Optimal Forbearance of Bank Resolution

Linda M. Schilling*

December 13, 2017

Abstract

This paper analyzes optimal strategic delay of bank resolution (forbearance) in a setting where a resolution authority (RA) observes withdrawals of deposits at the bank level. Withdrawals may accumulate to a bank run. RA needs to decide and commit ex ante when to optimally intervene (suspend convertibility), by this changing depositors’ behavior. Optimal forbearance depends on deposit insurance coverage and efficiency with which RA liquidates assets. Under low insurance coverage, the optimal policy can be to commit to not intervene at all during runs even though intervention was ex post efficient. Under high insurance coverage or under efficient liquidation of assets, it is optimal to intervene as soon as possible.

Key words: Bank resolution, Forbearance, Recovery Rates, Bank Run, Deposit Insurance, Global Games

JEL Classification: G28,G21,G33

*Utrecht University, l.m.schilling@uu.nl, and Becker Friedman Institute for Economics at University of Chicago. I thank Harald Uhlig, Benjamin Brooks, Mikhail Golosov, Zhiguo He, Joonhwi Joo, Eugen Kovac, Philip Schnabl, Jeremy Stein and Philipp Strack for very insightful comments on the paper.
1 Motivation

This paper addresses optimal strategic delay ('forbearance') of bank resolution in a setting where after arrival of bad information on bank returns (solvency shock), partially insured depositors fear for the uninsured part of their deposit and run on the bank. The bank is forced to pledge assets to raise cash for repayment to depositors. Since the bank exhibits a liquidity mismatch on her balance sheet, the bank’s assets are not sufficient to cover repayments to all depositors, thus the deposit insurance becomes liable. To protect the insurance, a resolution authority (RA) exists, which monitors bank withdrawals and has the legal authority to intervene during a run. By intervention, RA seizes bank assets and suspends convertibility of deposits. By this, she protects the deposit insurance fund since she prevents partially insured depositors to run at the expense of other depositors and the insurance, see Martin et al. (2017) for empirical evidence on such incidents.

The question we ask is, what is the welfare maximizing measure of withdrawals RA should tolerate before intervening ('forbearance policy'), given that she wants to protect the deposit insurance from losses and given that her forbearance policy impacts depositors behavior and by this feeds back into the likelihood of bank resolution.

Our analysis is motivated by the differences in the bank resolution procedures of the European Monetary Union versus United States and the consequences of these differences. In 2014, the European Union passed the 'Bank Recovery and Resolution Directive' (BRRD) which is supposed to unify the tools for bank resolution in Europe and resolve systemically relevant banks under the centralized 'Single Supervisory Mechanism' (SSM), supervised by the European Central Bank. If the European Council or Commission however objects a resolution scheme proposed by the Single Resolution Board, resolution of the bank in question will be implemented by national resolution authorities 'in accordance with national law’ transposing the Bank Recovery and Resolution Directive. That is, the BRRD deliberately implemented the option to delay bank resolution of systemically important banks and to resolve these banks at national terms (bankruptcy laws) as opposed to the centralized mechanism.1 In contrast, in the U.S., the Federal Deposit Insurance

---

1See the webpage of the European Banking Authority (EBA) for a complete list of the 42 resolution authorities of 33 countries, http://www.eba.europa.eu/about-
Corporation (FDIC) is appointed as receiver if an FDIC insured depository institution becomes critically undercapitalized and operates under the least cost resolution requirement to 'minimize the present value of net losses incurred by the deposit insurer, regardless of other factors' such as maintaining market discipline, restricting risk to the banking sector as a whole (Bennett, 2001). That is, FDIC’s objective is to resolve banks with minimum delay, by this supposedly minimizing costs. The question, whether the U.S. approach is optimal and whether the European option to strategically delay bank resolution (forbearance) is valuable has to the best of our knowledge not been addressed yet in the literature and is at the heart of this paper. We analyze this question under varying degrees of deposit insurance coverage and recovery rates at which the resolution authority effectively operates.

In our setting, a resolution authority (RA) monitors withdrawals at the bank level and can decide when to intervene once she observes abnormally high withdrawals. Depositors run on the bank when they observe bad news about the bank’s asset returns. The RA has the authority to stop a run by suspending convertibility of deposits and putting the bank into receivership. Receivership here means, RA takes over control, seizes and liquidates bank assets at a ‘recovery rate’ and evenly allocates proceeds among all remaining depositors. To depositors, the ‘haircut’, the difference between face value of deposit and the pro rata share obtainable under resolution, matters for their decision to run on the bank. The measure of withdrawals RA tolerates before intervening affects depositors in three ways. First, under early intervention RA seizes a larger proportion of the asset and thereby prevents some withdrawing depositors from also securing the uninsured fraction of their deposit which could otherwise be shared by all remaining depositors to increase the pro rata share under resolution. Thus early intervention decreases the haircut. Second, since RA liquidates assets at a different rate than the bank does, the intervention policy impacts overall proceeds realized from liquidating the asset which are available for distribution to depositors. Third, the tolerance level to which RA commits affects depositors’ behavior since given resolution may take place, uninsured depositors are better off when running on the bank before RA takes over control. If RA follows a low tolerance forbearance policy, the chance of bank resolution seems higher ex ante, thus running is optimal
more often since by withdrawing depositors may obtain the uninsured part of their deposit. Altogether, RA’s tolerance threshold creates variation in strategic uncertainty. Taking into account how RA’s forbearance policy affects depositors’ behavior, we determine ex ante optimal forbearance policies depending on deposit insurance coverage and efficiency of RA’s liquidation procedure (recovery rate).

As main contribution of the paper, we show that under low insurance coverage, if RA liquidates as efficiently as the bank does during the run, the optimal forbearance policy is to never intervene and let depositors run at the bank until the bank is illiquid. This holds, even though intervention may be ex post efficient since a stricter intervention policy would alter depositors’ behavior in a way that inefficient runs become ex ante more likely. The result stands in contrast to the result in Diamond and Dybvig (1983) that the first best demand deposit contract can be implemented when combined with suspension of convertibility since there exists a unique equilibrium in dominant strategies to not run on the bank. In our setting, it can be shown that the ex ante probability of bank resolution strictly decreases as RA commits to show more forbearance, i.e. tolerates more withdrawals. Our result here differs from Diamond and Dybvig (1983), since depositors’ behavior and thus likelihood of bank resolution alters as RA commits to different intervention policies ex ante. In particular, here, by withdrawing conditional on the event bank resolution a depositors has the chance to recover additionally the uninsured part of her deposit, thus the incentive to run exists as long as insurance coverage differs from total coverage. In contrast, in Diamond and Dybvig (1983) the incentive to run vanishes since given suspension withdrawing depositors get zero.

As second main result, we point out that the optimal forbearance policy crucially depends on the amount of insurance coverage. In our model, deposit insurance satisfies no purpose for risk sharing since investors are risk-neutral. Instead, insurance impacts welfare indirectly by changing depositors behavior since insurance coverage provides bounds to the maximum deviation loss ('haircut') depositors face when choosing the 'wrong' action. We show, under high coverage, if the resolution authority liquidates as efficient as the bank does in the market, immediate intervention is optimal. This result holds since

\[\text{The result holds for patient depositors. Patient depositors can consume in period two. In the paper here all depositors are patient.}\]
under high coverage, depositors become insensitive to bad news because the
deviation loss goes to zero. In our model, bank resolution triggered by runs
is the only mechanism to interrupt investment. If investment is inefficient,
welfare increases if bank resolution occurs more often i.e. if forbearance is
low.

As third main result, we show that RA’s recovery rate impacts the optimal
forbearance policy. In general, depositors’ behavior is non-monotone in
forbearance. If RA liquidates assets during resolution only as efficient or close
to as efficient as the bank does during the run, bank stability improves in for-
bearance. As RA’s recovery rate during resolution increases as opposed to the
liquidation value of assets the bank achieves, stability becomes hump-shaped
with an interior stability maximizing policy, see Figure 2 and Proposition 3.2.
As recovery rate increases further, stability can become strictly decreasing in
forbearance. The intuition behind the result is an apparent trade-off deposit-
itors face as forbearance of bank resolution goes up. If RA intervenes later,
more assets are liquidated at low liquidation value as opposed to RA’s higher
recovery rate. Thus, overall proceeds available to depositors under resolu-
tion go down, which increases the haircut and aggravates the coordination
problem. On the other, as forbearance increases, the posterior belief that the
bank is resolved decreases\(^3\) thus the incentive to withdraw goes down since by
rolling over depositors can earn a higher interest rate on their deposit if the
bank is not resolved.

As fourth result, we show that for a fixed level of forbearance, the optimal
amount of insurance coverage is interior. This is, since under full coverage
inefficient investment occurs while under too low coverage inefficient runs take
place. Finally, we discuss extensions such as inefficient resolution authorities,
Emergency Liquidity Assistance, diversion of forbearance from one authority
to another (cascading forbearance) and pricing of deposit insurance.

\(^3\)From the perspective of the marginal agent who observes information which makes her
indifferent between rolling over or withdrawing, the proportion of withdrawing depositors
is uniform (Laplacian belief,\((\text{Morris and Shin, 2001})\)), thus changes in tolerance level for
withdrawals are equivalent to changes in posterior belief of bank resolution of the marginal
agent.
1.1 Literature

Our study is motivated by the articles by Martin et al. (2017) and Iyer et al. (2016) demonstrate that depositors react to solvency shocks or bad regulatory news about their banks by withdrawing funds and show that sensitivity of depositors to react depends on the (credible) insurance coverage level of deposits, see also Goldberg and Hudgins (2002); Baer et al. (1986); Goldberg and Hudgins (1996). Despite the existence of (partial) deposit insurance in many countries, the possibility of bank runs persists since only about 59% of U.S. domestic deposits are insured as of 2016, see appendices (FDIC, 2016). The European option to divert bank resolution to national authorities has as further implication that recovery rates to creditors differ not only across bank asset classes Gupton et al. (2000) but also across national resolution authorities by bankruptcy code (Davydenko and Franks, 2008), see also (Bris et al., 2006).

The paper is connected to the literature stand on bank runs and liquidity risk (Diamond and Dybvig, 1983; Bryant, 1980) and is closest in the model to Goldstein and Pauzner (2005) who analyze optimal risk sharing under partial repay and Schilling (2017) who analyzes bank values under repricing of debt, both in global games. Also closely related are Morris and Shin (2016) and Rochet and Vives (2004) who consider credit risk, respectively interventions by a lender of last resort and Eisenbach (2016), Allen et al. (2017) and Matta and Perotti (2017) who analyze efficiency of asset liquidation, government guarantees respectively secured repo funding under roll over risk, all of them using global games to obtain an equilibrium selection (Carlsson and Van Damme, 1993; Morris and Shin, 2001).

Our paper further adds to the literature strand on banking crises and resolution. (Keister, 2015) studies how anticipation of bail outs ex ante changes financial fragility via liquidity choices. We instead analyze how timing of intervention changes fragility via change of haircuts and strategic uncertainty. Keister and Mitkow (2016) study interaction between bailout policies and bank’s choice to (not) bail in her investors. Li (2016) studies bank stability under liquidity regulation in connection with bailouts. Farhi and Tirole (2012) study the impact of anticipated bail outs on leverage choices, see also Chari and Kehoe (2016), Bianchi (2012) and Walther and White (2017).

\footnote{59\% of deposits of banks insured with the FDIC are covered.}
Important in our framework is that withdrawals occur and are observed gradually but depositors make their roll over decision simultaneously without yet knowing their position in the queue. As a consequence, the chance to recover the entire deposit is always strictly positive and the incentive to run exists, see also He and Manela (2016); Green and Lin (2003) and Peck and Shell (2003).

2 Model

We extend the model set out by Goldstein and Pauzner (2005). There are three time-periods and no discounting. A bank invests in a risky and illiquid long-term project and finances investment with equity and short-term debt. Depositors and equity investor are risk-neutral and are each endowed with one unit to invest. The bank follows a particular capital structure rule, the debt ratio $\delta \in (0, 1)$ is exogenous. For instance, the bank targets a particular amount of influence outside investors may exert on the bank. There are constant returns to scale, thus we normalize initial bank investment to one unit. The bank has limited liability. Depositors are small, symmetric and given by a continuum $D = [0, \delta]$ of measure $\delta$ (by normalization). There is free entry, thus the bank is in perfect competition to other banks and makes zero profit. The markets for equity and deposits are segmented. For instance, depositors have high participation costs such that they do not invest in equity. The opportunity costs of deposits in the deposit market is $u$. The bank competes for deposits, for given debt ratio $\delta$ she sets coupon $k$ such that utility from the debt contract equals $u$. All additional profits go to shareholders. Shareholders invest in equity if shareholder value exceeds $\tilde{e}$.

