+1}, l_{t+1}^d)\right]\right\}. \quad (36)$$
The capital price is determined by future values of dividends, discounted by subjective discounting factor. The right-hand side shows the return on capitals depends on the value of marginal product of capital and its re-selling price, which in turn relies on future price of $p_{t+1}^N$ and $q_{t+1}^K$. First, under the occasionally binding financial constraint, the capitals serve as collateral increasing the marginal benefit of buying the asset. Second, due to the currency mismatch, the fluctuation of future non-tradable good prices, not the current one, affects the current capital price. But since the current non-tradable good price is determined by supply and demand, it is also influenced by the current capital price. This creates the time-inconsistency problem, as discussed in the three-period model. Committing to the fiscal policy that appreciates local currency in the future can help the government win more financial flexibility today.

The relation between the financial constraint and asset prices can be examined by further exploring these equations. Define the rate of return on capital as $R_{t+1}^q = q_{t+1}^K + p_{t+1}^N z_{t+1} F_t(k_{t+1}, l_{t+1}) q_{t+1}^K$. Using equation 35 and 36, the expected excess return (i.e. equity premium) on capital can be written as

$$R_{t+1}^{EP} = E_t [R_{t+1}^q - R_t^*] = \frac{(1 - \kappa) \mu_t - cov(\Lambda_{t,t+1}, R_{t+1}^q)}{E_t \Lambda_{t,t+1}}.$$

$\Lambda_{t,t+j} = \beta^{uCT(t+j)} uCT(t)$ is the $j$-period stochastic discount factor. The first term increases when the current financial constraint becomes more binding. For the second term, the expectation of a binding collateral constraint suppresses rate of return on capital and pushes up the discounting factor. So, the covariance is more negative in economic downturns. Since $R_{t+1}^{EP}$ is a good proxy for the financial conditions of the economy, in the numerical exercise below, I use that to compare its counterpart in the data.

Combining labor demand and labor supply, we have

$$w_t^* = \frac{u_L(c_t, l_t)}{u_{CT}(c_t, l_t)} = \frac{p_t^N F_t(k_t, l_t)}{1 + \phi \mu_t}. \quad (37)$$

$w_t^*$ is the effective wage earned by the households. The financial constraint has both direct and indirect effect on labor hiring, and thus output production. When the financial constraint binds, $\mu_t$ goes up and this makes it more costly for the firms to pledge working capital. As a result, the effective wage earned by the households goes down. Apart from that, the indirect effect comes from the endogenous change of $p_t^N$. Upon sudden stop of capital inflows, the reduction of real absorption ($c_t^T$) depreciates local currency ($p_t^N$), suppressing labor demand and production as well.

### 4.2 Downward Nominal Wage Rigidity

Following Schmitt-Grohe & Uribe (2016), I allow the nominal wage to be potentially downward rigid. Formally, nominal wages are required to satisfy the following condition:

$$W_t \geq \gamma W_{t-1}. \quad (38)$$
where $\gamma$ controls the degree of wage rigidity. Simple manipulation implies that real wage follows,

$$
w_t \geq \gamma \frac{w_{t-1}}{\epsilon_t}, \tag{39}
$$

where $\epsilon_t = \frac{E_t}{E_{t-1}}$ is the growth rate of nominal exchange rate. One can see that current real wage depends on last period wage state and exchange rate policy if this constraint binds, or labor market equilibrium otherwise. The presence of downward wage rigidity implies that a negative international shock depresses labor demand and could lead to involuntary unemployment ($U_t$). In the model, the shocks driving the economy into unemployment crisis are almost the ones causing financial crisis. So, in the downturn, these two types of crisis usually happen at the same time and interact with each other, as will be illustrated below. Moreover, the mere possibility that wage constraint is binding tomorrow can change the effect of spending policy today. In normal times, a higher government spending pushes up wage rates and increases the probability of unemployment crisis in the next period. Since the capital price is defined recursively, the expected decline in production will make the collateral constraint more binding today. When $\gamma = 0$, the model has a flexible real wage.

### 4.3 The Government Spending and Exchange Rate Policy

The government spending policy follows a Taylor type of feed-back rule:

$$
\log \frac{g_t}{\bar{g}} = \rho^G g_{t-1} + \gamma^Y \log \frac{gdp_{t-1}}{gdp} + \gamma^B \log \frac{b_{t-1}/gdp_{t-1}}{b/gdp} + \epsilon^G_t. \tag{40}
$$

The real value of GDP in the model is $gdp_t = y_t^T + p_N^T y_N^T$. $\epsilon^G_t \sim N(-\frac{1}{2} \sigma^2_G, \sigma^2_G)$ is a normally distributed fiscal shock. $\rho^G$ indicates the persistence of spending policy. I will investigate the fiscal multiplier for different levels of $\rho^G$ below.

For simplicity, I assume government consumption is allocated to tradable and non-tradable sector in fixed proportion,

$$
g_t = \min \left\{ \frac{g_t^T}{\omega^G}, \frac{g_t^N}{1 - \omega^G} \right\}, \tag{41}
$$

where $\omega^G$ is the weight on tradable goods. Then, the relative price of government consumption is $p_t^G = \omega^G + (1 - \omega^G)p_N^T$.

The feed-back rule includes past GDP and external debt to account for systematic changes of fiscal policy. For example, automatic stabilizers built-in the advanced economies indicate the counter-cyclical fiscal policy ($\gamma^Y < 0$) that the government systematically reacts to low GDP by providing stimulus. On the other hand, the fiscal policy in emerging economies is found to be pro-cyclical. Also, recently, many advanced economies are required to initiate fiscal austerity programs in order to keep their government budget sustainable. $\gamma^B < 0$ is meant to capture that the government conducts spending cuts during high external debt episodes. For the countries I study in this paper, these two concerns coexist and I calibrate the government spending rule using
Mexican data.

To examine whether the mechanisms apply for countries under different exchange rate regimes, I extend the model to include the flexible exchange rate policies as the following:

$$\epsilon_t = \max \left\{ \frac{\gamma w_{t-1}}{w_t}, 1 \right\}^{\gamma_E},$$

(42)

with $\gamma_E \in [0, 1]$. $w_t^f$ is the wage rate at the full-employment status. $\gamma_E = 0$ implies a strict nominal peg (as “peg” below), while $\gamma_E = 1$ is the full-employment exchange rate policy (as “flex” below). In practice, the exchange rate policy in most countries lies between these two extremes and varies over time\textsuperscript{25}. In the baseline analysis, I compare the multipliers in the two extreme cases, and I check for other intermediate values of $\gamma_E$ in the sensitivity analysis.

4.4 Competitive Equilibrium

In the baseline model, I assume the aggregate stock of capital equals to unity: $K_t = 1$. The labor market condition is $l_t = l_t^d + U_t$, which is labor supply equals to labor demand plus involuntary unemployment. Taking the firm’s profits of equation 33 into and the household’s budget constraint 31, I get the resources constraints

$$c_t^T + g_t^T + b_t = \frac{1}{R^s} b_{t+1} + y_t^T,$$

(43)

$$c_t^N + g_t^N = y_t^N.$$  

(44)

Then, I define the competitive equilibrium as the following:

**Definition 4.1.** Given \{K_0, B_0, R_0, y_t^T, z_t, g_t\}_{t=0}^{\infty}, a sequence of exogenous process \{y_t^T, z_t, g_t\}_{t=0}^{\infty}, the competitive equilibrium of the full model consists of a sequence of allocations \{C_t^T, C_t^N, Y_t^N, L_t, L_t^d, B_t+1, K_t+1\}_{t=0}^{\infty} and prices \{w_t, q_t^K, p_t^N, \epsilon_t\}_{t=0}^{\infty}, such that (a) The firm-household solves the maximization problem 24 subject to 31, 32 taking prices and policies as given, (b) The DNWR constraint 39 holds, (c) The government sets fiscal policy based on the rule 40, (d) The central bank sets exchange rate policy based on 42, (e) The labor and capital market satisfy: $l_t = l_t^d + U_t$ and $K_t = 1$. The tradable and non-tradable goods market clear.

The solution of the model is characterized by the first order conditions, resources constraints, exogenous process of shocks, and two inequality constraints. The full description of model equations is in appendix A.

\textsuperscript{25}Ilzetzki et al. (2013) concludes fiscal multipliers are higher for the countries under fixed exchange rate system than those under flexible exchange rate system.
4.5 Analysis: The Fisherian Deflation and Unemployment Crisis

Before proceeding to the numerical results, it is useful to understand the interaction between financial constraint and DNWR, and build some intuition on the effect of government spending in this environment. How does the financial constraint affect involuntary unemployment? Is the government spending effect larger or smaller under DNWR?

The same as before, under a negative shock, the households’ borrowing is constrained and the tradable consumption declines. The capital return is discounted at a higher rate, depressing capital price and makes the financial constraint even tighter. The amplification mechanism creates a cycle of falling capital prices, lower borrowing, and more binding collateral constraint. At the same time, the declining real absorption also pushes down the exchange rate \( p^N_t \). So, the financial crisis is usually accompanied by local currency depreciation.

Figure 8: Occasionally Binding Constraints and Labor Market Equilibrium

How is the production determined under DNWR? To see this, combine equation 39 and 37 to write the labor market equilibrium as

\[
p^N_t \downarrow F_t(k_t, l^d_t, \downarrow) \geq 1 + \phi \mu_t \uparrow \gamma w_{t-1}.
\]

(45)

The binding financial constraint makes \( \mu_t \) increase and \( p^N_t \) decrease. When the real wage is rigid from downward, the labor is demand-determined and the higher-than-the-market effective wage induces firms to reduce labor-hiring. As a consequence, labor demand falls short of labor supply and the economy is driven into involuntary unemployment. The mechanism is illustrated in the right panel of figure 8. The choice is made at point A after a negative shock on \( y^T_t \). The left panel describes the binding areas of the financial constraint and wage constraint across the relevant state spaces. Notice that both constraints are more easily to get binding in bad states when debt levels are high or tradable endowments are low.

What is the effect of government spending during the “twin crisis”? Rewrite the non-tradable
citation: 26↑ means direct effect, and ↓ means induced effect.
The same as before, an increase in $g_t^N$ has both direct and indirect effects (through $b_{t+1}$) on $p_t^N$, relaxing the financial constraint. The difference here is with DNWR in place, government spending has a stronger stimulative effect on $y_t^N$ production, lifting the labor choice from point A to point B in the right panel of figure 8. The expansionary effect on $y_t^N$ is large enough so that the increase in $p_t^N$ is partially offset by the overproduction. Notice that this offsetting channel is even stronger with the higher degree of complementarity (lower $\theta$). Therefore, the presence of wage rigidity weakens the financial channel of government spending when the two constraints bind at the same time. Overall, as we can see in the quantitative analysis below, fiscal expansions can always mitigate the financial crisis by appreciating real exchange rate and capital price, but degree of that depends on whether DNWR is binding and the accompanied exchange rate policy.

