Asset Bubbles and Product Market Competition

Francisco Queirós*
Universitat Pompeu Fabra
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Abstract

This paper studies the effects of rational bubbles in an economy characterized by imperfect competition in product markets. It provides two main insights. The first is that imperfect competition relaxes the conditions for the existence of rational bubbles. When they have market power, firms restrict output and investment to enjoy supernormal profits. This depresses the interest rate, making rational bubbles possible even when capital accumulation is dynamically efficient. The second is that by providing a production or entry subsidy, asset bubbles may have a pro-competitive effect and force firms to expand and cut profit margins. However, once they get too large they can lead to overinvestment and sustain corporate losses.

I use anecdotal evidence from the British railway mania of the 1840s and the dotcom bubble of the late 1990s to support the model’s hypotheses and predictions.

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Key words: rational bubbles, competition, market power, dotcom bubble, British railway mania

*Address: Universitat Pompeu Fabra, Ramon Trias Fargas 25-27, 08005 Barcelona, Spain. e-mail: francisco.queiros@upf.edu. I am greatly indebted to my advisors, Fernando Broner and Jaume Ventura, for their guidance and motivation. I also thank Matt Delventhal, Luca Fornaro, Alberto Martin, Haozhou Tang, Tomas Williams and participants of the CREI International Lunch for helpful discussions and suggestions. I also thank Gareth Campbell and John Turner for sharing their data on railway share prices. I acknowledge financial support from the Portuguese Science Foundation (grant SFRH/BD/87426/2012). All errors are my own.
1 Introduction

“With valuations based on multiples of revenue, there’s ample incentive to race for growth, even at the cost of low or even negative gross margins.”

“Dotcom history is not yet repeating itself, but it is starting to rhyme”, Financial Times, March 12, 2015

Stock markets often experience fluctuations that seem too large to be entirely driven by fundamentals. Major historical events include the Mississippi and the South Sea bubbles of 1720 or the British railway mania of the 1840s. A more recent example is that of the US stock market in the late 1990s, during the so-called dotcom bubble: between October 1995 and March 2000, the Nasdaq Composite index increased by almost sixfold to then collapse by 77% in the following two years. One common aspect among some of these stock market boom/bust episodes is that they are associated with a particular market or good: for instance, the railway mania affected essentially the British railway industry and the dotcom bubble was an episode concentrated on a group of internet and high-tech industries. More recently, some policy-makers questioned the economic rationale behind the staggering increase in stock prices in the biotechnological and social media sectors: on 07/16/2014, Fed’s chair Janet Yellen argued that “Valuation metrics in some sectors do appear substantially stretched - particularly those for smaller firms in the social media and biotechnology industries”.

Being phenomena typically associated with a set of particular goods or industries, stock market overvaluation episodes are often accompanied by significant changes in the market structure. The dotcom bubble of the late 1990s constitutes perhaps a good example in this regard. In an environment characterized by a widespread excitement about the internet and soaring prices of technology stocks, many internet firms went public. Furthermore, in an attempt to maximize their market values, these firms often sought for rapid growth and engaged in aggressive commercial practices, such as advertisement overspending, excess capacity or extremely low penetration prices. For instance, some online delivery companies appearing around this period (such as Kozmo.com or UrbanFetch) provided their services completely for free. Some firms would even make money payments to attract consumers: the advertising company AllAdvantage.com paid internet users to display advertisements on their screens. Most of these companies incurred in extensive income losses and could not survive the stock market crash in 2000 (see section 6).

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1 Although there is no consensus among economists, a great deal of evidence suggests that technology stocks became overvalued in the late 1990s. For instance, Ofek and Richardson [2002] made simple calculations and showed that the stock market value reached by the entire internet sector at the peak of the dotcom bubble could only be accounted for, on the basis of the discounted value of future earnings, if estimated growth rates were unreasonably high (far above the rates historically observed for the fastest growing individual firms in the whole economy) and/or discount rates were absurdly low. Lamont and Thaler [2003] also documented clear examples of overpricing in a number of equity carve-outs. They studied in depth the case of 3Com (a profitable provider of network systems), which sold 5% of its subsidiary Palm (a computer producer) through an IPO in 2000. As it was documented, the price reached by the shares of Palm was so high that, if one were to subtract the implied value of the remaining 95% of Palm from 3Com, one would find that the non-Palm part of 3Com had a negative value.

2 Goldfarb and Kirsch [2008] report that between 1994 and 2001 “approximately 50,000 companies solicited venture capital to exploit the commercialization of the internet”; among these, around 500 companies had an initial public offering.
But even if lacking market expertise or following unsustainable business models, these firms often posed a serious threat to incumbent companies and in some cases forced them to expand and enter in the online market. For instance, the appearance of many online toy retailers such as eToys, Toysmart, Toytime or Red Rocket (all of which went bankrupt after the stock market crash) forced Toys"R"Us to enter in the internet market by means of a partnership with Amazon. Some other companies, on the other hand, decided to expand even before a new competitor appeared. A well-known example in this category involves the “Destroy Your Business” program launched by GE’s CEO Jack Welch in 1999. Welch asked managers from different divisions of GE to go through a collective exercise and think of different ways in which a new dotcom company could destroy their leadership in specific markets. The main idea consisted in identifying new production processes or business opportunities before other companies did. As part of the “Destroy Your Business” initiative, many divisions of GE (such as GE Plastics, GE Medical Systems or GE Appliances) adopted cost-cutting programs and started providing new services through the internet. A more detailed description of these and other examples is done in section 6.