Investment The risky investment (asset) pays off $H$ at time two with probability $\theta$ and zero otherwise, where $\theta \sim U[0, 1]$ is the unobservable, random state of the economy (asset return probability). At time one, the asset yields no cash flow but can be prematurely liquidated or used as collateral to borrow cash in the competitive money market for value $l < 1$ per unit. To raise short-term debt, the bank offers a debt contract which for each initially invested unit at time zero, promises to pay a coupon of one unit if the contract

\footnotesize
\begin{itemize}
\item Funding liquidity equals market liquidity.
\end{itemize}
is liquidated at time one ('withdraw') or a larger coupon $k > 1$ if the contract is 'rolled over' until time two. Equity of measure $1 - \delta$ remains invested for two periods.

**Signals and actions (interim)** Before depositors decide on their action they observe noisy private signals about the state of the world

$$\theta_i = \theta + \varepsilon_i$$

where the idiosyncratic noise terms are independent of state $\theta$ and iid distributed according to $\varepsilon_i \sim U[-\varepsilon, +\varepsilon]$. The signal contains information on how likely the asset pays off high return $H$ such that the bank can pay $k$ at time $t_2$. On the other hand, since signals are correlated, the signal also contains information on beliefs of other agents. Depositors can decide to not roll over and withdraw their deposit for two reasons, either they think that the asset will not pay off high, thus coupon $k$ will not be paid. Or they believe that many other depositors withdraw which triggers bank resolution, so it is optimal to also withdraw. The bank does not observe the state but monitors the measure of consecutive withdrawals at the interim stage. To serve withdrawing depositors, the bank pledges assets to a counterparty $C$ in the money market to prevent foregoing return $H$. Since the asset is risky, the bank needs to pay interest $i$ on the borrowed amount (see later how $i$ is priced).

**Insurance** Each single deposit is insured up to exogenous fraction $\gamma \in (0, 1)$ by a deposit insurance company. If the bank becomes illiquid or insolvent, the insurance repays depositors fraction $\gamma$ of the coupon the bank owes at the point in time the bank defaults. The bank is prone to runs. The bank’s debt ratio is such that overall debt claims exceed the amount of cash the bank can raise by liquidating/pledging the entire asset $\delta \in (l, 1)$. Since only fraction $\gamma$ of deposits are insured, bad news on asset return probability $\theta$ can trigger a run, since by withdrawing a depositor has the chance to recover the entire deposit instead of only a fraction. The deposit insurance may encounter losses when runs occur since then she has to pay the insured fraction of the coupon.

**Resolution Authority (RA)** Our model adds new to the literature a strategic *resolution authority* (RA). RA has the legal authority to protect the
deposit insurance by seizing bank assets (putting the bank into receivership) and by this stopping runs on the bank (suspension on convertibility).

Under receivership, the bank stops service of withdrawing depositors (suspension of convertibility and mandatory stay) and no longer pledges assets at funding liquidity \( l \) in the market. By this, intervention prevents more depositors to run at the expense of the deposit insurance. The RA takes over control of the remaining fraction of the asset, liquidates the remaining asset at exogenous recovery rate \( r \) and evenly allocates realized proceeds among all remaining bank depositors who were not paid so far. The RA liquidates more efficiently than the bank does in the market, \( r \geq l \) (to be relaxed later). For instance, \( l \) can be interpreted as a fire sales price or the RA has more time to resolve the bank and can thus find a buyer with higher willingness to pay or valuation for the asset.

RA cannot observe the state but monitors the bank closely. She perfectly observes all withdrawals which occur at the bank level, by this making inferences about the state. If at the interim period RA observes withdrawals in excess of a particular threshold, which she optimally sets, RA infers that the state is 'low' and intervenes. RA sets this threshold publicly observable and in a way that the deposit insurance makes no losses. More concrete, if the bank is forced to pledge a fraction larger than '\( a \)' of the entire asset to

\[
\begin{align*}
\text{Asset} \\
\alpha \quad 1 - \alpha
\end{align*}
\]

\[
\text{Fraction } \alpha \text{ of asset liquidated at } l \text{ to serve queue before suspension.} \\
\text{Realized cash: } l a \\
\text{Fraction } 1 - \alpha \text{ of asset liquidated at } r \text{ under bankruptcy proceedings.} \\
\text{Realized cash: } (1 - a) r
\]

\[
\text{1a depositors receive coupon 1 before suspension} \\
\text{\( \delta n - 1a \) depositors not served in queue} \\
\text{\( \delta - \delta n \) depositors did not withdraw} \\
\text{\( \delta - \delta n \) depositors have claim on proceeds } (1 - a) r \text{ after bankruptcy}
\]

\[dn: \text{Length of queue}\]

Figure 1: Forbearance-weighted liquidation procedure of assets: Forbearance determines the proportion of the asset liquidated during the run versus under bank resolution.
serve withdrawing depositors, RA steps in, seizes remaining assets and puts
the bank into receivership. We call $a \in (0, 1)$ the RA’s forbearance policy.
Forbearance $a$ is common knowledge among all agents and is set by the RA
at time zero before depositors decide whether to roll over. This policy can be
understood as optimal ‘timing’ of bank’s suspension of convertibility with fol-
lowing mandatory stay, see (Diamond and Dybvig, 1983). Denote by $n \in [0, 1]$
the endogenous equilibrium proportion of depositors who decide to withdraw
at the interim period, thus measure $\delta n$ of depositors withdraw and claim one
unit. For given forbearance policy, the event ‘bank resolution’ is triggered if
$n$ realizes such that the measure of claimed funds exceeds the liquidation and
collateral value of the critical fraction of the asset at which RA steps in,

$$\delta n \cdot 1 \geq al \quad \Leftrightarrow \quad \{\text{Bank resolution}\} \quad (2)$$

For $a = 1$, RA does not intervene at all and lets the bank fail due to a
run. For $a < 1$, RA intervenes and secures fraction $1 - a$ of the asset after
observing how the bank was forced to pay out measure $al$ in cash to measure
$al$ of depositors, see Figure 1. Liquidation of this remaining fraction leads to
proceeds $r(1 - a)$ realized under bank resolution which are evenly allocated to
measure $\delta - la$ of remaining depositors who were not served so far$. Denote
by

$$s(a) := \frac{r(1 - a)}{\delta - la} \quad (3)$$

the pro rata share RA recovers when resolving the bank. RA can set $a$ such
that the pro rata share depositors obtain under bank resolution exceeds the
claim they had towards the deposit insurance $(\frac{1-a}{\delta - la}) \geq \gamma$. Define the maximum
forbearance RA can grant such that insurance runs no loss as

$$\overline{a}(r, \gamma) := \frac{r - \delta \gamma}{r - l \gamma} \quad (4)$$

where the bank’s capital structure $\delta$ satisfies $\delta < \frac{r}{\gamma}$ such that $\overline{a} > 0.$
Forbearance level $\overline{a}(r, \gamma)$ increases in recovery rate $r$ and decreases in insurance
coverage $\gamma$. For higher forbearance $a > \overline{a}$, the pro rata share undercuts the
insured amount of the deposit and the insurance has to pay the difference
$\gamma - s(a) \in [0, \gamma]$ to each depositor. Under resolution, each depositor obtains

$^6$This formulation is equivalent to saying that proceeds are allocated pro rata to depositors holding remaining $\delta - al$ debt claims.
Further, we assume that RA obeys a forbearance minimum $a > 0$ which can be interpreted in the sense that RA observes withdrawals with a delay and cannot intervene immediately. This assumption is grounded in legal constraints since the bank has to be insolvent given bank resolution occurs, see later subsection.\footnote{Here, the assumption is of technical nature since otherwise, bank resolution was always triggered for $a = 0$, see equation (2). Bound $\underline{a}$ can be arbitrarily close but bounded away from zero.}

RA’s objective is to set the welfare maximizing forbearance policy $a^*(\gamma, r)$ under the constraint $a^*(\gamma, r) \in (\underline{a}, 1]$. Deposit insurance in this model does not serve risk sharing purposes but plays an indirect role by affecting depositors’ sensitivity to bad news and thus the probability of bank resolution. Further, insurance coverage affects the maximum forbearance RA can grant before imposing losses on the deposit insurance. We take RA’s recovery rate and deposit insurance here as exogenously given. This can be justified when seeing deposit insurance as a parameter determined by politics beyond RA’s authority and recovery rates as being asset and country specific depending on national bankruptcy laws.

**Payoffs** A depositor’s payoff from 'withdrawing’ conditional on bank resolution equals

$$\frac{la}{\delta n} \cdot 1 + (1 - \frac{la}{\delta n}) \cdot s_\gamma(a)$$

where $\frac{la}{\delta n}$ is the probability to be served when withdrawing. Here we assume, as in Goldstein and Pauzner (2005), that in order to withdraw depositors queue in front of the bank and are served the coupon of one unit one after another until RA intervenes where a depositors’ positions in the queue is random. Given bank resolution occurs, depositors who roll over receive pro rata share $s_\gamma(a)$ instead of the long-term coupon since a too large proportion of other investors enforced bank resolution by withdrawing. We define the haircut

$$H(a) = \max(1 - s_\gamma(a), 0) \in [0, 1]$$

Further,

$$s_\gamma(a) := \max\left(\frac{r(1-a)}{\delta - la}, \gamma\right) \geq \gamma$$
as the difference between the pro rata share obtainable under resolution and the face value of deposits. In case that \( r \leq \delta \), the haircut is always positive and we do not need to consider the max operator, for instance if \( r = \ell \).

In particular,

<table>
<thead>
<tr>
<th>Event/ Action</th>
<th>Withdraw</th>
<th>Roll-over</th>
</tr>
</thead>
<tbody>
<tr>
<td>No resolution ( n \in [0, la/\delta] )</td>
<td>1</td>
<td>( \begin{cases} \frac{k}{\delta n}, p = \theta \ \gamma, p = 1 - \theta \end{cases} )</td>
</tr>
<tr>
<td>Bank resolution ( n \in (la/\delta, 1] )</td>
<td>( \frac{ln}{\delta n} \cdot 1 + (1 - \frac{ln}{\delta n})s_\gamma(a) )</td>
<td>( s_\gamma(a) )</td>
</tr>
</tbody>
</table>

Implicit in the payoff table above is the assumption that the bank can repay depositors at time two given no resolution occurs at time one and the asset pays off. This holds, if given no resolution and success of investment, asset return \( H \) is sufficiently large such that the bank can repurchase the pledged fraction of the asset and pay the interest rate \( i \) (see extension). Until the discussion in the extension we will assume this is the case.

**Information structure** We follow Goldstein and Pauzner’s global games information structure to obtain a unique equilibrium. We assume there are states \( \theta \) and \( \bar{\theta} \) which mark the bounds to dominance regions to obtain a unique equilibrium: We impose that for states in the range \([0, \theta] \) withdrawing is dominant while for high states rolling over becomes dominant. Boundary state \( \theta \) is given by the equation \( k\theta + \gamma(1 - \theta) = 1 \). That is

\[
\theta = \frac{1 - \gamma}{k - \gamma} \quad (8)
\]

By \( k > 1 > \gamma \), for states below \( \theta \) the expected value of rolling over and either receiving high coupon or the insured fraction \( \gamma \) undercuts the payoff from withdrawing. For the upper dominance region, we assume that for states \( \theta > \bar{\theta} \) the asset pays \( H \) at time two for sure.\(^8\) Thus, the asset becomes risk

---

\(^8\)More intuitively, we can now rewrite the payoff from withdrawing as \( s_\gamma(a) + \frac{ln}{\delta n} \cdot H(a) \) where a withdrawing depositor receives \( s(a) \) for sure and with probability \( la/(\delta n) \) she receives the haircut on top.

\(^9\)To make this assumption work, the precise return probability of the asset would need to be \( p(\theta) = \begin{cases} \frac{\theta}{\bar{\theta}}, \theta \in [0, \bar{\theta}] \\ 1, \theta \in [\bar{\theta}, 1] \end{cases} \) with \( \theta \in [0, 1], \bar{\theta} \in (0, 1) \). The constant \( \frac{1}{\bar{\theta}} \) however does not alter incentives and as \( \bar{\theta} \to 1 \), all results apply and it is without loss of generality to consider \( p(\theta) = \theta, \theta \in [0, 1] \).
free and we assume that the bank can transfer the certain return across time via a repurchase agreement to time one at zero interest such that the coordination problem at time one vanishes by $H > 1 > \delta > \delta n$ for all $n \in [0, 1]$. For idiosyncratic noise $\varepsilon_i \sim U[-\varepsilon, \varepsilon]$ we see that for $\varepsilon$ sufficiently small, signals become precise. Since the true state has a maximum distance of $\varepsilon$ to the private signal, depositors can infer from their signals whether the state $\theta$ is located in either of the dominance regions: If $\theta_i < \theta - \varepsilon$, the state has to lie in the lower dominance region, for signal $\theta_i > \theta + \varepsilon$ the state has to lie in the upper dominance region. We assume that $\varepsilon$ is sufficiently small such that such signal observations realize with positive probability.