4.6 Solution Method

Since there are two occasionally binding constraints in the model and my goal is to analyze the state-dependent effect of government spending policy, I use global solution method. More specifically, I use collocation method with time iteration on Euler equations. The policy functions are approximated by multilinear functions on the discretized state-space. Linear interpolation is good to deal with kinks near the constraint region. The high dimension expectations are evaluated by the monomial quadrature. Starting from a set of initial guess of policy functions, I calculate the law of motion for states and use that to get conditional expectations. Then, at each state point, a system of equations are solved simultaneously by a Newton type solver and the policy functions are updated accordingly. In the next iteration, I use these new policy functions to evaluate the variables at $t+1$. I iterate on these policies until the difference between successive iteration is small enough.

To deal with the financial constraint, I use the “guess and verify” procedure as in Bianchi & Mendoza (2013). In each iteration, I first solve the model without the financial constraint, and then check whether the financial constraint is violated. If it is not, go to the next grid point. Otherwise, solve the model again assuming the financial constraint is binding. For DNWR, I create an auxiliary variable and also approximate the policy function of that. The computational details can be found in appendix B.

5 Calibration and Model Fit

The calibration strategy follows E. G. Mendoza (2010) and M. E. G. Mendoza & Bianchi (2011). The model is calibrated to a two-sector production economy using Mexico 1993Q1 - 2015Q4 data. On the production side, I use the labor share in their paper so that $\alpha^L = 0.64$. The output elasticity
of capital $\alpha_K = 0.05$ is taken from M. E. G. Mendoza & Bianchi (2011). On the preference side, relative risk aversion is set to a standard value of $\sigma = 1$. The elasticity of substitution $\theta$ is a key parameter of this paper because it governs the fluctuation of real exchange rate. E. G. Mendoza (2005) reports a wide range of estimates from 0.4 to 0.83, and I use the medium value of 0.65. The share of tradable goods in the consumption basket $\omega$ is set to match the average output ratio between two sectors, $\frac{y^T}{p^T y^N} = 0.614$, using the value-added data at annual frequency from World Development Indicators. Frisch elasticity parameter $\nu$ is given a standard value of 1 and $\chi$ is chosen to normalize the steady state labor.

The interest rate is set to the historical average of government bond rate $R^* = 1.02$. $\phi = 0.15$ so that the average working capital-GDP ratio equals to 9%, as in M. E. G. Mendoza & Bianchi (2011). The historical average of government spending to GDP ratio is 11.7%. The home-bias $\omega_G$ is an important parameter but difficult to find from data. E. G. Mendoza (2002) uses a shorter sample of sectoral data to get the tradable share of government purchase as 7.2% while other papers (e.g., Anzoategui, 2016; Bianchi et al., 2016) simply assume $\omega_G = 0$. I follow the later assumption. The degree of wage rigidity $\gamma = (0.99)^{1/4}$ follows Schmitt-Grohé & Uribe (2016). The remaining six parameters: the subjective discounting parameter $\beta$, collateral rate $\kappa$, standard deviation and persistence of structural shocks $\sigma_z$, $\sigma_y^T$, $\rho_z$, $\rho_y^T$ are jointly calibrated to match data moments: average debt service-GDP ratio, average equity premium (or sudden stop probability), the standard deviation and the first order auto-correlation of GDP and real exchange rate. Table 3 summarizes the calibrated parameters and the ones used to target data moments.

Because most of the external debt of Mexico have long maturities, but my model only has short term debt, I use the debt service rather than total external debt balance in the calibration. In a debt constrained economy, the debt service ratio better reflects the overall financial burden. The average debt-service to GDP ratio for Mexico is 20.77%. Many related papers (e.g., Schmitt-Grohé & Uribe, 2017) calibrate the process of sectoral shocks by directly running regressions on the sectoral outputs. Given that I have a production economy and this paper mainly focuses sudden stop crisis that is closely related to real exchange rate, I calibrate the exogenous process to match the statistics of real GDP and real exchange rate instead.

The fiscal rule of the government is estimated using a longer data sample from 1981 to 2016. The estimation result is

$$\log(g_t) = 0.96*** \log(g_{t-1}) - 0.02 \log(gdp_{t-1}) + 0.02 b_{t-1}/gdp_{t-1} + \epsilon^g. \quad (47)$$

---

27Since capital in the model is in fixed supply, I cannot set the capital share to the standard value of 1/3, which is based on capital income accrued to the entire capital stock. Instead, M. E. G. Mendoza & Bianchi (2011) sets $\alpha_K$ so that the model matches an estimate of fixed capital to GDP ratio based on the value of the housing stock.

28Generally consistent with the existing papers, the tradable sector is defined as manufacturing industry, agricultural and natural resources. I define the rest part of GDP as non-tradable sector.

29There is no agreement in the literature about how much proportion of the real exchange rate volatility is due to the changes of non-tradable good price. E. G. Mendoza (2005) reports that when Mexico was under a managed exchange rate regime, the fluctuations of its non-tradable good price account for between 50% and 70% of the change in Mexico-U.S. of CPI-adjusted real exchange rate. Here I use the lower bound of 50%.

30The external debt statistics comes from International Investment Portfolio database, where I add “debt security”, “debt instrument”, and “other debt instrument” together to get the number of external balance.
We can see that government spending process is highly persistent and both the lagged GDP and debt-GDP ratio are statistically insignificant. For this reason, in the numerical exercise below I assume $g_t$ follows an exogenous AR(1) process.

### Table 3: Parameter Values

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Description</th>
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<tr>
<td>From literature or simple moment match:</td>
<td></td>
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<tr>
<td>$\sigma$</td>
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<td>$\nu$</td>
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<td>The relative sector size</td>
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<td>$\alpha_l$</td>
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<td>$\gamma$</td>
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<tr>
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<td>Debt service-GDP ratio</td>
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<tr>
<td>$\sigma_y^{TE}$</td>
<td>0.023</td>
<td>REX standard deviation</td>
</tr>
<tr>
<td>$\rho_y^{TE}$</td>
<td>0.81</td>
<td>REX auto-correlation</td>
</tr>
</tbody>
</table>

### 5.1 Model Fit

Table 4 shows the data moments and their model counterparts when I stimulate the economy for 1,000,000 periods, discarding the first 10,000 periods. The baseline economy assumes the flexible exchange rate policy, while I also report the result of pegged exchange rate for comparison. The model does a decent job fitting the selected moments, especially for the average debt-service ratio and auto-correlations. Moreover, the shock of this size is able to generate sudden stops crisis with the correct frequency. As for other statistics, notice that the model is able to capture some distinctive features of small open economies: private consumption is more volatile than output, trade balance is counter-cyclical and consumption is highly correlated with output.

Aside from that, my model also reproduces a counter-cyclical spread and a pro-cyclical real exchange rate$^{31}$ that are widely observed among emerging market economies with sudden stop risk. When the economy is hit by a negative shock, the indebted agents deleverage and decrease consumption, depressing the capital price. If the shock is large enough, the financial constraint gets binding. The risk premium rises up to compensate for the loss on capital investment. At the same time, the decrease in tradable consumption suppresses the effective wage rate. If real wage is rigid from downward, large wage deflation creates involuntary unemployment. In general, the model

---

$^{31}$The unemployment rate is also counter-cyclical in the model under pegged exchange rate with $\rho(U, y) = -0.80$. 

33
provides a good environment to account for important features of sudden stop prone economies: deleveraging, currency depreciation, rising risk premium, and current account reversals.\(^{32}\)

### Table 4: Model Fit

<table>
<thead>
<tr>
<th>Targeted Moments:</th>
<th>Mexico 1993Q1-2015Q4</th>
<th>Baseline Model Flex</th>
<th>Peg</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma(y))</td>
<td>2.34</td>
<td>2.27</td>
<td>3.12</td>
</tr>
<tr>
<td>(\rho(y, y_{-1}))</td>
<td>0.77</td>
<td>0.79</td>
<td>0.70</td>
</tr>
<tr>
<td>(\sigma(rex) \times 50%)</td>
<td>3.93</td>
<td>4.12</td>
<td>2.53</td>
</tr>
<tr>
<td>(\rho(rex, rex_{-1}))</td>
<td>0.70</td>
<td>0.69</td>
<td>0.82</td>
</tr>
<tr>
<td>Debt service-GDP (%)</td>
<td>20.77%</td>
<td>20.62%</td>
<td>20.98%</td>
</tr>
<tr>
<td>Average spread (%)</td>
<td>6.04%</td>
<td>5.73%</td>
<td>5.65%</td>
</tr>
<tr>
<td>Prob. of Crisis (%)</td>
<td>3%-5%</td>
<td>4.1%</td>
<td>3.2%</td>
</tr>
</tbody>
</table>

| Other Statistics:                   |                       |                     |      |
| \(\sigma(c)/\sigma(y)\)            | 1.18                  | 1.13                | 1.18 |
| \(\sigma(tb)/\sigma(y)\)           | 0.53                  | 0.19                | 0.14 |
| \(\rho(c, y)\)                      | 0.82                  | 0.95                | 0.97 |
| \(\rho(tb, y)\)                     | -0.51                 | -0.23               | -0.39 |
| \(\rho(rex, y)\)                    | 0.58                  | 0.44                | 0.31 |
| \(\rho(spread, y)\)                 | -0.22                 | -0.55               | -0.72 |

\(^a\) For the data, when calculating the second moment, the logarithm of GDP, private consumption, government consumption, and the trade-balance ratio were detrended by HP filter with a smoothing parameter of 1600. We report deviations from trend. The interest rate spread is the difference between domestic lending rate and saving rate. The spread is in annual term.

\(^b\) Consistent with the empirical analysis before, the Sudden Stop in the simulation is defined as the state where the financial constraint is binding, GDP is one s.d. below its long-run trend, and the current account reversal is greater than two s.d..

\(^c\) The pegging exchange rate regime is the case with \(\gamma^E = 0\); while the flexible one is \(\gamma^E = 1\).

### 6 Spending Multipliers

This section discusses the state-dependent effect of government spending shock. First, I consider the definition of fiscal multiplier in the model environment.