**Timing** At time zero the random state realizes unobservably to all agents. RA sets her forbearance policy $a$, taking deposit insurance, recovery rate and behavior that follows in the subgame at her chosen forbearance policy as given. At time one, all depositors observe RA’s forbearance policy and noisy, private signals about the state. Then they decide whether to withdraw. RA monitors withdrawals. If withdrawals exceed threshold $a_l$ the bank is put into receivership and gets resolved. Otherwise the game proceeds to period two and the asset pays off or not.

The analysis proceeds as follows. Via backward induction, we first analyze the interim stage. Depositors take as given RA’s forbearance policy, recovery rate $r$ and insurance coverage $\gamma$. We analyze how depositors’ behavior alters as RA shifts her forbearance policy for various recovery rates and levels of deposit insurance coverage. Going to the ex ante stage, we consider socially optimal policies $a^*$ where RA takes as given the coordination behavior of depositors that will follow in the subgame determined by her own policy.

All proofs can be found in the appendix. The equilibrium concept is subgame perfect Nash, where for given forbearance policy depositors play Bayes Nash at the interim stage.

### 3 Equilibrium coordination game

Forbearance policy $a$ determines the delay until the point of intervention at which control switches from bank managers (shareholders) to the resolution
authority RA respectively to depositors who enforce resolution by running. By this, forbearance interferes with depositors’ incentives in three different ways. First, the forbearance policy determines the overall realized proceeds from liquidation

\[ T(a) = al + (1 - a)r \] (9)

available to depositors at the interim period should resolution occur.\(^\text{10}\) By \(l < r\), liquidation at value \(l\) is inefficient. Second, forbearance determines how liquidation proceeds are split between withdrawing depositors and depositors who roll over, given a run by influencing the size of the haircut. As RA’s forbearance increases, more depositors can run (withdraw) at the expense of depositors who roll over, since the haircut \(H(a)\) increases in \(a\). Third, forbearance impacts strategic uncertainty among depositors. For total withdrawals \(\delta n\) in the range \([aL, l]\) bank resolution is triggered but in absence of a resolution authority, the bank had survived the run. This means, there are states of the economy where withdrawing from the bank (running) is optimal since RA is ready to intervene while rolling over had been optimal if RA did not exist (we proof this below). Moreover, the number of withdrawals the RA tolerates depends on forbearance, thus depositors change their behavior depending on whether the forbearance policy is harsh or lenient to be able to run ahead of other depositors to secure the uninsured fraction of their deposit.

To solve the game at the interim stage, we start with the impact of forbearance on strategic uncertainty. Depositors take RA’s forbearance policy \(a\), the debt contract, deposit insurance coverage, recovery rate and asset liquidation value as given when deciding whether to roll over their deposit.

**Proposition 3.1.** The game played by depositors has a unique equilibrium which is in trigger strategies. All depositors withdraw if they observe a signal below threshold signal \(\theta^*\) and roll over otherwise.

This existence and uniqueness result was first derived in Goldstein and Pauzner (2005). The trigger signal at which a depositor’s beliefs are such

\(^{10}\) Here, we assume that as bank resolution is triggered, since the bank stops managing the pledged fraction of the asset, counterparty C sells the pledged fraction \(a\) of the asset immediately. This can be justified when considering that the bank is an investment expert and return likelihood and asset return would deteriorate under different management, see (Diamond and Rajan, 2001).
that she is indifferent between rolling over and withdrawing at the limit is explicitly given by

\[ \theta^* = \frac{(1 - \gamma) - H(a) \ln \left( \frac{la}{\delta} \right)}{k - \gamma} \]  

(10)

where \( H(a) = 1 - s_\gamma(a) > 0 \) is the haircut (payoff difference) depositors face when they missed to withdraw from the bank and the bank is resolved.

If the haircut goes to zero, we see how the trigger approaches the bound to the lower dominance region \( \theta \).

Bank resolution occurs, if the measure of depositors who observe signals below threshold \( \theta^* \), and consequently withdraw, exceeds the critical value \( al \). In a trigger equilibrium, since we have infinitely many depositors, the measure of withdrawing depositors \( n \) is a deterministic function of the random state, and given by (56). Denote by \( \theta_b \) the critical state such that bank resolution occurs if the true state realizes below \( \theta_b \). Then \( \theta_b \) is implicitly given by

\[ n(\theta_b, \theta^*) = \frac{la}{\delta} \]  

(11)

Since the random asset return is uniformly distributed and bank resolution occurs if the state realizes below the critical state, the probability that bank resolution occurs is just equal to \( \theta_b \). Therefore, we define

**Definition 3.1.** We say bank stability increases if the probability of bank resolution \( \theta_b \) goes down.

At the limit, the trigger \( \theta^* \) and critical state \( \theta_b \) coincide, as do their comparative statics, by (61).

We are interested in how the critical state changes as RA alters her forbearance policy for given deposit insurance and recovery rate under which RA liquidates or sells assets. It turns out that depositors’ behavior changes in forbearance policy depending on how efficiently RA liquidates assets given the bank is resolved.

**Proposition 3.2** (Comparative statics - critical state). Assume RA intervenes before the insurance runs a loss \( a \in (\underline{a}, \overline{a}) \).

1a) For recovery rate close to liquidation value in the market, bank stability
monotonically improves in forbearance. 1b) As RA’s recovery rate increases, stability can become non-monotone in forbearance with a unique interior, stability maximizing policy.
1c) As recovery rate increases further, stability becomes monotone decreasing in forbearance.
2) As RA sets forbearance such that the insurance runs a loss \(a \in (\bar{a}, 1]\), stability monotonically improves in forbearance.

In Figure 2, as recovery rate increases from plot to plot, the slope of the equilibrium trigger flips from negative to u-shaped to positive.
Figure 2: Monotonicity of trigger varies in forbearance as recovery rate changes for different levels of deposit insurance $\gamma$. As recovery rates increase, the trigger goes from decreasing in forbearance to increasing in forbearance (bank stability goes from improving in forbearance to deteriorating in forbearance). Since the trigger monotonically decreases in insurance coverage $\gamma$, monotonicity is preserved as $\gamma$ changes. Held fixed through all graphs: $H = 4, l = 0.5, d = 0.9, k = 1.8$

To give some intuition for the change in the slope, as RA’s forbearance to resolve a bank increases, two effects impact depositors’ behavior. First, the longer the RA waits to intervene, the larger becomes the haircut (payoff difference) depositors have to take if the bank is resolved. This is, since for a longer waiting period till intervention, the bank is exposed to the run for a longer time\footnote{In terms of depositors she has to serve until intervention.} and is therefore forced to pledge a larger proportion of the asset.
at low funding liquidity \( l \leq r \) to serve withdrawing depositors the full coupon. The proportion of the asset which is liquidated during bank resolution under higher recovery rate is therefore smaller which decreases the pro rata share. Shortly, as forbearance increases, more depositors obtain the full coupon when withdrawing at the expense of depositors who roll over. As the haircut increases, the coordination problem among depositors is aggravated and bank stability goes down. On the other hand, as RA's forbearance increases, at the roll over stage, the marginal depositors' posterior belief\footnote{She observes exactly the trigger signal, thus she has a uniform belief about \( n \). As \( al/\delta \) increases, her belief that \( n \) exceeds \( al/\delta \) goes down.} that the bank will be resolved in the future goes down. Depositors know that bank resolution is triggered if more than \( la \) depositors withdraw. As forbearance goes up, RA tolerates a larger number of withdrawals before intervening, implying that for intervention to take place the run has to be larger. The decrease in belief of bank resolution relaxes the coordination problem. Allover, more lenience with the bank has a non-monotone impact on bank stability. If the recovery rate at which RA liquidates is close to the asset's liquidation value, the decrease in belief that the bank will be resolved dominates the increase in haircut should bank resolution occur and bank stability improves ex ante as the RA shows more forbearance. As RA's recovery rate increases, the effect of a change in haircut becomes stronger since liquidation under bank resolution is more favourable compared to liquidation in the market. For high recovery rates, the increase in haircut may outweigh the decrease in belief that resolution occurs and bank stability deteriorates in forbearance, or there exists an interior, stability maximizing forbearance policy.

Here, our assumption that there exists an upper bound on forbearance depending on the level of insurance coverage and RA's recovery rate, \( a < \bar{\alpha}(r, \gamma) \) given in (4), plays a role. As RA's forbearance policy exceeds upper bound \( \bar{\alpha}(r, \gamma) \), the proceeds from liquidating remaining assets under resolution undercut the insured amount of deposits, and the deposit insurance runs a loss since she is obliged to compensate depositors for the insured fraction \( \gamma \). But even if RA sets forbearance at the highest possible value \( a = 1 > \bar{\alpha}(r, \gamma) \) and by this runs a loss should resolution occur, the coordination problem among depositors will prevail since by assumption the asset is not sufficiently liquid to cover the face value of debt at the interim stage \( l < \delta \). By setting her forbearance policy, RA balances the haircut and strategic uncertainty among
depositors. As she sets a level such that the insurance runs a loss, the haircut becomes constant in forbearance and just equals the uninsured fraction of the deposit $1 - \gamma$. Thus, the trade-off between an increase in haircut and decrease in belief that resolution occurs vanishes. As forbearance exceeds $\bar{\pi}$, only the belief that resolution occurs keeps decreasing, thus bank stability improves monotonically over $(\bar{\pi}, 1]$.

Deposit insurance and RA’s recovery rate also impact depositors’ behavior:

**Lemma 3.2.** *Bank stability monotonically increases in RA’s recovery rate and deposit insurance coverage.*

Higher deposit insurance coverage decreases the risk of rolling over and therefore lowers the incentive to withdraw. On the other hand, alterations in deposit insurance have an effect on forbearance since the maximum forbearance policy $\bar{\pi}$ declines in deposit insurance. Intuitively, as the insurance company has higher obligations, for RA to protect the insurance fund she has to intervene sooner and the maximum lenience RA can show towards the bank goes down. As RA’s recovery rate goes up, proceeds RA recovers to depositors go up, the haircut depositors have to take given resolution decreases which relaxes the coordination problem. Further, as recovery rate goes up, RA’s maximum forbearance level $\bar{\pi}$ increases since proceeds she recovers rise.

4 Welfare - Ex ante stage

We now proceed to the ex ante stage at which the RA sets her forbearance level, taking as given depositors’ behavior that follows in the subgame.

4.1 Efficiency

To study the effect of forbearance on welfare, first observe that the efficient liquidation cut-off state below which investment in the asset should be interrupted is given by

$$\theta_e = \frac{r}{H}$$

(12)

This is since RA liquidates at better terms than the bank does during a run $r \geq l$ and since continuation value from investment equals $\theta H$. As RA’s forbearance with the bank increases, more assets are liquidated during the
run at low liquidation value as opposed to being liquidated by RA during resolution at higher recovery rate. Denote by

$$\theta_r(a) = \frac{T(a)}{H}$$  \hspace{1cm} (13)$$

the state at which realized liquidation value equals continuation value and \( \theta_r(0) = \theta_e \) if RA could set forbearance equal to zero, \( a \to 0 \). By \( a \in (a, 1] \), realized proceeds always fall short of the first best case \( T(a) < r \). As RA’s forbearance with the bank increases, a direct deadweight loss occurs if the bank is resolved since proceeds \( r - T(a) > 0 \) are lost. In addition, the change in forbearance indirectly impacts welfare via the change in critical bankruptcy state \( \theta_b \). Here, the relative position of the critical bankruptcy state compared to the efficient liquidation cut-off becomes crucial. If the chosen forbearance policy \( a \) is such that the resulting bankruptcy state \( \theta_b \) undercuts the efficient liquidation cut-off, 'underliquidation' of assets occurs in the range \( [\theta_b, \theta_e] \) since depositors are not sufficiently responsive to bad news. Thus, stability improvements (decrease in critical bankruptcy state) can harm welfare if inefficient continuation of investment becomes more pronounced. In particular, the bank does not liquidate assets voluntarily at the interim stage since she cannot observe the state, only the sequence of withdrawals to which she responds by pledging assets to repay.\(^{13}\) If on the other hand the bankruptcy state exceeds the efficient liquidation cut-off, \( [\theta_e, \theta_b] \), depositors are overly sensitive to bad news and run too often. A raise in bank stability would therefore lower the chance of inefficient runs and increase welfare.