#### 6.1 Definition

The non-linear nature of this model and the state-dependence make it conceptually difficult to define the fiscal multipliers. Here, I use the procedure similar to that used to compute generalized impulse response functions (GIRF). In the model, the fiscal multiplier is a function of state of the economy, \(S = (b, w; y^T, z, g)\). Starting from \(S_0\), I simulate the economy for a large number of paths of shocks, and take it as benchmark. In the second time, I simulate it again with the same set

\(^{32}\)In order to show the model can account for similar sudden stop characteristics as in the data, in appendix E.4, I compare the 9-periods event window of sudden stop episodes from the data and the model. Similar to the data, sudden stop in the model is characterized by large real depreciation and consumption drop. However, the model can not match the pre-crisis expansion as in the data and also produces less persistence over the recession. Both of them are considered as drawbacks of sudden stop models with only transitory shock, as in Aguiar & Gopinath (2006).
of shocks but increase the government spending shock in the first period by a certain amount\(^{33}\). At each time horizon, the spending multiplier is the median difference of the outcomes between the paths with and without the fiscal shock. Specifically, the cumulative multiplier of horizon \(T\) is defined as,

\[
FM^T(S_0) = \frac{\sum_{t=1}^{T} \prod_{j=1}^{t} 1/R^*_j (X_t^{Shock} - X_t^{No\ Shock})}{\sum_{t=1}^{T} \prod_{j=1}^{t} 1/R^*_j (Spending_t^{Shock} - Spending_t^{No\ Shock})},
\]

where \(X\) is the variable of the interests. Because the model is in an open economy environment, I consider the multiplier on variables denominated in values or volumes respectively. When \(X\) is measured in tradable values (i.e. GDP-value, C-value), the denominator is also adjusted for the change of real exchange rate. When \(X\) is measured in final good volumes (i.e. GDP-volume, C-volume), the real exchange rate is kept as constant at its steady state. For example, to define the multiplier on the value of GDP, \(X_t = y_t^T + y_t^N p_t^N\) and \(Spending_t = p_t^G g_t\); To define the multiplier on output volume, \(X_t = y_t^T + y_t^N p_t^N\) and \(Spending_t = \overline{p^G} g_t\). The multiplier in equation 48 is called the cumulative multiplier in Ramey & Zubairy (2014), and it nests the impact multiplier when \(T = 1\). Because my baseline model does not have the component to create cumulative effect of government spending and the sudden stop event is featured with the sudden collapse of financial activities in one period or two, the discussion below mainly focuses on impact multipliers.

### 6.2 State-Dependent Effect of Government Spending

Government purchase is the canonical instrument of macroeconomic stabilization. For models in the literature, the stimulative effect of government purchase mostly comes from New-Keynesian nominal rigidity, consumption habit-persistence, search friction in the labor market, or unresponsive monetary policy, etc.. In this paper, these traditional channels are shut down and I focus on the novel effect of government spending during the financial crisis of small open economies.

Figure 9 plots the predicted multipliers from the model simulated data, where I apply the same estimation method as in the empirical part before. We can see the model is successful in producing the financial cycle-dependent fiscal multiplier effect. The multipliers on output and consumption are higher during sudden stop crisis than that during normal times. The real exchange rate also appreciates more during crisis. Although what my model predicts generally line up with the key channels of financial crisis, they do not necessarily match the quantitative features implied by the actual data. First of all, while in the data, an increase in government spending generates consumption multiplier close to 0 in normal times but significantly crowds-in consumption during sudden stops, the model generally predicts crowding-out effect except on impact during sudden stop periods\(^{34}\). Second, the model fails to generate the hump-shaped fiscal multiplier effect on

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\(^{33}\)Given that in a nonlinear environment, the effect of shocks is not additive, both the sign and size of shocks matter for the value of fiscal multiplier. To control this, for each simulation I use 50% of negative shocks and 50% of positive shocks. The size of the shock is always fixed at 0.25% of GDP unless otherwise informed.

\(^{34}\)Due to the strong RBC flavor of my baseline model, the consumption is crowded out after positive expenditure...
output and consumption like in the data. These failures are mainly because in this paper, I use a parsimonious model to understand the financial channels of government spending while abstracting from the traditional channels mentioned above that can generate sizable and empirically relevant fiscal multipliers.

From figure 10, we find the same channel applies to both exchange rate regimes, and the difference in spending multiplier is even larger for the case of pegged exchange rate. Under flexible exchange rate, the average impact multiplier on consumption is -0.31 in normal times and -0.21 in the identified sudden stop periods. Under pegged exchange rate, it increases further, from -0.18 in normal times to 0.37 in sudden stops. In the later case, consumption is crowded-in on impact when the shock hits. In contrast, the model predicts sudden stop has a larger impact on real exchange rate appreciation under flexible exchange rate policy.

In order to contrast fiscal multipliers between distinct states of nature more clearly, I define the economic “boom” and “bust” states, and then compare the theoretical impulse responses starting from different states. First, I start the economy from the stochastic steady state and hit it with a sequence of four positive \( y_t^T \) shocks of 1 standard deviation. At the end of the fourth quarter, the economy is highly leveraged with consumption and output above the trend. Besides, the ever-increasing productions drive-up wage state. The state is considered as the boom. Next, I get the bust by imposing large “sudden stop shocks” on the economy, which are extracted from long shocks, especially without DNWR. This is consistent with Lewis & Winkler (2017). They find even with endogenous firm entry, the model fails to generate consumption crowding-in for a variety of model specifications, except with strong complementarity between public and private consumption in utility or assuming public spending is productivity-augmenting. But in this paper, I emphasize the difference of multipliers between financial states.

From the event analysis in appendix E.4, we can see the sudden stop recession does not show much persistence as well.

Different authors define the “boom-bust cycle” in different way. Schmitt-Grohé & Uribe (2016)’s definition is based on exogenous process. The way I define the boom-bust leverage cycle (on output and current account change) is consistent with the empirical findings that the leverage and real activities usually go above the trend during periods prior to sudden stop crisis.
Figure 10: Model Simulation for Different ERR

Note: The graphs report the predicted fiscal multipliers from the model of different exchange rate regimes. The flexible exchange rate is the case of $\gamma^E = 1$, while the pegged exchange rate is $\gamma^E = 0$. The details are the same as figure 9.

run simulations\(^{37}\). This empirical strategy to generate “boom-bust cycles” is motivated by the observation that sudden stop crises are usually the periods when a highly leveraged economy is hit by large international shocks. Notice that this is different than simply starting the economy at the stochastic steady state and hitting it with a positive (negative) international shock, with the reason being that my procedure accounts for the evolution of endogenous states. This is particularly important for my purpose, as a leverage boom increases consumption today but potentially makes the economy more exposed to financial and unemployment crisis tomorrow.

The state-dependent effect of government spending is evident from table 5, which compares the median value of simulated multipliers between the financial states\(^{38}\). Generally, the fiscal multipliers on GDP and consumption are higher in the credit bust than in the credit boom, especially on impact. Moreover, due to DNWR, the difference in the volume-multipliers (C-volume or GDP-volume) is even more pronounced for the economy with pegged exchange rate. Lastly, model’s prediction about real exchange rate is different from the data. The real exchange rate always appreciates after the shock but its difference between the bust and the boom depends on exchange rate policy. The financial channel is the same as that in the three-period model. At the bust states where the financial constraint binds, a fiscal expansion increases the capital price and relaxes the constraint,

\(^{37}\)I simulate the economy for 100,000 periods and identify the sudden stop events by the standard described before. Event window analysis shows that the sudden stop is usually accompanied by a large negative shock on $y^T$ and a small positive shock on domestic TFP $z$. The combinations of exogenous processes are averaged across the crisis events and I call that “sudden stop shocks”.

\(^{38}\)In appendix E.4, I plot the median path of simulated multipliers from the impulse response for the economy under two distinct exchange rate systems
Table 5: State-Dependent Multipliers

<table>
<thead>
<tr>
<th></th>
<th>Baseline: Flexible Exchange Rate</th>
<th>Baseline: Pegged Exchange Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Boom-impact</td>
<td>Bust-impact</td>
</tr>
<tr>
<td>$c^T$</td>
<td>0.02</td>
<td>0.22</td>
</tr>
<tr>
<td>$c^N$</td>
<td>-0.58</td>
<td>-0.53</td>
</tr>
<tr>
<td>REX</td>
<td>0.24</td>
<td>0.27</td>
</tr>
<tr>
<td>C-volume</td>
<td>-0.60</td>
<td>-0.28</td>
</tr>
<tr>
<td>GDP-volume</td>
<td>0.42</td>
<td>0.47</td>
</tr>
<tr>
<td>C-value</td>
<td>0.29</td>
<td>0.77</td>
</tr>
<tr>
<td>GDP-value</td>
<td>1.28</td>
<td>1.59</td>
</tr>
<tr>
<td>C.E.</td>
<td>-0.17%</td>
<td>-0.13%</td>
</tr>
</tbody>
</table>

a. Given initial states, I simulate the economy for 10,000 times with and without the government spending shock and take the difference. The numbers in the table are the median cumulative multipliers from the simulation (except for REX and C.E.).

b. C.E. is the certainty equivalence moving from an economy without shock to an economy with one unit government spending shock. The C.E. is the amount of final consumption goods that households are willing to pay so that the finite-periods utility sums of the two economies are equalized. This is a way to measure the state-dependent welfare effect of government spending shock, similar to the “welfare multiplier” in Rendahl (2016).

thus it promotes borrowings. Greater ability to borrow increases tradable consumption and relaxes the financial constraint by even more. Besides, the increase in government spending also has larger stimulative effect on production in recessions than in expansions. This comes from two reasons. From the working-capital requirement, the relaxed financial constraint reduces the firm’s labor-hiring cost. When the real wage is rigid, the higher government spending also reduces involuntary unemployment, further stimulating non-tradable good production.

It is interesting to know how much of the state-dependence comes from DNWR and how much it comes from the financial constraint. This is easy to tell by comparing GDP-volume multipliers between the two exchange rate regimes. Notice that the difference of GDP-volume multipliers between financial states is relatively small under flexible exchange rate (0.42 in the boom and 0.47 in the bust), which means the working-capital constraint only contributes marginally to the stimulative effect of government spending on the production. On the other hand, under pegged exchange rate, the multipliers increase from 0.67 in the boom to 0.97 in the bust. This larger difference is due to DNWR.

However, we know from the discussion above that under pegged exchange rate, overproduction
of \( y^N \) exerts an counteractive force on financial constraint and may worsen the real activities. This is evident by considering the multiplier effects of government spending on either REX, \( c^T \) or GDP-values. First, we find in the lower panel of the table that, due to the large increase of non-tradable good production, the real exchange rate appreciates less in the bust than that in the boom. That limits the financial acceleration effect of government spending. So, the difference in \( c^T \) multipliers also gets smaller. The GDP-value can be written as,

\[
\text{GDP-value} = y^T + \frac{1 - \omega}{\omega} \left( \frac{c^T}{y^N - g^N} \right)^{\frac{1}{\theta}} y^N.
\]

(49)

When the financial constraint binds, \( c^T \) depends on the credit available from foreign investors and thus the price of capital. It increases because the fiscal expansion relaxes the financial constraint. However, when \( \frac{1}{\theta} > 1 \), overproduction of \( y^N \) has an offsetting effect on the non-tradable good price and makes the difference of GDP-value multipliers also relatively small.

C.E. is the certainty equivalence households are willing to pay for moving from a no-policy economy to an economy with a unit government spending shock. Here, it measures the net welfare effect of a policy shock in finite periods. Notice that C.E. is larger when the shock hits the economy at bust state. This is consistent with the implication of the three-period model. The higher government spending increases capital price and mitigates the severity of crisis at bust states. When the wage is rigid, part of the benefit also comes from the stimulative effect on output. Therefore, the welfare multipliers are generally larger in the economic bust than that in economic boom, and under pegged exchange rate, a wasteful government spending shock can even produce an instant net welfare gain.

Overall, the model can capture the key empirical findings of this paper: spending multiplier is higher when the shock hits during financial recessions than during financial expansions; and the dependence of multiplier on financial states is more pronounced for an economy with pegged exchange rate.

The result is consistent with Ilzetzki et al. (2013)’s empirical finding that the multipliers are larger when the economy is in financial distress. My model contains this non-linear feature due to the occasionally binding constraints and the mechanisms described before. Ilzetzki et al. (2013) also concludes that the countries with currency pegs have larger fiscal multipliers than the countries with flexible exchange rate. Different from that, this paper mainly focuses on how fiscal multiplier depends on financial states.

### 6.2.1 Multipliers across Different Levels of Indebtedness

To see the mechanisms of the model more clearly, figure 11 displays the landscape of fiscal multipliers across the debt levels. It is evident from this figure that the fiscal multiplier effect is highly non-linear and is characterized by the two thresholds where financial constraint and wage constraint start to bind, respectively. When the financial constraint binds, the government spending increases borrowing and consumption (upper left panel), but its effect on production is relatively small (lower
left panel). If the wage constraint also binds, the multipliers on consumption and GDP volumes (left panels) increase further by a larger amount. However, this strong effect from the wage rigidity in turn reduces real exchange rate and damps the multiplier on consumption and GDP values (right panels). As a result, even though the multiplier effect of government spending on GDP volumes is monotone across the debt levels, the net effect on equilibrium GDP values may not be.

![Figure 11: Multipliers across the State-Space](image)

**Figure 11: Multipliers across the State-Space**

Note: The figures plot the median values of impact multipliers at different debt-GDP ratios. The wage state is fixed at 95% of its mean because that allows me to separate out effects of the two constraints in the figures. The exogenous states are kept at their steady states.

Using U.S. data, Bernardini & Peersman (2015) finds that in private debt over-hang periods, crowding-in effect drives multipliers to be higher than unity (1.45), whereas significant crowding-out of personal consumption and investment in low debt states make multipliers below one. Based on different mechanisms, my model predicts for small open economies, external indebtedness is also an important factor in determining fiscal multipliers.
6.3 Asymmetric Effect of Government Spending Shock

Since I have a non-linear model, the **marginal multiplier** (for a small government spending shock) should be different from the **average multiplier** (for a sizable government spending shock)\(^{39}\). The intuition is that large government spending shocks can potentially put the economy in different financial regimes or change the probability of regime-shifting. Moreover, given the two inequality constraints, positive shocks are supposed to have different impacts on the economy from negative shocks, especially when the shocks are sizable. To understand this result, we return to the asset price channel discussed before. A positive government spending shock creates a wealth effect through the real appreciation and boosts up the capital price. But the capital price plays a smaller role if the financial constraint is less likely to bind. On the opposite, a negative spending shock crowds-out consumption initially, and at the same time, tightens the financial constraint. This accelerates the decline of private activities resulting in a fiscal multiplier larger in size.

Table 6 shows that both the size and sign of government spending shock matter. Generally, positive shocks have smaller multiplier effects than that of negative shocks. From the middle column to the right, notice that the average multiplier on both consumption and production becomes smaller when the positive shock gets larger. From the middle to the left, on the other hand, a more sizable negative shock creates the bigger multiplier effect on average. Comparing the upper and lower panels, however, we can see the DNWR amplifies the multiplier effect of a large negative shock on production (GDP-volume, C-volume), but weakens its effect on the financial constraint \((c^T)\). Notice that in the model of flexible exchange rate, the decrease in production after a negative shock is due to the working-capital constraint, which is relatively small.

<table>
<thead>
<tr>
<th></th>
<th>Baseline: Flexible Exchange Rate</th>
<th>Baseline: Pegged Exchange Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>(c^T)</td>
<td>-2% -1% -0.25% +0.25% +1% +2%</td>
<td>-2% -1% -0.25% +0.25% +1% +2%</td>
</tr>
<tr>
<td>C-volume</td>
<td>0.170 0.148 0.123 0.118 0.108 0.098</td>
<td>0.200 0.191 0.188 0.160 0.132 0.118</td>
</tr>
<tr>
<td>GDP-volume</td>
<td>0.451 0.446 0.439 0.439 0.437 0.436</td>
<td>0.759 0.693 0.627 0.613 0.581 0.548</td>
</tr>
</tbody>
</table>

Note: “+0.25%” means government spending increases by 0.25 percent of GDP; while a negative number means government spending decrease. The numbers in the table are the median of impact multipliers. For comparison, the multipliers of positive and negative shocks have the same meaning. The simulation starts from the model’s long-run stochastic steady state.

\(^{39}\)The size effect of government spending shock is also explored in non-linear work of Fernández-Villaverde et al. (2015)
indicates spending cuts during recessions. My model predicts asymmetric effect of government spending, especially during sudden stop crisis. This calls for the necessity to understand and explore the asymmetric effect of government spending in fiscal policy research. It also casts doubt on the empirical estimation that uses linear VAR to identify the effect of government spending shock. Using threshold VAR approach, Pragidis et al. (2017) claims that the multiplier of a negative spending shock is higher than that of a positive shock during financially stressful periods and the asymmetry is absent in financially tranquil periods. Failure to account for the asymmetric effect of government spending shock may result in bias of estimation, and could be a potential explanation for the wide range of fiscal multiplier estimates in the literature.

6.4 Persistence of Government Spending Shock

As implied by the three-period model, if the government can commit the certain path of fiscal policies in the future, the current financial constraint can be relaxed. This motivates me to investigate the effect of government spending at different shock persistence. In this section, I solve the model for different values of $\rho^G$ and for each $\rho^G$ I calculate the median impact multipliers at both the boom and bust states.

From figure 12, we can see the government spending is more expansionary for the higher level of shock persistence, especially during crisis. At the relatively low persistence, the financial channel is weak so that the median multipliers on output (lower left panel) in economic bust are smaller than those in the boom. Only when the shock persistence is greater than a threshold (around 0.73), output multipliers in the bust can exceed the ones in the boom. The intuition comes from the currency mismatch channel. Promised fiscal expansions create an expectation of real appreciations, which helps in relaxing the current financial constraint and increasing real activities. At the same time, more persistent fiscal expansions also impose a stronger negative wealth effect. The result comes in because the financial channel dominates.

6.5 Investment Economy

The baseline models do not have physical investment and assume aggregate amount of capital always equals to one. But, we know in a canonical RBC model, the existence of investment heavily affects the value of spending multipliers, and capital adjustment also influences the financial accelerator channel emphasized in this paper. So, it is interesting to see how the above mechanism works in an investment economy.

Suppose there is a capital good producer with investment technology,

$$k_{t+1} - (1 - \delta)k_t = k_t \Phi(\frac{i_t}{k_t}),$$

(50)
Figure 12: Multipliers for Different Shock Persistence

Note: The figures plot the median value of impact multipliers for different degree of shock persistence. Here, I use the baseline model with flexible exchange rate. The definition of boom and bust states are the same as before. The corresponding figures for the model of pegged exchange rate is in appendix E.4.

where $i_t$ is the same composite good as the final consumption\textsuperscript{40},

$$i_t = \left( \omega i^T_t + (1 - \omega) i^N_t \right)^{\frac{\alpha}{1 - \gamma}}.$$  \hspace{1cm} (51)

The capital good producer purchases the used capitals from the firm-households, decorates them and resells them in the financial market. It solves the following static problem

$$\max_{(i_t^T, i_t^N)} \left\{ q^K_t [k_{t+1} - (1 - \delta)k_t] - rex_t i_t \right\},$$  \hspace{1cm} (52)

subject to \textsuperscript{50} and \textsuperscript{51}. $\delta$ is the depreciation rate of physical capitals. The equilibrium conditions are listed in appendix A.\textsuperscript{43}

\textsuperscript{40}Another extreme used in the literature is that investment goods are sector-specific. In my model, that implies: $i_t = i_t^N$. Given that non-tradable sector mainly consists of service industry, this assumption is not realistic in practice. But I still find the value of investment multiplier heavily depends on the parameter of sector share $\omega$ in the investment good composite.
Following Bocola (2016), the investment technology is \( \Phi(\frac{k_t}{K}) = a_1 + a_2 (\frac{k_t}{K})^{1-\xi} \), where \( \xi \) is the capital price elasticity of investment demand. Three parameters need to be re-calibrated in this environment: \( \xi = 0.46 \) is used to match \( \sigma(i)/\sigma(y) = 3.04 \) in the data; \( \delta \) is set to 0.88 following E. G. Mendoza (2010)\(^{41}\); Different from the baseline model, \( \alpha^K \) here is set to the standard value of 0.33 as the capital share.

### Table 7: Investment Economy

<table>
<thead>
<tr>
<th></th>
<th>Flexible Exchange Rate</th>
<th>Pegged Exchange Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Boom-impact</td>
<td>Bust-impact</td>
</tr>
<tr>
<td><strong>C-volume</strong></td>
<td>-0.58</td>
<td>-0.23</td>
</tr>
<tr>
<td><strong>I-volume</strong></td>
<td>0.15</td>
<td>0.27</td>
</tr>
<tr>
<td><strong>GDP-volume</strong></td>
<td>0.44</td>
<td>0.52</td>
</tr>
</tbody>
</table>

Note: Given initial states, I simulate the economy for 10,000 times with and without the government spending shock and take the difference. The numbers are the median cumulative multipliers from the simulation.

As can be seen from the first order condition of capital price (as shown in appendix A), the fluctuation of Tobin’s Q and so the strength of financial channel highly depends on elasticity \( \xi \). In this environment, an increase in government spending has two competing effects under play. On one hand, similar to a closed economy RBC model, the higher government spending increases domestic interest rate (the interest rate in non-tradable sector is \( 1/\left[\beta \mathbb{E}_t \frac{u_{CN}((t+1))}{u_{CN}(t)}\right] \) and crowds-out investment through the traditional inter-temporal substitution effect. On the other hand, in this open economy with collateralized debt, the Fisher’s debt-deflation mechanism is in place. The higher government spending appreciates real exchange rate and capital price, and thus increases the investment good purchase. Here, both of these two effects exist and investment multipliers depend on model calibrations\(^{42}\).