The position of the bankruptcy state in relation to the efficient liquidation cut-off is determined not only by forbearance but also by deposit insurance and recovery rate.

**Lemma 4.1.** If deposit insurance is low, inefficient asset liquidation enforced by runs occurs for every recovery rate and forbearance policy. If deposit insurance is high, inefficient continuation of investment occurs for every recovery rate and forbearance policy.

Figure (3) shows, while inefficient runs occur at insurance level \( \gamma = 0.7 \)\(^{12}\). Even if she could observe the state she would not liquidate voluntarily (when depositors roll over), since for every unit she liquidates she realizes proceeds \( l \) which are not sufficient to cover claims \( \delta k > \delta > l \). This is result is due to a lack of reinvestment opportunities and the bank’s liquidity mismatch, see Schilling (2017).
for recovery rates in the range $[0.6, 0.8]$, the range shrinks to $[0.6, 0.73]$ under higher insurance coverage since depositors are less sensitive to bad news. The range of inefficient continuation widens from $[0.8, 1]$ under $\gamma = 0.7$ to $[0.73, 1]$ under higher coverage.

![Figure 3: As recovery rate increases, the efficient liquidation cut-off increases while the critical state goes down. This is since under higher recovery rate, asset liquidation is efficient for a greater range of states while bank stability improves in recovery rate since haircuts decrease. For sufficiently low recovery rate, the critical state $\theta_b$ exceeds the efficient cut-off $\theta_e$ and for states inbetween $[\theta_e, \theta_b]$ inefficient runs exist. As recovery rate goes up, this range narrows, respectively for high recovery rate, the critical state undercut the efficient liquidation cut-off and inefficient continuation of investment occurs for states in $[\theta_b, \theta_e]$. As insurance coverage increases, inefficient continuation already occurs for lower recovery rates as opposed to under low insurance coverage since depositors become less sensitive to bad news.

Define *welfare* as the total value from investment realized at forbearance policy $a$. For states below the critical state the bank is resolved which results in realized liquidation value $T(a)$, while for states above the critical state investment is continued.

$$W = T(a) \theta_b(a) + \int_{\theta_b(a)}^{\theta_e} \theta H \, d\theta$$  \hspace{1cm} (14)

In the first best case, liquidation takes place only for states below $\theta_e$,

$$W^{FB} = r \theta_e + \int_{\theta_e}^{1} \theta H \, d\theta$$  \hspace{1cm} (15)
Define the *deadweight loss at forbearance policy* \( a \) as the difference between first best and welfare at forbearance policy \( a \)

\[
D_{(r,\gamma)}(a) = W_{FB} - W = (r - T(a)) \theta_b(a) + \int_{\theta_b}^{\theta_e} (\theta H - r) d\theta
\]  

(16)

By \( a > a \), first best is not attainable since \( r - T(a) > 0 \). If RA can match the efficient liquidation state for given deposit insurance coverage \( |\theta_e - \theta_b| = 0 \) by setting \( a \) correspondingly, this may additionally increase the difference \( r - T(a) \). Further, matching the efficient liquidation state is not stable under variation of deposit insurance by Lemma 4.1.

Figure 4: Deadweight loss at forbearance policy \( a \) is given by the dark shaded area, welfare achieved equals the light shaded area.

Case 1: Inefficient continuation of investment for states in \( [\theta_b, \theta_e] \) where additionally the critical state undercut the state \( \theta_r \) at which switching from liquidation to continuation was optimal if forbearance \( a \) was exogenously fixed.

Figure 5: Case 2: Inefficient continuation of investment for states in \( [\theta_b, \theta_e] \) where the critical state exceeds the state \( \theta_r \).
Define the optimal forbearance policy $a^*(r, \gamma)$ as

$$a^*(r, \gamma) \in \arg \min D(r, \gamma)(a) \quad \text{subject to feasibility} \quad a^*(r, \gamma) \in (a, 1] \tag{17}$$

The dark shaded areas in Figures (4), (5) and (6) depict the deadweight loss under over- and under liquidation of assets. The light shaded areas show welfare realized under the particular forbearance policy.

### 4.2 Deadweightloss and Forbearance

To see how the change of deadweight loss in forbearance depends on changed coordination behavior and shifts in liquidation proceeds, it is instructive to analyze the derivative of the deadweight loss directly

$$\frac{\partial}{\partial a} D(a) = \left( \frac{\partial}{\partial a} (r - T(a)) \right) \theta_b(a) + (r - T(a)) \frac{\partial \theta_b}{\partial a} + \frac{\partial \theta_b}{\partial a} (\theta_b H - r)$$

A: Direct efficiency loss

B: change in direct loss due to change in stability

C: Change in efficiency of enforced liquidation due to change in stability

(18)

Positive term A is the change of the direct loss due to forbearance when liquidating a fraction of the asset at $l$ and thus realizing $T(a)$ instead of the higher value $r$. This loss is realized each time the state realizes below the critical bankruptcy state $\theta_b$ which triggers resolution, that is, with ex ante probability $\theta_b$. Term B is the change in direct loss due to a change in stabil-
ity (first indirect effect). If stability improves, the critical state goes down, bank resolution becomes less likely ex ante meaning that the direct loss is realized less often, thus term B is negative and decreases the deadweight loss. If stability deteriorates in forbearance, the term is positive. Importantly, the sign only depends on the change of stability and is therefore independent of whether liquidation is efficient or inefficient, i.e. independent of the relative position of the critical bankruptcy state compared to the efficient liquidation state. Not so for term C. Term C describes the change in efficiency due to over or underliquidation of assets. The sign thus depends on the change of bank stability and whether inefficient liquidation exists or not. By Lemma 4.1, the sign of term C is thus determined by the amount of insurance coverage and the magnitude of RA’s recovery rate. If insurance coverage is low, depositors are overly sensitive to bad news and withdraw too often such that inefficient runs exist. If bank stability improves in forbearance, i.e. if the recovery rate is sufficiently low, inefficient runs become less likely and the deadweight loss shrinks. As insurance coverage goes up, depositors become insensitive to bad news since their deposits are almost safe. Therefore, they do not withdraw despite observing bad news on asset returns, investment is continued inefficiently often and the deadweight loss enlarges as stability increases further in forbearance. Considering all terms together,

**Theorem 4.1.** Assume RA liquidates as efficient as the market \( r = l \). If deposit insurance is low, the deadweight loss monotonically decreases in forbearance and is minimized by not intervening at all \( a^* = 1 \). If deposit insurance coverage is close to one, the deadweight loss monotonically increases in forbearance and is minimized by intervening as soon as possible \( a^* = a \).

Note, the Theorem considers the entire range of forbearance policies, \((a, \bar{a}] \cup [\bar{a}, 1]\) and thus may impose losses on the insurance. To understand the first result, under low insurance depositors are sensitive to bad news and inefficient runs exist. Since RA’s recovery rate is low, stability decreases in forbearance, thus inefficient runs become less likely as RA commits to more forbearance ex ante. The result means, given that inefficient runs occur and thus intervention is ex post efficient, a stricter intervention policy \( a < 1 \) set at time zero would change depositors’ incentives to run on the bank in a way, that inefficient runs and thus bank resolution becomes ex ante more likely. RA minimizes the deadweight loss by committing to a policy ex ante that she will watch
every run happening in full length without intervention. Our result stands in
sharp contrast to a result in (Diamond and Dybvig, 1983) which states that
a suspension of convertibility policy not only prevents bank runs (the bad
equilibrium) but has a unique Nash equilibrium in dominant strategies to not
run on the bank if the depositor is patient\(^{14}\), as are all depositors in the model
here. By this, the demand deposit contract with suspension of convertibility
achieves optimal risk sharing in Diamond and Dybvig (1983) since patient
types no longer have an incentive to withdraw. In Diamond and Dybvig, this
is achieved by setting payoffs to depositors who run but are not served in
the queue to zero while in the model here these depositors are subject to a
mandatory stay and receive the pro rata share recovered by RA. Therefore,
depositors in our model still have an incentive to run despite partial insurance
since by withdrawing depositors may additionally receive the uninsured part
of their deposit. In addition, one feature our (global game) model accomplishes as opposed to Diamond and Dybvig is that the ex ante probability
of bank resolution arises as optimal aggregate response of depositors to the
intervention policy (suspension threshold) to which RA commits at time zero
while in Diamond and Dybvig bank runs occur as sunspots and a likelihood or
changes in likelihood for the run equilibrium cannot be attached endogenously
from within the model as function of the suspension policy.

If insurance coverage is high, the result reverts. Under high coverage,
depositors are insensitive to bad news on bank solvency. Thus, investment is
continued inefficiently often. The deadweight loss goes down if runs enforced
liquidation more frequently. This holds for low forbearance policies since
stability monotonically increases in forbearance. By setting forbearance as low
as possible, depositors anticipate bank resolution to occur for small measures
of withdrawals already. Thus, it is optimal to run on the bank for smaller
solvency shocks, by this decreasing the chance of inefficient continuation of
investment. The U.S. forbearance policy to intervene fast given that the bank
is insolvent can thus be explained, and is optimal, under either high recovery
rates and low insurance achieved by the FDIC.

To prove Theorem 4.1, note that terms A and B in (18) are zero and the
\(^{14}\)In our model here, all depositors are patient while in Diamond and Dybvig (1983)
measure \(\lambda \in (0, 1)\) of depositors are impatient and thus have to withdraw to consume in
period one.
partial derivative of the critical state in forbearance is negative by Proposition 3.2. The amount of deposit insurance determines the sign of the bracket in term C by Lemma 4.1. The bracket is positive if and only if the critical state is such that inefficient runs exist.

**Theorem 4.2.** Given low deposit insurance, as recovery rate increases away from the value at which the bank liquidates the deadweight loss goes from monotonically decreasing to monotonically increasing in forbearance, as long as the insurance runs no losses $a \in (a, \overline{a})$. Immediate intervention $a^* = \underline{a}$ minimizes the deadweight loss locally.

In the U.S., the FDIC’s forbearance policy to intervene fast given that the bank is insolvent can thus be explained, and is optimal, under either high recovery rates and low insurance or under recovery rates close to the bank’s efficiency level and high levels of deposit insurance.

We show in the appendix that under certain conditions, for fixed forbearance level $a$, there exists a unique recovery rate $r^*(a)$ such that the deadweight loss decreases in forbearance if RA’s true recovery rate undercuts $r^*(a)$ but increases in forbearance if her recovery rate exceeds $r^*(a)$. In general, the slope of deadweight loss in forbearance becomes zero for at least one recovery rate $r^*$ such that the deadweight loss decreases in forbearance for $r < r^*$ and increases for some $r > r^*$. There may however exist further recovery rates $r^{**}$ for which the slope of deadweight loss in forbearance switches sign again.

The results are demonstrated in Figures (7) and (8) and hold for insurance coverage levels up to $\gamma = 0.8$. To give some intuition for the result, as recovery rates are low, terms A and B are small since realized proceeds are close to RA’s recovery rate. Further, bank stability improves in forbearance. Thus, the small, direct loss occurs less often, term B is negative. Since insurance coverage is low, inefficient runs exist, and the increase in stability makes these runs less likely. The main term C is negative, which lowers the deadweight loss.

As recovery rates go up, stability starts deteriorating in forbearance since the change in haircut becomes more pronounced, term B becomes positive as does the first factor of term C. If insurance remains low, since now stability deteriorates in forbearance, inefficient runs become more likely, term C becomes positive and together with term B extends the deadweight loss.
If for \( r >> l \) and insurance low we allow for forbearance levels which impose losses on the insurer \( a \in (\bar{a}, 1] \), the change in deadweight loss becomes less clear. Since the haircut is not constant in forbearance, stability monotonically improves and makes inefficient runs less likely. On the other hand, as the RA liquidates more efficient than the bank does in the market, the direct loss term A is strictly positive and grows in \( a \).

Under higher insurance coverage and high recovery rates the case becomes even less clear. For intervention policies for which the insurance runs no loss, under high recovery rates stability deteriorates in forbearance which lowers the deadweight loss since under high insurance investment is continued inefficiently often. On the other hand, more forbearance increases the direct loss term A. If RA considers to intervene such that she imposes losses on the insurer, stability becomes monotonically increasing in forbearance which increases the deadweight loss through inefficient investment but lowers the deadweight loss by decreasing the direct loss, term B is negative.

Thus, under high insurance as opposed to low insurance, the deadweight loss does not have to increase in forbearance as recovery rates go up, we see this effect in Figure 7,8 and 9, for high insurance the slope hardly reacts as recovery rates increase.