I find with proper parameter values, government spending increases capital price and crowds-in investment. For the same reasons discussed above, the crowding-in is even stronger at states where financial constraint is binding. The upper panel of table 7 shows the multiplier effect of flexible exchange rate and the lower panel shows the case of pegged exchange rate. Generally, the baseline results about consumption and GDP multipliers still hold here. The multipliers are larger during contractions than that during expansions, and the presence of wage rigidity amplifies the multiplier effect on productions. The difference is that due to the crowding-in of investment, the difference

\(^{41}\)E. G. Mendoza (2010) uses the perpetual inventory method to back out a series of capital stock and a depreciation rate. Although our data sample extends the one he used, I believe the depreciation rate doesn’t vary too much over time. Investment to GDP ratio is set to sample average of 20.3%.

\(^{42}\)The impulse response of the higher domestic interest rate, and at the same time, the higher capital price after a government spending shock are show in appendix E.4.
in consumption multipliers between financial states become smaller. Even with pegged exchange rate, consumption is still crowded-out by government spending during the credit bust.

### 6.6 Sensitivity Analysis

This section checks whether the baseline results can be extended to other model specifications. Firstly, the baseline model has a separable utility function that is popular in the two-sector environment. Another commonly used preference is the GHH preference (i.e., Greenwood et al., 1988), which eliminates the wealth effect on labor\(^{43}\). The utility function is modified as

\[
u(c_t, l_t) = \left[ \frac{c_t - \chi l_t^{\nu + 1}}{1 + \nu} \right]^{1-\sigma} - 1.
\]  

(53)

Frisch elasticity parameter \(\nu\) is the same as in the baseline model and I re-calibrate \(\chi\) to normalize the steady state of labor. The first order condition w.r.t. labor decision becomes

\[
\frac{\chi l_t^\nu}{\omega c_t^\nu l_t^{T-\frac{1}{\beta}}} = \frac{p_t^N F_t(k_t, l_t^N)}{(1 + \phi \mu_t)} = w_t^* \geq \gamma w_{t-1},
\]  

(54)

where \(w_t^*\) is still the effective wage. In the upper panel of table 8, we can see due to the smaller wealth effect, the multipliers on GDP-volume are smaller than those in the baseline case. Due to the complementarity between consumption and labor in GHH utility, consumption multipliers are also smaller. Sudden stop crisis still increases the fiscal multiplier of government spending, but due to the same reason, the financial channel is also weakened. In the upper panel, consumption multipliers increase from -0.60 in the boom to -0.28 in the bust for the baseline economy, but only increase from -0.72 to -0.49 in the economy with GHH preference. In the lower panel, when the economy is also subject to DNWR, the strong increases of non-tradable good production depreciates real exchange rate and hurts the borrowing constraint. Therefore, the consumption multipliers in the boom and the bust get even closer.

My analysis so far assume the tradable as an endowment process. Next, I extend the model to include a tradable good production as

\[y_t^T = z_t^T l_t^{a_T^T},\]

(55)

and the non-tradable good production is re-written as

\[y_t^N = z_t^N l_t^{a_K^N} l_t^{a_K^N} a_N^N.\]

(56)

Here, I assume the labor shares are the same \((\alpha^T = \alpha^N = 0.64)\) and the labor services are flexible to move between sectors. As in the literature, there are reasons to believe the tradable sector, which

\[^{43}\text{The consumption's wealth effect on labor supply equals to 0 in the one-good environment. In this two-goods set up, the wealth effect on labor is still present actually.}\]
Table 8: Sensitivity Analysis

<table>
<thead>
<tr>
<th></th>
<th>Flexible Exchange Rate</th>
<th></th>
<th>Pegged Exchange Rate</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C-volume</td>
<td>C-value</td>
<td>GDP-volume</td>
<td>GDP-value</td>
</tr>
<tr>
<td></td>
<td>Boom</td>
<td>Bust</td>
<td>Boom</td>
<td>Bust</td>
</tr>
<tr>
<td>Baseline</td>
<td>-0.60</td>
<td>-0.28</td>
<td>0.29</td>
<td>0.77</td>
</tr>
<tr>
<td>GHH</td>
<td>-0.72</td>
<td>-0.49</td>
<td>0.36</td>
<td>0.59</td>
</tr>
<tr>
<td>$Y^T$ Production</td>
<td>-0.65</td>
<td>-0.58</td>
<td>-0.22</td>
<td>-0.18</td>
</tr>
<tr>
<td>$\gamma_E = 0.3$</td>
<td>-0.42</td>
<td>-0.23</td>
<td>0.26</td>
<td>0.32</td>
</tr>
</tbody>
</table>

Note: The numbers in the table are the median of impact multipliers for each variable. The states of “boom” and “bust” are simulated in the same way as in the baseline case.

mainly refers to manufacturing industry, is less labor-intensive than the non-tradable sector, which mainly consists of services. That leads to $\alpha^T < \alpha^N$ and implies the smaller sector reallocation effect after an increase of non-tradable government spending. Besides, existing papers (e.g., Shen et al., 2015) also use the model with labor market segmentation and hold the view that labor services are not perfectly mobile between sectors. This provides another friction on labor adjustment. In that sense, the modeling choice here provides a lower bound of fiscal multiplier and makes it easy for us to consider whether the baseline mechanisms still work when I include tradable productions. As can be imagined, either a smaller labor share in tradable sector or the adjustment cost of labor should make the result closer to the baseline case. From the upper panel of table 8, we see the tradable production indeed weakens the financial channel of government spending. An increase in non-tradable government spending attracts labor supply to non-tradable sector, reducing the proportion supplied to the tradable sector. So, GDP-volume multipliers only increase marginally from 0.33 in the boom to 0.36 in the bust. The decrease of tradable goods in turn deteriorate the value of collateral and tightens the financial constraint. As a result, the borrowing condition is only slightly improved by the government spending increase. Compared to the baseline case, the difference of consumption multiplier also gets smaller which is -0.65 in the boom and -0.58 in the bust.

Lastly, given the fact that most countries are under the managed-float exchange rate system and their exchange rate policies vary across time, I check the intermediate value of exchange rate coefficient $\gamma_E = 0.3$. A higher value of $\gamma_E$ implies a more accommodating exchange rate policy following the government spending shock. Consistent with the intuition, table 8 shows that the values of fiscal multiplier when $\gamma_E = 0.3$ lie between the baseline economies with perfectly pegged

\[44\] That means a contemporaneous nominal exchange rate depreciation happens at the time of the shock if the full-employment wage is below the wage state.
exchange rate and perfectly flexible one. Fiscal expansions still stimulate production more in the
financial bust (0.71) than in the boom (0.58). But since the real wage rigidity is partly offset by
nominal depreciation, the stimulative effect is smaller compared to the perfectly pegged case (0.67
in the boom and 0.97 in the bust). The financial channel is still present here and consumption
multipliers increase from -0.42 in the boom to -0.23 in the bust. In appendix E.4, I show the results
for a continuum of exchange rate coefficients $\gamma^E \in [0, 1]$, where we can see the exchange rate policy
is an important factor of spending multiplier in small open economy. It governs relative strength
of the two channels of a government spending shock: demand channel and financial channel.

7 Conclusion

In this paper, I provide the empirical evidence and a theoretical framework to explain that the
financial channel plays an important role in the transmission of government spending shock for
small open economies. Using local projection method, I find a positive government spending shock
has larger stimulative effect on private consumption during sudden stop episodes than that during
normal times, and the dependence of fiscal multiplier on financial states is more significant for the
counties under pegged exchange rate regime.

To understand the mechanism, I build a two-sector three-period small open economy model
with collateral constraint. In the model, the debt is denominated in foreign currency units while
the domestic capital is used for collateral. When financial constraint binds, the higher government
purchase of non-tradable goods appreciates real exchange rate, increases the price of collateral,
and promotes real absorption. Since the value of collateral today depends on the expectation of
future real exchange rate, the government’s commitment on future stimulus can further relax the
current financial constraint and stimulate real activities. Analysis of the optimal spending policy
shows that a fiscal expansion ameliorates the severity of financial crisis when the constraint binds.
The mitigated pecuniary externality during crisis in turn encourages private borrowing in normal
times. Next, I extend the analysis to an infinite-horizon environment and show that the model’s
predictions are generally consistent with my empirical findings. The simulation result shows that
under flexible (pegged) exchange rate, the consumption multiplier on impact is -0.31 (-0.18) during
sudden stop periods and is -0.12 (0.37) during normal times. I also show this baseline result is
robust to a variety of model specifications.

To keep the non-linear solution manageable, my model abstracts from several important aspects
that can be important to quantify the effects of government spending shock, such as rule-of-thumb
consumers, domestic real and nominal rigidities, or unresponsive monetary policy. Incorporating
these components can help to generate empirically-relevant multipliers, at the cost of making the
key mechanism less clear. Despite the limitation, the result of this paper suggests that Fisher’s
debt-deflation is an important channel to consider when we study fiscal multiplier in small open
economies. For future research, it is interesting to move it to a quantitative environment with
financial intermediation and domestic monetary policy and examine how it interacts with other
traditional channels.
References


Appendix

A Equilibrium Conditions

The first order conditions of the baseline model are listed as follows,

\[ p_t^N = \frac{1 - \omega c_t^T}{\omega c_t^N} \frac{1}{\theta} \] (57)

\[ (1 - \mu_t) = \beta \mathbb{E}_t \Lambda_{t,t+1} R_t^* \] (58)

\[ (1 - \kappa \mu_t) q_t^K = \beta \mathbb{E}_t \{ \Lambda_{t,t+1} [ q_{t+1}^K + r_{t+1}^K ] \} \] (59)

\[ \Lambda_{t,t+1} = \frac{u_{CT}(c_{t+1}, l_{t+1})}{u_{CT}(c_t, l_t)} \] (60)

\[ \frac{1}{R_t^*} b_{t+1} + \phi w_t l_t \leq \kappa q_t^K k_{t+1} \] (61)

\[ w_t = \frac{u_L(c_t, l_t)}{u_{CT}(c_t, l_t)} \frac{p_t^N z_t F_L(k_t, l_t^d)}{1 + \phi \mu_t} \] (62)

\[ r_t^K = p_t^N z_t F_K(k_t, l_t^d) \] (63)

\[ y_t^N = z_t F(k_t, l_t^d) \] (64)

\[ w_t \geq \gamma \frac{w_{t-1}}{\epsilon_t} \] (65)

\[ c_t = \left( \omega c_t^T \frac{\theta - 1}{\theta} + (1 - \omega) c_t^N \frac{\theta - 1}{\theta} \right)^{\frac{\theta}{1 - \theta}} \] (66)

\[ k_t = 1 \] (67)

\[ c_t^T + \omega^G g_t + b_t = \frac{1}{R_t^*} b_{t+1} + y_t^T \] (68)

\[ c_t^N + (1 - \omega^G) g_t = y_t^N \] (69)

\[ \log(z_t) = \rho^T \log(z_{t-1}) + \epsilon_t^z \] (70)

\[ \log(y_t^T) = \rho^{YT} \log(y_{t-1}^T) + \epsilon_t^{YT} \] (71)

Spending policy: \[ \log \frac{g_t}{\bar{g}} = \rho^G \frac{g_{t-1}}{\bar{g}} + \gamma^Y \log \frac{gd_{t-1}}{gd_{t}} + \gamma^B \log \frac{b_{t-1}/gd_{t-1}}{b/gd_{t}} + \epsilon_t^G \] (72)

Exchange rate policy: \[ \epsilon_t = \max \left\{ \frac{\gamma w_{t-1}}{w_t^E}, 1 \right\} \] (73)
In the model with investment technology, we also have the following equations:

\[ k_{t+1} - (1 - \delta)k_t = k_t \Phi \left( \frac{i_t}{k_t} \right) \]  
\( (74) \)

\[ i_t = \left( \frac{\omega_i T^{\frac{\theta - 1}{\sigma}}}{\theta} + (1 - \omega) N^{\frac{\theta - 1}{\sigma}} \right)^{\frac{\sigma}{\theta - \sigma}} \]  
\( (75) \)

\[ q_t^K \Phi'(\frac{i_t}{k_t}) \omega_i T^{\frac{1}{\theta}} i_t^{\theta - \frac{1}{\theta}} = 1 \]  
\( (76) \)

\[ q_t^K \Phi'(\frac{i_t}{k_t})(1 - \omega) i_t^{\frac{1}{\theta}} t^{\frac{\theta - 1}{\theta}} = p_t^N \]  
\( (77) \)

B Computational Method

The model is solved using global method. I use time iteration and linear interpolation. The high dimensional expectation is evaluated using monomial quadrature. The state space of the baseline economy is:

\[ S_t = [b_t, w_{t-1}, y_t^T, z_t, g_t] \]  
\( (78) \)

Firstly, I discretize the state space and approximating a list of control variables (policy functions) \( C_t \), by multi-linear functions on \( S_t \). Taking these functions as initial guess, I can back out other model variables at time \( t \) and the state vector at the beginning of next period \( S_{t+1} \). Then, I evaluate the control variables at time \( t+1 \) using the guessed policy functions and linear interpolation. The conditional expectations are calculated. Next, a system of equilibrium conditions are jointly solved backward in order to update the value of policy functions on each grid point. Lastly, I iterate until the policy functions converge on the finite number of grid points.

In my model environment, the algorithm proceeds as follows,

1. Use piece-wise linear functions to approximate a set of policies on the state space \( C_t^0 = \{ c_t^0, l_t^0, \eta_t^w, q_t^K, \} \). As the initial guess, I use the model’s non-stochastic steady state. \( \eta_t^w \) is an auxiliary variable to deal with the wage constraint. See below.

2. Given the initial guess \( C_t^0 \), back out other variables in the current period \( \{ y_t^N, w_t, r_t^K, c_t^N, b_{t+1}, p_t^N \} \) using static equilibrium conditions. Notice that \( b_{t+1} \) and \( w_t \) are the endogenous state of the next period’ problem.

3. Use the law of motion for the exogenous process to iterate the state vector. On each quadrature point, the next period state is \( S_{t+1} = [b_{t+1}, w_t, y_{t+1}^T, z_{t+1}, g_{t+1}] \). Then, I use the guessed policy functions to evaluate control variables at time \( t+1 \): \( \{ c_{t+1}^T, l_{t+1}, \eta_{t+1}^w, q_{t+1}^K \} \), and back out other variables at time \( t+1 \): \( \{ p_{t+1}^N, c_{t+1}^N, l_{t+1} \} \).

4. For each point on the grid, jointly solve a system of equations 58, 59, 61, 62, 65 and update the value of policies: \( C_t^1 = \{ c_t^{T,1}, l_t^1, \eta_t^{w,1}, q_t^{K,1} \} \) (also the financial multiplier \( \mu_t^1 \). See below.).
5. Compute the distance between the new policy functions and the original ones, stop if

$$\max |C^0 - C^1| < 1e^{-7}. \quad (79)$$

Otherwise, take the new policy functions as the guess and start from the second step.

There are two occasionally binding constraints and I use different methods to deal with them. For the wage constraint, I apply the auxiliary variable method in Benigno et al. (2013). Suppose $\eta^w_t$ is an auxiliary variable and it is positive if and only if the wage constraint is binding. In that case, there is involuntary unemployment in equilibrium and I define,

$$l^s_t - l^d_t = U_t = \max(\eta^w_t, 0)^3 \quad (80)$$

If the wage constraint is not binding, $\eta^w_t$ is negative and its absolute value is the difference between full-employment wage and wage state from the last period,

$$w_t - \frac{\gamma w_{t-1}}{\epsilon_t} = \max(-\eta^w_t, 0)^3 \quad (81)$$

The method allows me to transform the inequality constraint into equality when I can apply Newton-type solver to get the solutions.

To deal with the financial constraint, I use the “guess and verify” procedure as in Bianchi & Mendoza (2013). In each iteration, I first solve the model without the financial constraint, and then check whether the financial constraint is violated. If it is not, go to the next grid point. Otherwise, solve the model again assuming the financial constraint is binding.

C Empirical Appendix

This paper collects quarterly data on GDP components, real exchange rate and current account for a set of small open economies. The use of quarterly data is crucial to define sudden stop crisis episodes and is also essential for the identifying assumption.

Most of the data comes from IMF’s International Financial Statistics, except for the missing values of current account balance for several countries where I interpolate the World Development Index’s annual data to quarterly frequency. The real variables are the nominal variables deflated by GDP deflators if they exist or CPI index if they do not. The variables measured in US dollars are converted using period average exchange rate. For most of the countries, I use the CPI-adjusted real effective exchange rate index to represent the real exchange rate level. For those countries these indexes do not exist, I construct real exchange rate using nominal exchange rate adjusted by relative CPI levels.

Sudden stops: I define the sudden stop as a big current account reversal happening amid of deep economic recession. This definition of sudden stop is similar to that in Calvo et al. (2004).
Specifically, I count the sample-episodes that satisfying the following two criterion as sudden stop periods:

1. Year-To-Year current account reversal is larger that 1.5 standard deviations of the current account to GDP ratio;

2. The real GDP is at least 1.5 standard deviations below its long-run trend.

To compare the fiscal policy effect during the fixed sudden stop episodes, for each identified sudden stop period, I create a 8-quarters event window with 2 periods before and 6 periods after it. This clear-cut way to define sudden stop event generally captures the spirit of “big and sudden” capital flow stop as in Calvo et al. (2004) and at the same time makes it suitable to do serious empirical analysis. Different from Calvo et al. (2004)’s definition, my sudden stop definition is also based on economic cycle of GDP, which makes our identified sudden stop dates not necessarily overlap. But I find most of the identified sudden stop episodes in their paper also belong to my sample.

**Exchange rate regime:** The way I define exchange rate regime follows Ilzetzki et al. (2013) and Ilzetzki et al. (2017). More specifically, I include the country with no legal tender, hard pegs, crawling pegs, and de facto or pre-announced bands or crawling bands with margins no larger than ±2% as pegged exchange rate regime. While all other countries are classified as flexible regime.

Based on the above definitions, I construct a sample data-set of 30 small open economies with each country having at least one sudden stop event in the history. The sample covers a wide range of countries having the financially distressful periods that are frequently studied in the literature. I successfully identifies most of the well-known sudden stop events in the last two decades. These include the Tequila crisis in Latin American countries, Asian financial crisis, Russian financial crisis and European debt crisis, etc. Overall, I find 38 sudden stop episodes, and 15 countries are classified as pegged exchange change rate while the rest is believed to take flexible exchange rate regime.

### C.1 Econometric Methodology

A key issue in fiscal multiplier research is the identification of fiscal shocks. In this paper, I combine the identification assumption of Ilzetzki et al. (2013) with Jordà et al. (2005)’s local projection method to generate impulse responses. Ilzetzki et al. (2013) applies the SVAR approach in Blanchard & Perotti (2002) to a panel-data environment. The restriction behind this method is that fiscal policy requires at least one period to respond to innovations to economic states. To account for the non-linearity in my model, the econometric technique in this paper follows Jordà et al. (2005)’s local projection method, modified by Ramey & Zubairy (2014). Compared with SVAR, the local projection method has two advantages: 1. it is more robust to misspecification because it does not impose implicit dynamic restrictions on the shape of the impulse responses; 2. it is easy to accommodate state-dependence and to study non-linear objects.

Ramey & Zubairy (2014) modifies the “three steps method” in Jordà et al. (2005) to create impulse response and calculate multipliers into the “one step method”, and they argue that these two methods gives the same result if there is no data loss. I mainly use the “one step method” in
my paper but I also confirmed that the other way gives similar result for the baseline case. The methodology can be summarized as the following steps:

- **Data transformation:** Before running the regression, I convert each quantity variables as proportion of trend GDP.
  
  \[ x_{i,t} = \frac{X_{i,t}}{\bar{Y}_{i,t}} \]  
  
  (82)

  where \( X_{i,t} \) is respectively GDP or one of its component and \( \bar{Y}_{i,t} \) is hp-filtered trend of GDP. After this transformation, all economic variables are in the same unit. That makes it convenient to get fiscal multipliers directly from the regression.

- **SVAR regression:** In order to identify the government spending shock and use that as an instrument in the multiplier estimation, I extract the residuals from a SVAR regression for each country and call that “BP shock”. More specifically, I estimate a system of recursively-identified vector auto-regression equations:

  \[ A Z_{i,t} = \sum_{k=1}^{4} C_k Z_{i,t-k} + B u_{i,t} \]  

  (83)

  where \( Z_{i,t} = [g_{i,t-1}, GDP_{i,t-1}, drex_{i,t-1}, CA_{i,t-1}/GDP_{i,t-1}]' \) is the vector of economic variables. \( drex_{i,t-1} \) is the difference of log real exchange rate. \( A \) and \( C_k \) are the estimated coefficient matrices featuring contemporous and dynamic effects. \( B \) is an orthogonal matrix with the diagonal elements showing the variance of structural shocks. Following Blanchard & Perotti (2002), I assume that changes in government consumption require at least one quarter to respond to innovations in other macroeconomic variables. The remaining identifying assumptions include a Cholesky decomposition ordering current account ratio after GDP but before the real exchange rate.