As coverage goes up, the upper bound \( \bar{a} \) decreases, in Figure (7) we see how the plotted lines become shorter as coverage goes up and become single points. In Figure 7 plot (a) for instance, we have \( \bar{a} = 0.4 \) for insurance coverage of \( \gamma = 0.6 \) already. Further, when considering lower recovery rates, the insurance coverage levels we may analyze are more constrained since \( \bar{a} \) needs to be positive, i.e. \( \gamma < r/\delta \). To put this in perspective, note that as of 2016, insurance coverage of US domestic deposits is 59% (FDIC, 2016).
(a) Case: $r = 0.5$
(b) $r = 0.6$
(c) $r = 0.7$
(d) $r = 0.8$
(e) $r = 0.9$
(f) $r = 1$

Figure 7: Change of deadweight loss in forbearance for varying deposit insurance coverage. Across the graphs, for fixed insurance coverage $\gamma$, the change of the deadweight loss in forbearance reverts as recovery rate increases. Parameters: $l = 0.5$, $d = 0.7$, $k = 1.8$, $H = 4$. For the first plot $r = 0.5 = l$

The optimal level of forbearance attainable under certain insurance coverage may nevertheless not be attainable under higher coverage, again by the constraint that RA has to intervene before losses occur. In Figure 7 plot (a), forbearance $a = 0.8$ is attainable and optimal under coverage $\gamma = 0.3$ but not attainable under $\gamma = 0.6$. 
Figure 8: Change of deadweight loss in forbearance for varying recovery rate functions across different deposit insurance coverage. By RA’s constraint to intervene before insurance runs a loss $\frac{r(1-a)}{\delta-t_a} \leq \gamma$, $\delta < r/\gamma$, the maximum forbearance RA may show decreases in insurance coverage, thus the plotted lines become ‘shorter’ in $\gamma$ and longer in $r$. The slope of the deadweight loss in forbearance alters as RA’s recovery rate $r$ increases. Parameters: $t = 0.5, d = 0.7, k = 1.8, H = 4$

Proposition 4.1. For given forbearance level, the deadweight loss is minimized when insurance coverage is such that the critical state matches

$$\theta_b = \frac{T(a)}{H}$$

For every forbearance level there exists a unique coverage level $\gamma^*(a)$ which satisfies (19) and the deadweight loss strictly decreases in insurance for $\gamma < \gamma^*(a)$ and increases in insurance for $\gamma > \gamma^*(a)$. For given forbearance level, the deadweight loss decreases in recovery rate if insurance coverage is low but increases in recovery rate if insurance coverage is high.

The results are due to a stability improvement as insurance coverage and recovery rates increase. More insurance makes depositors less sensitive to sol-
vency shocks while higher recovery rates lower the deviation loss (haircut), both effects increase stability. As insurance In addition, if insurance coverage is low, inefficient runs exist while as insurance coverage goes up, underliquidation of assets occurs. As the critical state equals the efficient liquidation cut-off \( \theta_e \), neither inefficient runs nor inefficient liquidation occurs. Interestingly, the deadweight loss is not minimized as the critical state matches \( \theta_e \) but as it takes the lower value \( T(a)/H \). This is, because the deadweight loss has a second component, namely the direct loss from showing forbearance, see the first term in (22). A similar intuition holds for the second result. Under low insurance coverage inefficient runs exist. As recovery rate increases, bank stability improves and inefficient runs become less likely. Under higher insurance coverage however, underliquidation occurs and more stability increases the deadweight loss, see Figures 7 and 8.

4.3 Inefficient RA (\( r < l \))

We assumed in the model section that RA liquidates more efficiently than the market, \( r \geq l \). Consequently, we defined \( \theta_e = r/H \) as the benchmark state for efficient asset liquidation. Here, we relax this assumption and consider the case where the market liquidates more efficiently \( r < l \). Under French bankruptcy law for instance, the state has the explicit objective of preserving the firm and maintaining employment. By this, bankruptcy courts are not mandated to sell firm assets to the highest bidders, see Davydenko and Franks (2008).

Consequently, we need to redefine the efficient liquidation cut-off

\[
\hat{\theta}_e = \frac{l}{H} > \theta_e
\]

(20)

Then, for \( \gamma \) sufficiently small\(^{16} \) inefficient asset liquidation exists, \((\hat{\theta}_e, \theta_b)\) is non empty. The first best case changes to

\[
\tilde{W}^{FB} = l \hat{\theta}_e + \int_{\hat{\theta}_e}^{1} \theta H \, d\theta
\]

(21)

---

\(^{15}\) Note, that some of these effects may not be observable in the plots. Under the constraint that RA imposes no losses on the insurer, the upper bound on forbearance \( \pi \) monotonically decreases in insurance coverage, such that the latter effects may only be visible when allowing RA to intervene late, by this imposing losses on the insurance company, see later section.

\(^{16}\) We have \( \hat{\theta}_e < \bar{\theta} \) if and only if \( \gamma < (H - kl)/(H - l) \in (0,1) \) and we know that the critical state has to exceed the bound to the lower dominance region.

---
Redefine the deadweight loss at forbearance policy $a$ as

$$\hat{D}_{(r,\gamma)}(a) = W^{FB} - W = (l - T(a)) \theta_b(a) + \int_{\theta_a}^{\theta_b(a)} (\theta H - l) d\theta$$

The comparative statics of bank stability remain the same for all variables except for

**Lemma 4.2.** Bank stability monotonically improves in forbearance, if $r \leq l$.

The trade-off between the increase in haircut but decrease in belief that the bank will be resolved still exists but the effect of the haircut is always dominated since the recovery rate is small.

The comparative statics for the deadweight loss become

$$\frac{\partial}{\partial a} D(a) = \left( \frac{\partial}{\partial a} (l - T(a)) \right) \theta_b(a) + (l - T(a)) \frac{\partial \theta_b}{\partial a} \left. \right| \quad \text{B: change in direct gain due to change in stability}$$

$$+ \frac{\partial \theta_b}{\partial a} (\theta_b H - l) \left. \right| \quad \text{C: Change in efficiency of enforced liquidation due to change in stability}$$

$$= \text{A: Direct efficiency gain}$$

A further change to the case $r \geq l$ from previous sections is that the change in direct loss becomes a change in direct gain $\frac{\partial}{\partial a} (l - T(a)) < 0$. Term A becomes negative, thus more forbearance decreases the deadweight loss since RA liquidates less efficiently than the market. Since stability monotonically improves in forbearance for all $a \in (a, 1]$, term B is negative too. As stability goes up, the bank is resolved less often, thus inefficient liquidation under resolution occurs less often by $l - T(a) > 0$. Term C is negative under inefficient asset liquidation, i.e. under low deposit insurance but positive under high insurance. Under low insurance depositors run too often which enforces inefficient liquidation. As stability improves in forbearance, these losses are avoided.

**Proposition 4.2.** Under an inefficient RA, $r < l$, if deposit insurance is low, maximum forbearance is optimal $a^* = 1$.

Thus, together with Theorem 4.1, committing ex ante to not intervene during runs is optimal for all recovery rates $r \leq l$ if insurance is low. Under high insurance inefficient continuation becomes more likely as stability
increases and term C has thus opposing signs to term A and B. Thus under higher insurance the rise in forbearance either decreases the deadweight loss slower or the loss may even increase in forbearance.

4.4 Cascading Forbearance

So far we have analyzed the model from the perspective that there exists one resolution authority which can decide whether and how much forbearance to exercise. In Europe however, the case is that national authorities have the option to divert and by this cascade resolution from one central authority (European Stability Mechanism) to a national resolution authority. In more detail, national authorities have no impact on forbearance and recovery rate of the central resolution authority, but given that one wishes to divert resolution to national authorities, the bank is resolved under the recovery rate of the national authority and forbearance increases.

Denote by \((r_1, a_1)\) the recovery rate and forbearance level chosen by the centralized resolution authority and let \((r_n, a_n)\) the recovery rate and forbearance level of the national resolution authority. Given that the national resolution authority diverts resolution to the national authority, the bank is resolved at recovery rate \(r_n\) and forbearance level \(a_2 = a_1 + a_n > a_1\). Exercising the option to delay is optimal if and only if \(a_n\) can be chosen such that

\[
D(r_1, a_1) > D(r_n, a_1 + a_n)
\]  

(24)

Application 1 (Theorem 4.2) Assume there exists a centralized resolution authority RA with forbearance level \(a_1\) and recovery rate \(r_1\). Further assume, national politics have the option to delay and divert resolution to the national RA at an equivalent high recovery rate \(r_n = r_1\). When exercising the option to delay, overall forbearance increases to \(a_2 > a_1\). Assume insurance coverage is low to medium high. If recovery rate \(r_1\) is low, it is strictly optimal to exercise the option to delay and divert resolution to national authorities. If recovery rates are high though, is is strictly optimal to not delay and let the centralized resolution authority resolve the bank at lower forbearance level, see Figure 7 plot (a) versus (e) or any plot of Figure 8.
Application 2 (Proposition 4.1) Assume again, there exists a centralized resolution authority RA with forbearance level $a_1$ and recovery rate $r_1$. National politics have the option to delay and divert resolution to the national RA which works highly effective such that $a_2$ is close to $a_1$ and resolution takes place at higher recovery rate $r_n > r_1$. Then exercising the option to delay and divert resolution is strictly optimal if insurance coverage is low but not optimal if insurance coverage is high.

Application 3 (Theorem 4.1) Assume a national authority can divert and delay resolution away from a centralized resolution authority, by this increasing forbearance from $a_1$ to $a_2 > a_1$. Assume both resolution authorities liquidate as efficiently as the market, then exercising the option to delay is not optimal if insurance coverage is low, but is strictly optimal under high coverage.

4.5 Legal constraints

In the model section we have assumed that the bank can repay depositors at time two if no resolution occurs and the asset pays off high. We could meet this assumption by assuming that the bank has access to the money market at time one where she can borrow cash by pledging the asset as collateral. To give an intuition why this assumption is important, assume the bank was forced to sell (not pledge) assets in the market. Then, the bank can be liquid but insolvent simultaneously in period one if she is forced to liquidate a too large fraction of the asset to repay depositors - although no bank resolution occurs. To repay $n\delta$ in cash at time one, the bank liquidates $n\delta/l$ of the asset. Given the asset pays off at time two, the bank earns a return on remaining investment of $H(1 - n\delta/l)$ but faces claims at time two of $(1 - n)\delta k$. If $n$ exceeds threshold

$$h^* := \frac{H - \delta k}{\delta(H - k)}$$

remaining claims exceed her return on investment for sure and the bank is insolvent. Since the bank can pledge assets, she can overcome this insolvency point and continue to pledge assets in excess of $n = h^*$. Still, for $n > h^*$ the

\footnote{Further, in the definition of welfare we assume that no assets are liquidated if no resolution occurs, i.e. depositors make no mistakes}
bank is technically insolvent since return on her remaining, unpledged assets undercuts debt claims.

Next observe, that the resolution authority cannot set forbearance in a way that $la/\delta < h^*$. If this was the case, for withdrawals $n \in [la/\delta, h^*]$ the bank is resolved but is solvent in absence of resolution which raises legal issues. For instance, in September 2017 bondholders of failed Banco Popular filed an appeal against Spain’s banking bailout fund which followed European authorities (Single Resolution Board) and wiped out equity and junior bondholders before selling the bank to Banco Santander, see Bloomberg (2017) and Reuters (2017). To guarantee $h^* < la/\delta$, forbearance needs to satisfy

$$a > \frac{H - \delta k}{H - kl} = a'$$

which gives us the lower bound we worked with in the model section.

To ensure that the bank can repay depositors at time two when she is not resolved the bank accesses the money market to raise additional cash $x \in [0, la)$ using the asset as collateral by pledging fraction $x/l$ of the asset to some counterparty C. Since the bank cannot observe the state but withdrawals $n$, we assume that the bank pledges assets of overall measure $al$, and then stops to be put in to receivership. Our assumption that the bank observes $n$ immediately, but still pledges proportion $a$ of the asset before being resolved can be justified by interpreting that the bank observes withdrawals gradually accumulating over time. Thus, as the bank accesses the money market, the bank and C who does not observe withdrawals, can only condition on the information $n > 0$, not on $n < al$.

Since the asset is risky, C demands interest $i$. The bank can repay interest if no resolution occurs and the asset pays off. If the bank is put into receivership, we assume that C liquidates the asset immediately since the bank does not continue to manage the investment which negatively affects returns (the bank is an investment expert, see Diamond and Rajan (2001)). Since given no resolution the bank continues to operate the asset, the return likelihood of the asset and asset return remain the same. The competitive interest rate satisfies

\footnote{If the bank could observe the state, she could infer $n(\theta)$ immediately and would not need to pledge assets given she knew that she will be resolved $n > al/\delta$.}
\[ x = E[\theta \cdot 1_{\{n \in [0, l_a/\delta]\}} | n > 0] \left(1 + i\right) x + \mathbb{P}(n > \frac{l_a}{\delta} | n > 0) l \cdot \frac{x}{l} \]  

(27)

The formula simplifies to

\[ (1 + i) = \frac{1 - \mathbb{P}(n > \frac{l_a}{\delta} | n > 0)}{E[\theta \cdot 1_{\{n \in [0, l_a/\delta]\}} | n > 0]} \]  

(28)

\[ = \frac{\mathbb{P}(n \in [0, \frac{l_a}{\delta}); n > 0)}{E[\theta \cdot 1_{\{n \in [0, l_a/\delta]\}}; n > 0]} \]  

(29)

\[ = \frac{\mathbb{P}(n \in (0, \frac{l_a}{\delta}))}{E[\theta \cdot 1_{\{\theta \in (\theta_b, 1)\}}]} \]  

(30)

\[ = \frac{\mathbb{P}(\theta \in (\theta_b, 1))}{E[\theta \cdot 1_{\{\theta \in (\theta_b, 1)\}}]} \]  

(31)

\[ = \frac{\theta^* + \varepsilon - \theta_b}{\int_{\theta_b}^{\theta^*+\varepsilon} \theta d\theta} = \frac{2}{\theta^* + \varepsilon + \theta_b} \rightarrow \frac{1}{\theta_b} \]  

(32)

(33)

by law of iterated expectations. Interest depends on the critical state and in particular forbearance level \( a \).

### 4.6 Emergency Liquidity Assistance (ELA)

A different situation compared to the previous subsection emerges if the bank taps ELA instead of borrowing in the market directly. In Europe, ELA is paid to banks which are illiquid but solvent by the national central bank if the bank in question has trouble to raise cash in the market. In return, the bank has to provide assets as collateral which may be of 'inferior' quality, i.e. of quality not accepted by the market. If for withdrawals in excess of \( h^* \) the bank accesses emergency liquidity assistance, the institution with whom the bank pledges the asset is the same as the resolution authority, since ELA is paid under supervision of the European Central Bank. In that case, the process of pledging the asset already increases proceeds available to depositors under resolution, should bank resolution occur. Therefore, the weighted liquidation value of the asset changes since only fraction \( h^* \delta/l \) of the asset is liquidated at \( l \) while fraction \( 1 - h^* \delta/l \) is first pledged to but then liquidated by RA at rate \( r \) if withdrawals are such that resolution is triggered. Thus, weighted liquidation...
proceeds become independent of forbearance $T(a) = T = h^* \delta + (1 - h^* \delta/l)r$
and the pro rata share to depositors under resolution changes to

$$s(a) = \frac{(1 - h^* \delta)r}{\delta - la}$$

which is now increasing in $a$, thus the haircut now decreases in forbearance but was increasing under the previous model set-up. As a consequence, bank stability would become monotonically increasing in forbearance since the trade-off between haircuts and belief that bank resolution occurs vanishes.

**Lemma 4.3.** Under ELA, for $r \geq l$, bank stability monotonically improves in forbearance.

As a consequence, for recovery rate close to asset liquidity, if insurance is low, as before the optimal forbearance policy is to intervene as late as possible. This is since in (18) for $r$ close to $l$ term A is small, term B is small but negative and term C is negative if deposit insurance is sufficiently low. For high recovery rates, positive term A becomes larger, and there exists a trade-off between the direct loss and stability improvements which make both the direct loss and inefficient liquidation less likely.

Banks access ELA typically when they can no longer raise cash using their assets as collateral in the market. To make the model consistent with the assumption that the market refuses to accept the asset as collateral, we can assume that the market, i.e. C can observe $n$ using the 'gradual' interpretation from above. C accepts the asset as collateral as long as $n < h^*$, i.e. as long as the state is high enough and thus the bank is solvent. For $n > h^*$ C refuses to lend any further by accepting the asset as collateral and the bank has to ask for ELA. The competitive interest rate can then condition on observable withdrawals which have realized so far $n > \hat{n} > 0$ for $\hat{n} < h^*$:

36
\[(1 + i)(\hat{n}) = \frac{1 - P(n > \frac{la}{\delta}|n > \hat{n})}{E[\theta \cdot 1_{\{n \leq \hat{n}/la\}}|n > \hat{n}]} \quad (35)\]
\[
= \frac{P(n \in (\hat{n}, \frac{la}{\delta}))}{E[\theta \cdot 1_{\{n \in (\hat{n},\frac{la}{\delta})\}}]} \quad (36)
\]
\[
= \frac{P(\theta \in (\theta_b, \theta_n))}{E[\theta \cdot 1_{\{\theta \in (\theta_b, \theta_n)\}}]} \quad (37)
\]
\[
= \frac{\theta_n - \theta_b}{\int_{\theta_b}^{\theta_n} \theta d\theta} = \frac{2}{\theta_n + \theta_b} \quad (38)
\]

where we implicitly define \(\theta_n\) as \(n(\theta_n) = \hat{n}\), see (61). By (61) we see,

**Lemma 4.4.** The competitive interest rate \(1 + i\) monotonically increases in the amount of withdrawals \(\hat{n}\) observable in the money market.

Thus, the assumption that the bank accesses ELA as withdrawals \(n\) exceed a threshold \(h^*\) is reasonable.

### 4.7 Pricing of deposit insurance

In the current set up, given bank resolution, the deposit insurance incurs losses if and only if the bank is not resolved but the asset does not pay off. The fair premium charged to the bank equals

\[
P(a, \gamma) = \int_{\theta_b}^{\theta^* + \varepsilon} (1 - \theta)\delta(1 - n)\gamma d\theta + \int_{\theta^* + \varepsilon}^{1} (1 - \theta)\delta \gamma d\theta \quad (39)
\]
\[
\rightarrow \int_{\theta_b}^{1} (1 - \theta)\delta \gamma d\theta \quad (40)
\]

Since insurance is only paid for high states \(\theta \geq \theta_b\), if at all, \(P\) is small with an upper bound, at the limit, of

\[
P(a, \gamma) \leq (1 - \theta_b(a))^2\delta \gamma \leq (1 - \theta)^2\delta \gamma \quad (41)
\]

where \(\theta\) only depends on the contract \(k\) and insurance \(\gamma\).

If RA becomes more lenient and considers forbearance policies \(a > a\) for which the insurance would run a loss given a run, the premium changes and
increases to

$$C(a, \gamma) = \int_0^{\theta_b} \delta(\gamma - s(a))d\theta + \int_{\theta_b}^{\theta^* + \varepsilon} (1 - \theta)\delta(1 - n)\gamma d\theta + \int_{\theta^* + \varepsilon}^{1} (1 - \theta)\delta \gamma d\theta$$

$$\rightarrow \delta \left[ (\gamma - s(a))\theta_b + \int_{\theta_b}^{1} (1 - \theta)\gamma d\theta \right]$$

where the deposit insurance needs to compensate depositors for the difference of insurance coverage and pro rata share recovered by RA, $\gamma - s(a)$. As RA shows more forbearance, the critical bankruptcy state alters according to Proposition 3.2 and the pro rata share decreases in $a$.

As forbearance approaches one, RA does not intervene during a run and $s(a) \rightarrow 0$ by $\delta > l$. As a consequence, under maximum forbearance the deposit insurance compensates for the fully insured amount $\gamma\delta$ if the state falls below the critical bankruptcy state.

5 Conclusion

The paper analyzes optimal strategic delay of bank resolution. A resolution authority monitors withdrawals at the bank level. If withdrawals are 'abnormally high', she has the authority to suspend conversion of deposits by putting the bank into receivership and seizing bank assets to protect a deposit insurance fund. Optimal forbearance (measure of tolerated withdrawals until intervention) depends on deposit insurance coverage and efficiency of the resolution authority (RA) to liquidate assets. The paper is motivated by the discrepancy in the bank resolution directives of U.S. versus Europe. The paper can explain the incorporation of the option to delay and divert bank resolution to national authorities, found in the European bank resolution directive BRRD but also finds conditions under which the U.S. approach to intervene as soon as possible is optimal. As main result, we show that under low deposit insurance the optimal policy can be to commit ex ante to never intervene during a bank run even though intervention is ex post efficient.
6 Appendix A: Plot

Figure 9: Change of deadweight loss in forbearance for varying recovery rate functions across different deposit insurance coverage. \( l = 0.3, d = 0.7, k = 1.8, H = 4 \)

7 Appendix B: Proofs

7.1 Proof: Existence and Uniqueness

Proof. [proof Proposition 3.1]

We first show equivalence of this game to a version of the game in Goldstein Pauzner:

Conditional on a run occurring, the payoff difference from rolling over
versus withdrawing equals

\[
\Delta = \frac{r(1-a)}{\delta - la} - \left[ \frac{la}{\delta n} \cdot 1 + \left( 1 - \frac{la}{\delta n} \right) \frac{r(1-a)}{\delta - la} \right]
\]

(44)

\[
= -\frac{la}{\delta n} + \frac{la}{\delta n} \frac{r(1-a)}{\delta - la}
\]

(45)

\[
= -\frac{la}{\delta n} \left( 1 - \frac{r(1-a)}{\delta - la} \right)
\]

(46)

(47)

set \( f(l, \delta, r, a) = a l \left( 1 - \frac{r(1-a)}{\delta - la} \right) > 0 \), then

\[
\Delta = -\frac{f(l, \delta, r, a)}{\delta n}
\]

(48)

and the game described is equivalent to a game which has close similarity to the model analyzed in Goldstein and Pauzner (2005)

<table>
<thead>
<tr>
<th>Event/Action</th>
<th>Withdraw</th>
<th>Roll-over</th>
</tr>
</thead>
<tbody>
<tr>
<td>no Run ( n \in [0, \frac{la}{\delta}] )</td>
<td>1</td>
<td>( \begin{cases} k, &amp; p = \theta \ 0, &amp; p = 1 - \theta \end{cases} )</td>
</tr>
<tr>
<td>Run ( n \in (\frac{la}{\delta}, 1] )</td>
<td>( \frac{f}{\delta n} )</td>
<td>0</td>
</tr>
</tbody>
</table>

A: Existence and uniqueness of a trigger equilibrium

For fixed contract \((1, k)\), capital structure \(\delta\), recovery rate \(r\) and forbearance policy \(a\), a Bayesian equilibrium is a strategy profile such that each investor chooses the best action given her private signal and her beliefs about other players actions and strategies of other players. In equilibrium, an investor decides to withdraw when her expected payoff from rolling over versus withdrawing given her signal is negative, decides to roll over when it is positive and is indifferent if the expected payoff is zero. Since investors are identical ex ante, investors strategies can only differ at signals that make an investor indifferent between rolling over and withdrawing.

In a trigger/threshold equilibrium around trigger signal \(\theta^*\), all investors withdraw when they observe signals below \(\theta^*\) and roll over if they observe signals above \(\theta^*\). In case of directly observing \(\theta^*\) investors are indifferent and we specify here that they will roll over.

A threshold equilibrium around trigger \(\theta^*\) exists if and only if given that all
other investors use a trigger strategy around signal \( \theta^* \) an investor finds it optimal to also use a trigger strategy around trigger \( \theta^* \).

If all investors follow the same strategy, the proportion of investors who withdraw at each state is deterministic. If all investors further follow the same threshold strategy around trigger signal \( \theta^* \), define \( n(\theta, \theta^*) \) as the proportion of investors who observe signals below signal \( \theta^* \) and thus withdraw if the state is \( \theta \), \( n(\theta, \theta^*) = P(\theta_i < \theta^*|\theta) \). Thus we can explicitly calculate \( n(\theta, \theta^*) \) using the distribution function of noise as given in (56).

Note that if a continuum of investors but one single investor follow the same strategy this result continues to hold.