- **One-step estimation:** For each variable and each horizon, I estimate the following panel IV regression with fixed effect:

  \[ \sum_{j=0}^{h} x_{i,t+j} = c + \alpha_i + m^{x,h} \sum_{j=0}^{h} g_{i,t+j} + \Phi_h(L) z_{i,t-1} + u_{i,t+h} \]  

  where \( x \) is the variable of our interest, in this context, GDP, consumption, investment and real exchange rate change. \( z_{i,t-1} \) controls for lagged economic fundamentals and fiscal variables \( Y_{i,t-1} \). The endogenous regressor is instrumented by the estimated SVAR residual from the second step: \( shock_{i,t} = u^g(g_{i,t}) \), which is the orthogonalized part of current government spending observations. The benefit of this one step method comes from the fact that the estimates of \( m^{x,h} \) directly represents the spending multiplier on variable \( x \) in horizon \( h \). To study the state-dependent spending multiplier during sudden stop crisis, I run the following
non-linear regression:

\[
\sum_{j=0}^{h} x_{i,t+j} = I_{i,t-1} \times \left[ c_A + \alpha_{A,i} + m_{A,h}^{x} \sum_{j=0}^{h} g_{i,t+j} + \Phi_{A,h}(L) z_{i,t-1} \right] \\
+ (1 - I_{i,t-1}) \times \left[ c_B + \alpha_{B,i} + m_{B,h}^{x} \sum_{j=0}^{h} g_{i,t+j} + \Phi_{B,h}(L) z_{i,t-1} \right] + u_{i,t+h}
\]

“A” denotes sudden stop episodes and “B” denotes normal times. \( I_{i,t-1} \) is a dummy variable that indicates whether the economy is in sudden stop recession or not. In this regression, the government spending is instrumented by \( I_{i,t-1} \times \text{shock}_{i,t} \) and \( (1 - I_{i,t-1}) \times \text{shock}_{i,t} \) in respective regimes. The estimates of \( m_{A,h}^{x} \) and \( m_{B,h}^{x} \) are spending multipliers during sudden stop crisis and normal times.

In order to account for the interaction of sudden stop with exchange rate regime and the sign effect of fiscal shocks, I further extend the baseline regression. I divide my sample into the country-episodes of pegged exchange rate “P” or flexible exchange rate “F”, and fiscal expansion “+” or fiscal contraction “−”. First, let \( 1_{i,t-1}^{P} \) (\( 1_{i,t-1}^{F} \)) denote the economy is under predetermined (flexible) exchange rate regime, then I run the four-regime regression,

\[
\sum_{j=0}^{h} x_{i,t+j} = I_{i,t-1} \times 1_{i,t-1}^{P} \times \left[ c_{P,A} + \alpha_{P,A,i} + m_{P,A,h}^{x} \sum_{j=0}^{h} g_{i,t+j} + \Phi_{P,A,h}(L) z_{i,t-1} \right] \\
+ (1 - I_{i,t-1}) \times 1_{i,t-1}^{P} \times \left[ c_{P,B} + \alpha_{P,B,i} + m_{P,B,h}^{x} \sum_{j=0}^{h} g_{i,t+j} + \Phi_{P,B,h}(L) z_{i,t-1} \right] \\
+ I_{i,t-1} \times 1_{i,t-1}^{F} \times \left[ c_{F,A} + \alpha_{F,A,i} + m_{F,A,h}^{x} \sum_{j=0}^{h} g_{i,t+j} + \Phi_{F,A,h}(L) z_{i,t-1} \right] \\
+ (1 - I_{i,t-1}) \times 1_{i,t-1}^{F} \times \left[ c_{F,B} + \alpha_{F,B,i} + m_{F,B,h}^{x} \sum_{j=0}^{h} g_{i,t+j} + \Phi_{F,B,h}(L) z_{i,t-1} \right] + u_{i,t+h}
\]

The comparison between \( m_{P,A,h}^{x} \) and \( m_{P,B,h}^{x} \) (\( m_{F,A,h}^{x} \) and \( m_{F,B,h}^{x} \)) shows the relative effectiveness of fiscal policy during sudden stops or normal times for the countries under pegged (flexible) exchange rate regime. Second, when I explore the asymmetric effect on fiscal multiplier, I define the indicator for a positive shock \( 1_{i,t-1}^{+} = [u^{g}(g_{i,t-1}) \geq 0] \), where \( u^{g}(...) \) is the estimated SVAR residual. Similarly, the indicator
of a negative shock is $1_{i,t-1} = [u^i(g_{i,t-1}) < 0]$. Then I run the regression,

$$
\begin{align*}
\sum_{j=0}^{h} x_{i,t+j} = & I_{i,t-1} \times 1_{i,t-1}^+ \times \left[ c_{+,A} + \alpha_{+,A,i} + m_{+,A}^x \sum_{j=0}^{h} g_{i,t+j} + \Phi_{+,A,h}(L)z_{i,t-1} \right] \\
+ & I_{i,t-1} \times 1_{i,t-1}^- \times \left[ c_{-,A} + \alpha_{-,A,i} + m_{-,A}^x \sum_{j=0}^{h} g_{i,t+j} + \Phi_{-,A,h}(L)z_{i,t-1} \right] \\
+ & (1 - I_{i,t-1}) \times 1_{i,t-1}^+ \times \left[ c_{+,B} + \alpha_{+,B,i} + m_{+,B}^x \sum_{j=0}^{h} g_{i,t+j} + \Phi_{+,B,h}(L)z_{i,t-1} \right] \\
+ & (1 - I_{i,t-1}) \times 1_{i,t-1}^- \times \left[ c_{-,B} + \alpha_{-,B,i} + m_{-,B}^x \sum_{j=0}^{h} g_{i,t+j} + \Phi_{-,B,h}(L)z_{i,t-1} \right] + u_{i,t+h}
\end{align*}
$$

The difference between $m_{+,A}^x$ and $m_{-,A}^x$ (between $m_{+,B}^x$ and $m_{-,B}^x$) shows the asymmetric effect of fiscal policy during sudden stop episodes (normal times).

### C.2 Robustness Check

#### C.2.1 Testing instrument relevance

I test the relevance of the BP shock as the instrument in the baseline linear and non-linear model. The Kleinpergan-Paap rk F statistic is used to test whether we have a weak instrument. Following Ramey & Zubairy (2014), a statistic greater than 23 leads us to reject the null hypothesis and conclude the instrument is relevant. The result is shown in appendix E.

#### C.2.2 Sensitivity to the “three steps method”

Following Jordà et al. (2005)’s seminal paper, most of empirical studies using local projection method adopt the “three steps method” to estimate fiscal multipliers. They estimate the impulse responses from a series of forecasting regressions and then bootstrap fiscal multipliers using simulations. In order to see whether our baseline result is sensitive to this alternative, I also estimate the state-contingent multipliers using the “three steps method”. The process is that, for each horizon $h = 1, 2, ..., H$, I estimate the regression model,

$$
\begin{align*}
x_{i,t+h} = & I_{i,t-1} \times \left[ c_A + \alpha_{A,i} + \beta_A^x \text{Shock}_{i,t} + \Phi_{A,h}(L)z_{i,t-1} \right] \\
+ & (1 - I_{i,t-1}) \times \left[ c_B + \alpha_{B,i} + \beta_B^x \text{Shock}_{i,t} + \Phi_{B,h}(L)z_{i,t-1} \right] + u_{i,t+h}
\end{align*}
$$

$x_{i,t+h}$ include both the variables of the interest and government spending. The collection of the coefficient $\beta_A^x$ ( $\beta_B^x$) provides the dynamic response of variable $x$ at time $t + h$ after the shock hit the economy at time $t$ supposing the economy stays in the same financial regime A (B). The cumulative multipliers of horizon $H$ that is measured as the cumulative change of $x$ per cumulative
change in government spending is calculated as,

\[ m_{x,H}^A = \sum_{h=0}^{H} \beta_{x,h}^A, \]
\[ m_{x,H}^B = \sum_{h=0}^{H} \beta_{x,h}^B, \]

The confidence bands can be get by Monte Carlos simulations based on the Newey-West variance-covariance matrix from the impulse response estimation. The result is in appendix E.

C.2.3 Sensitivity to alternative instruments

Instead of taking SVAR residuals as fiscal instrument in the panel data regression, Ramey & Zubairy (2014) directly uses the current government spending observations (after the transformation). They argue that controlling for past economic variables in the right hand side, the BP shocks are already embedded as a part of spending observations. However, SVAR imposes extra restrictions on the residuals in addition to the timing of fiscal response. These restrictions could provide additional information in identifying the effect of government spending shock. In appendix E, I also compare the baseline result with the one directly taking spending observations as the instruments.

D Additional tables

<table>
<thead>
<tr>
<th>Table 9: Data Sources</th>
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| **GDP:** Definition: real GDP over trend GDP  
Data sources: IFS: Nominal GDP adjusted by GDP deflator or CPI (NGDP_XDC) |
| **Government spending:** Definition: real government consumption over trend GDP  
Data sources: IFS: Public Final Consumption Expenditure, General Government (NCGG_XDC) |
| **Private consumption:** Definition: real private consumption over trend GDP  
Data sources: IFS: Household Consumption Expenditure, incl. NPISHs (NCP_XDC) |
| **Private investment:** Definition: real fixed capital formation over trend GDP  
Data sources: IFS: Gross Capital Formation, Gross Fixed Capital Formation, Corporations, Households, and Non-profit Institutions Serving Households (NFI_XDC) |
| **Current account-GDP ratio:** Definition: Current account balance divided by GDP  
Data sources: IFS: Balance of Payments, Supplementary Items, Current Account, Net (excluding exceptional financing) (BCAXF_BP6_USD) or WDI: Current account balance (BN.CAB.XOKA.CD) |
| **Real exchange rate:** Definition: CPI-based real effective exchange rate or nominal exchange rate divided by consumer price index  
Data sources: IFS: Real Effective Exchange Rate, based on Consumer Price Index (EREER_JX) or IFS: Domestic Currency Per US Dollar, Period Average (ENDA_XDC_USD_RATE) |
E Additional Figures

E.1 Data Coverage

Figure 13: Peg and Flex Sample

![Diagram showing data coverage for various countries with sudden stop events: Peg and Flex over the years 1980 to 2015. The countries listed include Argentina, Bolivia, Bulgaria, Czech Republic, Ecuador, Estonia, Greece, Hungary, Italy, Latvia, Lithuania, Malaysia, Peru, Portugal, Spain, Turkey, Thailand, Switzerland, Russia, Colombia, Brazil, Chile, Hungary, Poland, Romania, and South Africa. The diagram indicates the periods of sudden stop events for both Peg and Flex systems.](image-url)
E.2 Empirical Regression