Denote by \( D(\theta_i, n(\cdot, \theta^*)) \) the expected payoff difference from rolling over versus withdrawing when the investor observes signal \( \theta_i \), and other investors follow a trigger strategy around \( \theta^* \). Since a run is triggered if the measure of withdrawing depositors \( \delta n \) exceeds \( \alpha l \), we have

\[
 D(\theta_i, n(\cdot, \theta^*)) = \frac{1}{2\varepsilon} \int_{\theta_i - \varepsilon}^{\theta_i + \varepsilon} (k\theta - 1) \mathbf{1}_{\{n(\theta, \theta^*) \leq \frac{\varepsilon}{\sqrt{\pi}}\}} - \frac{f}{\delta n(\theta, \theta^*)} \mathbf{1}_{\{n(\theta, \theta^*) > \frac{\varepsilon}{\sqrt{\pi}}\}} d\theta
\]  

(49)

For existence of a trigger equilibrium we need to show

\[
 D(\theta_i, n(\cdot, \theta^*)) < 0 \quad \text{for all } \theta_i < \theta^*  \tag{50}
\]

\[
 D(\theta_i, n(\cdot, \theta^*)) > 0 \quad \text{for all } \theta_i > \theta^*  \tag{51}
\]

and existence and uniqueness of a signal \( \theta^* \) for which an investor is indifferent between rolling over and withdrawing (payoff indifference equality)

\[
 0 = D(\theta^*, n(\cdot, \theta^*)) = \frac{1}{2\varepsilon} \int_{\theta^* - \varepsilon}^{\theta^* + \varepsilon} (k\theta - 1) \mathbf{1}_{\{n(\theta, \theta^*) \leq \frac{\varepsilon}{\sqrt{\pi}}\}} - \frac{f}{\delta n(\theta, \theta^*)} \mathbf{1}_{\{n(\theta, \theta^*) > \frac{\varepsilon}{\sqrt{\pi}}\}} d\theta
\]  

(52)

To prove existence and uniqueness of \( \theta^* \) such that (52) holds, observe that the function \( D(\theta^*, n(\cdot, \theta^*)) \) is continuous in \( \theta^* \). By existence of dominance regions, we have \( D(\theta^*, n(\cdot, \theta^*)) < 0 \) for \( \theta^* < \theta - \varepsilon \) and \( D(\theta^*, n(\cdot, \theta^*)) > 0 \) for
\( \theta^* > \overline{\theta} + \varepsilon \). Thus, together with continuity by the Intermediate Value Theorem there exists at least one \( \theta^* \in [\overline{\theta} - \varepsilon, \overline{\theta} + \varepsilon] \) for which (52) holds.

To see uniqueness, since all other agents use a threshold strategy around \( \theta^* \), we can substitute for \( n(\theta, \theta^*) = \frac{1}{2} + \frac{\theta^* - \theta}{2\varepsilon} \) and derive

\[
D(\theta^*, n(\cdot, \theta^*)) = \int_0^{\frac{\varepsilon}{2}} (k\theta(n, \theta^*) - 1) \, dn - \int_0^{1} \frac{f}{\delta n} \, dn
\]

(53)

where \( \theta(n, \theta^*) = \theta^* + \varepsilon(1 - 2n) \), \( \theta^* \in [\overline{\theta} - \varepsilon, \overline{\theta} + \varepsilon] \) is the inverse of the function \( n(\theta, \theta^*) \).

For uniqueness, observe that \( D(\theta^*, n(\cdot, \theta^*)) \) is strictly increasing in signal \( \theta^* \) for \( \theta^* < \overline{\theta} + \varepsilon \) which gives us single-crossing.

Next we need to show that withdrawing is a best response if the private signal of an investor realizes below the trigger played by other investors \( \theta_i < \theta^* \), that is we need to show (50). Following Goldstein and Pauzner (2005), let \( \theta_i < \theta^* \). Decompose the intervals \( [\theta_i - \varepsilon, \theta_i + \varepsilon] \) and \( [\theta^* - \varepsilon, \theta^* + \varepsilon] \) over which the integrals \( D(\theta_i, n(\cdot, \theta^*)) \) and \( D(\theta^*, n(\cdot, \theta^*)) \) are calculated into a potentially empty common part \( c = [\theta_i - \varepsilon, \theta_i + \varepsilon] \cap [\theta^* - \varepsilon, \theta^* + \varepsilon] \) and the disjoint parts \( d_i = [\theta_i - \varepsilon, \theta_i + \varepsilon] \setminus c \) and \( d^* = [\theta^* - \varepsilon, \theta^* + \varepsilon] \setminus c \). Then,

\[
D(\theta_i, n(\cdot, \theta^*)) = \frac{1}{2\varepsilon} \int_{\theta \in c} v(\theta, n(\theta, \theta^*)) \, d\theta + \frac{1}{2\varepsilon} \int_{\theta \in d_i} v(\theta, n(\theta, \theta^*)) \, d\theta
\]

(54)

\[
D(\theta^*, n(\cdot, \theta^*)) = \frac{1}{2\varepsilon} \int_{\theta \in c} v(\theta, n(\theta, \theta^*)) \, d\theta + \frac{1}{2\varepsilon} \int_{\theta \in d^*} v(\theta, n(\theta, \theta^*)) \, d\theta
\]

(55)

Considering (55), the integral \( \int_{\theta \in c} v(\theta, n(\theta, \theta^*)) \, d\theta \) has to be negative since by (52) \( D(\theta^*, n(\cdot, \theta^*)) \) = 0 and since the fundamentals in range \( d^* \) are higher than in \( c \). This is, since we assumed \( \theta_i < \theta^* \) and because in interval \( [\theta^* - \varepsilon, \theta^* + \varepsilon] \) the payoff difference \( v(\theta, n) \) is positive for high values of \( \theta \), negative for low values of \( \theta \) and satisfies single-crossing. In addition, the function \( n(\theta, \theta^*) \) equals one over the interval \( d^i \), since \( d^i \) is below \( \theta^* - \varepsilon \) and thus all other investors withdraw. Therefore, the integral \( \int_{\theta \in d^i} v(\theta, n(\theta, \theta^*)) \, d\theta \) is
negative too which with (54) implies that \( D(\theta_i, n(\cdot, \theta^*)) \) is negative. The proof for \( \theta_i > \theta^* \) proceeds analogous.

\[ \text{B No existence of non-monotone equilibria} \]

See Goldstein and Pauzner, proof of Theorem 1, first page of part C.

\[ \square \]

### 7.2 Proof: Comparative statics Trigger

**Proof.** [Lemma 3.1]

First, by uniqueness of a trigger equilibrium and since we have a continuum of depositors, the proportion of withdrawing depositors \( n \) is a deterministic function of the state and the trigger and is by the error distribution given by

\[
n(\theta, \theta^*) = P(\theta_i < \theta^*|\theta) = P(\varepsilon_i < \theta^* - \theta|\theta) = \begin{cases} \frac{1}{2} + \frac{\theta^* - \theta}{2\varepsilon}, & \theta_i \in [\theta^* - \varepsilon, \theta^* + \varepsilon] \\ 1, & \theta_i < \theta^* - \varepsilon \\ 0, & \theta_i > \theta^* + \varepsilon \end{cases}
\]

(56)

Given signal \( \theta_i \) a depositors posterior on \( \theta \) is uniform on \( [\theta_i - \varepsilon, \theta_i + \varepsilon] \). The payoff difference between rolling over and withdrawing given bank resolution equals

\[
\Delta = \frac{r(1-a)}{\delta - la} - \left( \frac{la}{\delta n} \cdot 1 + (1 - \frac{la}{\delta n}) \cdot \frac{r(1-a)}{\delta - la} \right) \quad (57)
\]

\[
= -\frac{la}{\delta n} + \frac{la}{\delta n} \cdot \frac{r(1-a)}{\delta - la} \quad (58)
\]

\[
= -\frac{la}{\delta n} \left( 1 - \frac{r(1-a)}{\delta - la} \right) \quad (59)
\]

\[
= -\frac{la}{\delta n} H(a) \quad (60)
\]

where \( H(a) \) is the haircut depositors have to take under bank resolution. Thus expected payoff difference from rolling over versus withdrawing given a signal equal
0 = \frac{1}{2\varepsilon} \int_{\theta_i - \varepsilon}^{\theta_i + \varepsilon} (k\theta + (1 - \theta)\gamma - 1) 1_{\{n \in [0, \frac{\theta}{\delta}]\}} - \frac{la}{\delta n} H(a) 1_{\{n \in [\frac{\theta}{\delta}, 1]\}} d\theta

substituting using the function \(n(\theta, \theta^*)\), payoff indifference is equivalent to

\begin{align*}
0 &= \int_0^\frac{la}{\delta} ((k - \gamma)\theta(n, \theta^*) - (1 - \gamma)) dn - H(a) \int_{\frac{la}{\delta}}^1 \frac{la}{\delta n} dn \\
&= \int_0^\frac{la}{\delta} ((k - \gamma)\theta^* - (1 - \gamma)) dn + \int_0^\frac{la}{\delta} (k - \gamma)\varepsilon(1 - 2n) dn + \frac{la}{\delta} H(a) \ln\left(\frac{la}{\delta}\right) \\
&= \frac{la}{\delta} ((k - \gamma)\theta^* - (1 - \gamma)) + (k - \gamma)\varepsilon(1 - \frac{la}{\delta}) + \frac{la}{\delta} H(a) \ln\left(\frac{la}{\delta}\right)
\end{align*}

where

\[\theta(n, \theta^*) = \theta^* + \varepsilon(1 - 2n), \theta^* \in [\theta - \varepsilon, \theta + \varepsilon]\] (61)

is the inverse of the function \(n(\theta, \theta^*)\). Canceling terms, we obtain

\[0 = (k - \gamma)\theta^* - (1 - \gamma) + (k - \gamma)\varepsilon(1 - \frac{la}{\delta}) + H(a) \ln\left(\frac{la}{\delta}\right)\]

Solving the equation for the trigger \(\theta^*\) yields

\[\theta^* = \frac{(1 - \gamma) - H(a) \ln\left(\frac{la}{\delta}\right)}{k - \gamma} - \varepsilon(1 - \frac{la}{\delta})\] (62)

with \(H(a) = 1 - \frac{r(1-a)}{\delta - la}\). Since the noise term enters linearly, we can take partial derivatives directly from the limit of the trigger.

\[\text{Proof. [Proposition 3.2]}\] By (61), we have \(\theta_b = \theta^* + \varepsilon(1 - 2\frac{la}{\delta})\). Thus, at the limit \(\varepsilon \to 0\), we have \(\theta_b = \theta^*\) and also the partial derivatives coincide.

Set

\[n^* = \frac{la}{\delta}\] (63)
\[
\frac{\partial}{\partial a} \theta^* = -\frac{1}{k - \gamma} \left( H'(a) \ln(n^*) + H(a) \frac{1}{n^*} \frac{\partial n^*}{\partial a} \right) \tag{64}
\]

where

\[
\frac{\partial n^*}{\partial a} = \frac{l}{\delta} \tag{65}
\]

and thus

\[
H(a) \frac{1}{n^*} \frac{\partial n^*}{\partial a} = \frac{1}{a} H(a) > 0 \tag{66}
\]

while

\[
H'(a) = -\frac{r(\delta - la) + lr(1 - a)}{(\delta - la)^2} = \frac{r(\delta - l)}{(\delta - la)^2} > 0 \tag{67}
\]

that is the haircut increases in forbearance. But \(\ln(n^*) < 0\), thus in general, the change of the trigger in forbearance can be non-monotone. For \(r = l\), using the logarithm inequality \(\ln(1 + x) > x/(x + 1)\) for \(x > -1\) we obtain

\[
\frac{\partial}{\partial a} \theta^* = -\frac{1}{k - \gamma} \left( \frac{\delta - l}{\delta - la} \left( \frac{l}{\ln(n^*) + \frac{1}{a}} \right) \right) < 0 \tag{68}
\]

\[
< -\frac{1}{k - \gamma} \left( \frac{\delta - l}{\delta - la} \left( \frac{l}{(\delta - la)} \frac{la - \delta}{la} + \frac{1}{a} \right) \right) \tag{69}
\]

\[= 0 \tag{70}\]

Thus, for \(r\) sufficiently close to \(l\), we have \(\frac{\partial}{\partial a} \theta^* \leq 0\).

The cross derivative of the trigger with respect to forbearance and recovery rate is positive:

\[
\frac{\partial}{\partial r} \theta^* = -\frac{1}{k - \gamma} \left[ \left( \frac{\partial}{\partial r} \frac{\partial}{\partial a} H \right) \cdot \ln(n^*) + \frac{1}{a} \left( \frac{\partial}{\partial r} H(a) \right) \right] \tag{68}
\]

\[= -\frac{1}{k - \gamma} \left[ \frac{(\delta - l)}{(\delta - la)^2} \cdot \ln(n^*) + \frac{1}{a} \left( -\frac{1 - a}{\delta - la} \right) \right] > 0 \tag{70}\]

since \(\ln(n^*) < 0\), while the cross derivative of the trigger with respect to forbearance and deposit insurance has the same sign as the partial derivative
$$\frac{\partial}{\partial a} \theta^* :$$

$$\frac{\partial}{\partial \gamma} \frac{\partial}{\partial a} \theta^* = -\frac{1}{(k-\gamma)^2} \left( H'(a) \ln(n^*) + H(a) \frac{1}{n^*} \frac{\partial n^*}{\partial a} \right) \quad (71)$$

since signs depend on the bracket. Thus, if $\frac{\partial}{\partial a} \theta^* > 0$, as $\gamma$ increases, the trigger increases faster in $a$ while if $\frac{\partial}{\partial a} \theta^* < 0$, the trigger falls faster in $a$ as deposit insurance goes up.

$$\frac{\partial}{\partial \delta} \theta^* = -\frac{1}{k-\gamma} \left( \left( \frac{\partial}{\partial \delta} H(a) \right) \ln(n^*) + H(a) \frac{1}{n^*} \frac{\partial n^*}{\partial \delta} \right) \quad (72)$$

with

$$\frac{\partial n^*}{\partial \delta} = -\frac{la}{\delta^2} \quad (73)$$

thus

$$H(a) \frac{1}{n^*} \frac{\partial n^*}{\partial \delta} = -\frac{1}{\delta} H(a) < 0 \quad (74)$$

and

$$\frac{\partial}{\partial \delta} H(a) = \frac{r(1-a)}{(\delta - la)^2} > 0 \quad (75)$$

but $\ln(n^*) < 0$. Thus altogether,

$$\frac{\partial}{\partial \delta} \theta^* > 0 \quad (76)$$

$$\frac{\partial}{\partial r} \theta^* = -\frac{\ln \left( \frac{ra}{\delta} \right)}{k-\gamma} \cdot \left( \frac{\partial}{\partial r} H(a) \right) < 0 \quad (77)$$

since

$$\frac{\partial}{\partial r} H(a) = -\frac{1-a}{\delta - la} < 0 \quad (78)$$

and $-\ln \left( \frac{ra}{\delta} \right) > 0$ by $\delta > l > la$ and $K > 1 > \gamma$

Last,
\[
\frac{\partial \theta^*}{\partial \gamma} = \frac{-(k - \gamma) + (1 - \gamma) - H(a) \ln(n^*)}{(k - \gamma)^2} = \frac{1 - k - H(a) \ln(n^*)}{(k - \gamma)^2}
\]  

Plugging in the equilibrium condition (83) for \(-H(a) \ln(n^*)\), at the limit we obtain

\[
\frac{\partial \theta^*}{\partial \gamma} = \frac{1 - k + (k - \gamma)\theta^* - (1 - \gamma)}{(k - \gamma)^2} = \frac{\theta^* - 1}{(k - \gamma)} < 0
\]  

\[7.3\) Proof: Efficiency

Proof. [Lemma 4.1] We have for every \(r \in (l, 1)\)

\[
\lim_{\gamma \to 0} \theta^* = \frac{1 - H(a)}{k} > \frac{1}{k} > \frac{r}{k} > \frac{r}{H} = \theta_e
\]  

since \(-\ln(la/\delta) > 0\), \(k < H\) and \(r < 1\).

On the other hand, independently of whether \(a < \bar{a}\) or \(a \in (\bar{a}, 1)\). For every recovery rate \(r \geq l\),

\[
H(a) = 1 - s(a) \leq 1 - \gamma
\]  

Thus, by \(-\ln(la/\delta) > 0\) and \(k > 1\),

\[
\theta^* = \frac{(1 - \gamma) + H(a) (-\ln(la/\delta))}{k - \gamma} \leq \frac{(1 - \gamma)(1 - \ln(la/\delta))}{k - \gamma} \to 0 \quad \text{as } \gamma \to 1
\]  

Thus, we have found an upper majorante for the trigger which converges to zero. By the sandwich lemma therefore \(\lim_{\gamma \to 1} \theta^* = 0 < r/H = \theta_e\).
7.4 Proof: Change Deadweight loss

Proof. [Proposition 4.1]

\[
\frac{\partial}{\partial r} D = \frac{\partial}{\partial r} (r - T(a)) \theta_b \\
+ (r - T(a)) \frac{\partial \theta_b}{\partial r} + \frac{\partial \theta_b}{\partial r} (\theta_b H - r) \\
- \frac{\partial \theta_e}{\partial r} (\theta_e H - r) - (\theta_b - \theta_e) \\
= (\theta_b H - T(a)) \frac{\partial \theta_b}{\partial r} - (\theta_b (1 - a) - \theta_e) \\
\]

(86)

(87)

(88)

(89)

(90)

since \( \theta_e H = r \). We have \( \frac{\partial \theta_b}{\partial r} < 0 \) and for insurance coverage sufficiently small inefficient runs exist, \( \theta_b H - T(a) > \theta_b H - r > 0 \) and \( \theta_b > \theta_e \). Thus, under low insurance coverage, \( \frac{\partial}{\partial r} D < 0 \). Under high insurance however, the result can revert, since underliquidityation occurs. If coverage is sufficiently high, such that \( \theta_b < T(a)/H < r/H \) it follows \( \theta_b < \theta_e \) and thus \( \theta_b (1 - a) < \theta_e \), thus both terms become positive, \( \frac{\partial}{\partial r} D > 0 \).

\[
\frac{\partial}{\partial \gamma} D = (r - T(a)) \frac{\partial \theta_b}{\partial \gamma} + \frac{\partial \theta_b}{\partial \gamma} (\theta_b H - r) \\
= (\theta_b H - T(a)) \frac{\partial \theta_b}{\partial \gamma} \\
\]

(91)

(92)

where the critical state monotonically decreases in coverage \( \frac{\partial \theta_b}{\partial \gamma} < 0 \). For \( \gamma \) small, inefficient runs occur thus \( (\theta_b H - T(a)) > 0 \) and \( \frac{\partial}{\partial \gamma} D < 0 \). For coverage high, underliquidityation occurs, \( (\theta_b H - T(a)) < 0 \) and thus \( \frac{\partial}{\partial \gamma} D > 0 \). The deadweight loss is thus minimized if insurance is such that the critical state equals

\[
\theta_b = \frac{T(a)}{H} \\
\]

(93)

Since the critical state is continuous and monotone in insurance, \( \frac{\partial \theta_b}{\partial \gamma} < 0 \), and \( \frac{T(a)}{H} \) is constant in insurance, there is maximum one insurance level for given forbearance \( a \) which satisfies this condition. To show existence, by ( ) the trigger and thus critical state go to zero as \( \gamma \to 1 \) and exceeds \( \theta_e \) for \( \gamma \to 0 \),
by \( r \geq l \) we know \( \frac{T(a)}{H} < \theta_c \), thus for every \( a \) there exists a unique insurance level \( \gamma^*(a) \) such that

\[
\theta_b(a, \gamma^*(a)) = \frac{T(a)}{H}
\]

which minimizes the deadweight loss. Further, for \( \gamma < \gamma^*(a) \), \( \theta_b(a, \gamma) > \frac{T(a)}{H} \) and thus the deadweight loss is decreasing on \((0, \gamma^*(a))\), \( \frac{\partial D}{\partial \gamma} < 0 \), while for \( \gamma > \gamma^* \) the deadweight loss is increasing.

\[\Box\]

**Proof.** [Theorem 4.2] We have

\[
\frac{\partial}{\partial r} \frac{\partial}{\partial a} D = \theta_b + (r - l) \frac{\partial \theta_b}{\partial r}
\]

(95)

\[
+ \frac{\partial \theta_b}{\partial a} (a + \frac{\partial \theta_b}{\partial r} H - 1)
\]

(96)

\[
+ \frac{\partial}{\partial r} \frac{\partial \theta_b}{\partial a} (\theta_b H - T(a))
\]

(97)

by previous results, the first term is positive, the second term is negative by \( \frac{\partial \theta_b}{\partial r} < 0 \) but small for recovery rate sufficiently close to liquidation value, the derivative \( \frac{\partial \theta_b}{\partial a} \) is negative for \( r \) small and positive for \( r \) large, the bracket \( (a + \frac{\partial \theta_b}{\partial r} H - 1) \) is always negative by \( \frac{\partial \theta_b}{\partial r} < 0 \) and \( a < 1 \), \( \frac{\partial}{\partial r} \frac{\partial \theta_b}{\partial a} \) is always positive and \( \theta_b H - T(a) \) is positive for deposit insurance sufficiently small. Thus, for \( r \) sufficiently small, and deposit insurance low, only the second term is negative but small and \( \frac{\partial}{\partial r} \frac{\partial}{\partial a} D > 0 \).

To show that \( \frac{\partial}{\partial a} D \) satisfies single-crossing in \( r \) for \( \gamma \) low, fix \( a \) and consider \( r \to l \) in (18). Then, \( \frac{\partial \theta^*_a}{\partial a} \leq 0 \) by Proposition 3.2. Further, \( T'(a) = (l - r) \to 0 \), thus term A in (18) is small since \( \theta_b \in [0, 1] \) is bounded. Also \( (r - T(a)) \to 0 \) as \( r \to l \) and \( \frac{\partial \theta^*_a}{\partial a} \) is bounded since the derivative is a continuous function on a compact interval \( r \in [l, 1] \). Therefore also term B in (18) is small. The sign of term C depends, for low deposit insurance we know that inefficient runs take place, \( \theta_b > \theta_c \), thus \( \theta_b H > r \) and term C becomes negative, thus \( \lim_{r \to l} \frac{\partial}{\partial a} D < 0 \) for \( \gamma \) low. On the other hand, as \( r \) increases, the comparative statics of the trigger flip. For given \( a \) consider the unique value \( r^*(a) \) such that \( \frac{\partial \theta^*_a}{\partial a} (r^*) = 0 \) and \( \frac{\partial \theta^*_a}{\partial r} (r) \geq 0 \) for all \( r > r^*(a) \). Then, in (18) for all \( r \geq r^* \), \( -T'(a) = -(l - r) > 0 \), thus term A in (18) is positive. By \( r - T(a) > 0 \), term B is positive if too. Term C is positive under inefficient runs (low
deposit insurance), thus under low deposit insurance, all terms are positive, \( \lim_{r \to 1} \frac{\partial}{\partial a} D > 0 \). Thus, by continuity of \( \frac{\partial}{\partial a} D \) there exists at least one value \( \hat{r} \) at which \( \frac{\partial}{\partial a} D(\hat{r}) = 0 \) If in addition, \( \frac{\partial^2}{\partial r \partial a} D \geq 0 \) even for \( r > r^* \), then \( \hat{r} \) is unique.

Thus, for \( \gamma \) fixed and sufficiently low, by the Intermediate value Theorem for every forbearance level there exists a unique \( \hat{r} \) such that \( \frac{\partial}{\partial a} D(a, \hat{r}) = 0 \) and \( \frac{\partial}{\partial a} D(a, r) < 0 \) for \( r < \hat{r} \), \( \frac{\partial}{\partial a} D(a, r) > 0 \) for \( r > \hat{r} \).

The issue is here, that for \( r > r^* \) the third term in (95) switches sign and becomes negative which either slows down the increase of deadweightloss in forbearance or reverts it. Under reversion, single-crossing can get lost, i.e. there can exist a larger \( r^{**} > r^* \) such that \( D \) decreases again in \( a \) for \( r > r^{**} \).

Under high insurance coverage, the bracket in term four becomes negative \( \theta_b H - T(a) < 0 \). Thus for low recovery rates, terms one and three are positive, term two and four are negative which slows down the change of \( \frac{\partial}{\partial a} D(a, r) \) in \( r \). As \( r \) increases, term three turns negative too such that the slope of deadweight loss in forbearance may even decrease as \( r \) goes up.

\[ \tag{98} \frac{\partial}{\partial a} \theta^* = - \frac{1}{k - \gamma} \left( \frac{r(\delta - l)}{(\delta - la)^2} \ln(la/\delta) + \frac{1}{a}(1 - \frac{r(1 - a)}{\delta - la}) \right) \]

\[ \tag{99} < - \frac{1}{k - \gamma} \left( \frac{r(\delta - l)}{(\delta - la)^2} \frac{la - \delta}{la} + \frac{1}{a}(1 - \frac{r(1 - a)}{\delta - la}) \right) \]

\[ \tag{100} = - \frac{1}{k - \gamma} \left( 1 - \frac{r}{l} \right) < 0 \]

since \( r \leq l \).

7.5 Proof: Inefficient RA

Proof. [Lemma 4.2] From (64), using the logarithm equation again

\[ \frac{\partial}{\partial a} \theta^* = \frac{1}{k - \gamma} \left( \ln(la/\delta) + \frac{1}{a}(1 - \frac{r(1 - a)}{\delta - la}) \right) \]

\[ < \frac{1}{k - \gamma} \left( \frac{r(\delta - l)}{(\delta - la)^2} \frac{la - \delta}{la} + \frac{1}{a}(1 - \frac{r(1 - a)}{\delta - la}) \right) \]

\[ = - \frac{1}{k - \gamma} \left( 1 - \frac{r}{l} \right) < 0 \]

References