Figure 14: Fiscal Multiplier of S.S. and N.T. (in US$)

Note: The variables in the regression are denominated in US$, except real change rate as indexed numbers and current account ratio as percentage. Red curves are cumulative multipliers during sudden stop episodes; blue curves are cumulative multipliers in normal times. Dashed curves represent 80% confidence bands based on HAC standard deviations.
Figure 15: Fiscal Multiplier of S.S. and N.T. (in LCU)

Note: The variables in the regression are denominated in Local Currency Units (LCU), except real change rate as indexed numbers and current account ratio as percentage. Red curves are cumulative multipliers during sudden stop episodes; blue curves are cumulative multipliers in normal times. Dashed curves represent 80% confidence bands based on HAC standard deviations.
Figure 16: Fiscal Multiplier of S.S. and N.T. in different ERR (in US$)

Note: The variables in the regression are denominated in US$, except real change rate as indexed numbers and current account ratio as percentage. Red curves are cumulative multipliers during sudden stop episodes; blue curves are cumulative multipliers in normal times. Dashed curves represent 80% confidence bands based on HAC standard deviations. The definition of exchange rate regime follows Ilzetzki et al. (2017).
Figure 17: Fiscal Multiplier of S.S. and N.T. in different ERR (in LCU)

Note: The variables in the regression are denominated in LCU, except real change rate as indexed numbers and current account ratio as percentage. Red curves are cumulative multipliers during sudden stop episodes; blue curves are cumulative multipliers in normal times. Dashed curves represent 80% HAC confidence bands based on HAC standard deviations. The definition of exchange rate regime follows Ilzetzki et al. (2017).
Figure 18: Fiscal Multiplier of positive and negative shocks (in US$)

Note: The variables in the regression are denominated in US$, except real change rate as indexed numbers and current account ratio as percentage. Red curves are cumulative multipliers associated with positive shocks; blue curves are cumulative multipliers associated with negative shocks. The sign of shocks comes from the SVAR residuals for each country. Dashed curves represent 80% confidence bands based on HAC standard deviations.
Figure 19: Fiscal Multiplier of positive and negative shocks in S.S. and N.T. (in US$)

**FM under positive and negative shocks for S.S. vs N.T. (in US$)**

Note: The variables in the regression are denominated in US$, except real change rate as indexed numbers and current account ratio as percentage. Red curves are cumulative multipliers associated with positive shocks; blue curves are cumulative multipliers associated with negative shocks. The sign of shocks comes from the SVAR residuals for each country. Dashed curves represent 80% confidence bands based on HAC standard deviations.
Figure 20: Fiscal Multiplier of positive and negative shocks (in LCU)

Note: The variables in the regression are denominated in LCU, except real change rate as indexed numbers and current account ratio as percentage. Red curves are cumulative multipliers associated with positive shocks; blue curves are cumulative multipliers associated with negative shocks. The sign of shocks comes from the SVAR residuals for each country. Dashed curves represent 80% confidence bands based on HAC standard deviations.
Figure 21: Fiscal Multiplier of positive and negative shocks in S.S. and N.T. (in LCU)

Note: The variables in the regression are denominated in LCU, except real change rate as indexed numbers and current account ratio as percentage. Red curves are cumulative multipliers associated with positive shocks; blue curves are cumulative multipliers associated with negative shocks. The sign of shocks comes from the SVAR residuals for each country. Dashed curves represent 80% confidence bands based on HAC standard deviations.
Figure 22: The effect of shock persistence (in US$)

Note: The figures in the left compare fiscal multipliers for the countries with high (red) and low (blue) shock persistence. The middle and right panel shows respectively within certain persistence group, the fiscal multipliers during N.T. and S.S. episodes. All variables in the regression are denominated in US$. The countries with auto-correlation higher than 0.7 are classified as high persistence group.
Figure 23: The effect of shock persistence (in LCU)

Note: The figures in the left compare fiscal multipliers for the countries with high (red) and low (blue) shock persistence. The middle and right panel shows respectively within certain persistence group, the fiscal multipliers during N.T. and S.S. episodes. All variables in the regression are denominated in LCU. The countries with auto-correlation higher than 0.7 are classified as high persistence group.
Figure 24: Weak Instrument Test

![Graph showing Kleibergen-Paap rk Wald F statistic (in US$) vs. horizon]

Note: The variables in the regression are denominated in US$, except real change rate as indexed numbers and current account ratio as percentage. The figure shows the Kleibergen-Paap rk F statistics minus 23 in both the linear (black curve) and nonlinear regressions (red curve for S.S. and blue curve for N.T.).
Note: The variables in the regression are denominated in LCU, except real change rate as indexed numbers and current account ratio as percentage. The figure shows the Kleibergen-Paap rk F statistics minus 23 in both the linear (black curve) and nonlinear regressions (red curve for S.S. and blue curve for N.T.).
Figure 26: Sensitivity to the “three steps method” (in US$)

Note: The variables in the regression are denominated in US$, except real change rate as indexed numbers and current account ratio as percentage. The upper panel is the 1st and 2nd step regressions. In the lower panel, I construct the fiscal multipliers by bootstrap simulation. The standard error is derived using the Newey-West variance-covariance matrix from the impulse response estimations. Red curves are cumulative multipliers during sudden stop episodes; blue curves are cumulative multipliers in normal times. Dashed curves represent 80% confidence bands based on HAC standard deviations.
Figure 27: Sensitivity to the “three steps method” (in LCU)

IRFS: three steps method (in LCU)

Note: The variables in the regression are denominated in LCU, except real change rate as indexed numbers and current account ratio as percentage. The upper panel is the 1st and 2nd step regressions. In the lower panel, I construct the fiscal multipliers by bootstrap simulation. The standard error is derived using the Newey-West variance-covariance matrix from the impulse response estimations. Red curves are cumulative multipliers during sudden stop episodes; blue curves are cumulative multipliers in normal times. Dashed curves represent 80% confidence bands based on HAC standard deviations.
Figure 28: Alternative Instrument (in US$)

Note: The variables in the regression are denominated in US$, except real change rate as indexed numbers and current account ratio as percentage. In this exercise, I directly use the current government spending observations $g_t$ as the instrument for endogenous regressors in the “one step estimation”. Red curves are cumulative multipliers during sudden stop episodes; blue curves are cumulative multipliers in normal times. Dashed curves represent 80% confidence bands based on HAC standard deviations.
Figure 29: Alternative Instrument (in LCU)

Note: The variables in the regression are denominated in LCU, except real change rate as indexed numbers and current account ratio as percentage. In this exercise, I directly use the current government spending observations $g_t$ as the instrument for endogenous regressors in the “one step estimation”. Red curves are cumulative multipliers during sudden stop episodes; blue curves are cumulative multipliers in normal times. Dashed curves represent 80% confidence bands based on HAC standard deviations.
E.3 The Three-Period Model

Figure 30: The Optimal Borrowing in Period 1: Marginal Benefits and Marginal Costs

Note: The marginal benefit of borrowing in period 1 is $u_{CT}(1)$. The private marginal cost is $u_{CT}(2)$. The social marginal cost, on the other hand, incorporates the possibility of financial constraint binding in period 2, with externality $u_{CT}(2) \frac{\kappa_2^S}{1-\kappa_2^S} \frac{\partial \Lambda_2}{\partial \delta}$. The government spending mitigates the severity of crisis, thus reduce the social marginal cost at all debt states.

Figure 31: The Spending Multiplier in the Three-Period Model

Note: These figures show the fiscal multipliers on $c_2$ and $y_2^N$ in the three-period model. The multipliers here (defined as $\frac{\Delta x_2}{\Delta g_2}$) depend on the state at the beginning of period 2: $(b_2, y_2^T, g_2)$. The non-binding (binding) state is the one when $y_2^T = 1$ ($y_2^T = 0$) while $b_2$ and $g_2$ are kept the same at their mean values. The benchmark parameter values are $\rho_G = 0.95$, $\theta = 0.45$, $\omega^G = 0$, $\omega = 0.35$, and I change one parameter at a time. I assume with the persistent effect of government purchase: $g_3 = \bar{g}^{1-\rho_G} g_2^{\rho_G}$ where $\bar{g}$ is the steady state.
E.4 Full Model

Figure 32: Sudden Stop Event Analysis

Note: The figure plots the averaged 9-periods event window of sudden stop episodes from the data and the model. The sudden stop is defined in the same way as before. The dashed series show the average of 6 most typical and severe sudden stop crises: Argentina 2002 Q1, Mexico 1995 Q1, Ecuador 1999 Q1, Malaysia 1998 Q1, Korea 1998 Q1, and Thailand 1998 Q1. For the data, the left axis is the percentage deviation from their HP-filtered trends; for the model, the right axis shows each variable’s deviation from its mean.
Figure 33: Cumulative Multipliers: Flex

Note: These are the median paths of theoretical multipliers starting from the “boom” (red dashed) and the “bust” (black solid) states for an economy with flexible exchange rate $\gamma^E = 1$. The simulation of fiscal multipliers is the same as generating GIRF where I compare the simulation path with and without a unit government spending shock. The upper panel is the cumulative GDP and consumption multiplier. The real exchange rate is in term of cumulative percentage differences between the path with a government spending shock and the path without the shock. The welfare multiplier is the certainty equivalence that equalizes agents’ utility between these two paths.

Figure 34: Cumulative Multipliers: Pegged

Note: These are the median paths of theoretical multipliers starting from the “boom” (red dashed) and the “bust” (black solid) states for an economy with pegged exchange rate $\gamma^E = 0$. See figure 33 for detailed information.
Figure 35: Multipliers for Different Shock Persistence

Note: The figure plots the median value of impact multipliers under different degree of shock persistence. This is the model with DNWR and pegged exchange rate regime. The boom and bust state are defined by simulation, the same as before.
Figure 36: Multipliers for the investment economy: Flexible exchange rate

Note: The figures plot the median paths of simulation starting from the boom (red dashed) and bust (black solid) states for an investment economy with flexible exchange rate $\gamma^E = 1$. The numbers for in the upper left panel is the increase of domestic interest rate (in percentage) after a government spending shock. The numbers in the right panel reports the cumulative change of capital price between the path with and without government spending shocks. The numbers in the bottom panels are cumulative multipliers of investment in local and foreign currencies.
Figure 37: Impact Multipliers for Different $\gamma^E$

Note: The figure plots the median value of impact multipliers under different exchange rate policy coefficients. The boom and bust state are defined by simulation, the same as before.
Figure 38: Cumulative Multipliers for Different $\gamma^E$

Note: The figure plots the median value of cumulative multipliers under different exchange rate policy coefficients. The dashed (solid) curves are multipliers when the economy starts from the bust (boom). The boom and bust states are defined by simulation, the same as before.