But what role did the stock market play in fostering competition in product markets? Was it indeed behind the aggressive commercial strategies pursued by the new internet firms? To answer these questions, it is important to notice that valuation models are often based on multiples of revenues or market shares and not on profits (Liu, Nissin and Thomas [2002] and Kim and Ritter [1999]). This is especially true in the case of young firms: they typically start with low or even negative profit margins, which makes it difficult to project future cash flows from current earnings. For instance, Hong and Stein [2003] provide detailed evidence that equity analysts offering valuations for Amazon in the 1997-1999 period tended to emphasize its growth path (in terms of sales) and highly disregarded operating margins. This type of valuation methods has obvious consequences on firms’ behavior, as it can induce managers to focus on revenue targets at the expense of profits (Aghion and Stein [2008]). It can therefore have a positive and pro-competitive effect, but can also force firms to expand excessively and incur in income losses. As noted in the context of the recent Silicon Valley boom: “With valuations based on multiples of revenue, there’s ample incentive to race for growth, even at the cost of low or even negative gross margins. The many taxi apps and instant delivery services competing for attention, for example, are facing huge pressure to cut prices in the hope of outlasting the competition”.

Income losses were indeed prevalent during the dotcom boom. As Ofek and Richardson [2002] document, public Internet firms had aggregate revenues of $ 27.429 billion in 1999, but negative EBITDA and net income of (minus) $5.750 billion and (minus) $9.888 billion respectively (the aggregate stock market capitalization for the sector was $ 942.967 billion). Negative earnings could also be detected at the aggregate-industry level. Figure 1 shows economy-aggregate earnings and revenues for three industries at the center of the dotcom boom - ‘Publishing Industries (Software)’, ‘Information and

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3There is also ample evidence that this technique induced firms to choose aggressive revenue recognition practices during the dotcom years. These included the reporting of barter transactions and grossed-up (as opposed to net) revenue. Barter transactions were frequent among internet firms as they used to exchange advertising space in their websites, leaving managers with the faculty to assess the fair value of their revenues and expenses. See Bowen, Davis and Rajgopal [2002].

4“Dotcom history is not yet repeating itself but it is starting to rhyme” (03/12/2015), Financial Times
Data Processing Services’ and ‘Computer Systems Design and Related Services’. It also shows the price-sales ratio for the universe of publicly listed firms in those industries. As one can see, two of these industries exhibited negative aggregate profits at the peak of the boom in 2000. Furthermore, there seems to be a negative relationship between aggregate profits and stock prices in the three industries: aggregate profits decline in the boom period 1998-2000, but start increasing after the stock market crash.5

![Figure 1: Revenues and Profits at the Industry Level](image)

This figure shows economy-aggregate profits and revenues for three different industries (‘Publishing Industries (Software)’, ‘Information and Data Processing Services’ and ‘Computer Systems Design and Related Services’) and the price-sales for the universe of publicly listed firms in those industries. ‘Profits’ refers to ‘Corporate Profits Before Tax by Industry’ (from BEA - NIPA Table 6.17D) and ‘Revenues’ refers to ‘Gross Output by Industry’ (from BEA). ‘Price-Sales’ is the ratio of total stock market capitalization (stock price times common shares outstanding, from CRSP) to total sales (COMPUSTAT item #12), constructed at the beginning of the corresponding year.

The recurrence and magnitude of recent stock market boom/bust episodes (such as the dotcom bubble of the late 1990s) has somehow prompted revived interest in the old theory of rational bubbles, which dates back to the seminal contributions of Samuelson [1958] and Tirole [1985]. In particular, recent macroeconomic models have been developed to explain how asset bubbles may mitigate the existence of financial frictions and promote economic growth.6 In spite of the

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5 Due to changes in industry classification, data on aggregate profits and revenues is not available for earlier years.

6 As it is well known, the presence of agency problems may limit firms’ capacity to borrow and hence to undertake efficient investment plans.

It is within this context that asset bubbles may be useful. Either by being a liquidity instrument (Fahri and Tirole [2011], Hirano and Yanagawa [2017], Kocherlakota [2009], Miao and Wang [2012]) or a source of collateral (Martin and Ventura [2011, 2012, 2016] and Tang [2017]), bubbles may help constrained firms increase investment and therefore be associated with economic expansions.
undisputed relevance of credit market imperfections, these models are however silent about the effects of asset bubbles on the market structure and on the degree of competition in product markets. Furthermore, they fail to explain how overvaluation may sustain overinvestment and generate income losses. As I shall argue, these have been important aspects of the dotcom bubble of the late 1990s, but could be also found in other historical episodes, such as the British railway mania of the 1840s (see section 6).

In this paper, I aim at studying the interaction between asset bubbles and product market competition. To this end, I develop a standard multi-industry model with imperfect competition. In each industry there is a productive firm (the leader) which faces competition from a fringe of relatively inefficient competitors (the followers). Absent the formation of bubbles, the leader’s optimal decision is to set a limit price that prevents the entry of the followers (therefore enjoying supernormal profits).

I first show that imperfect competition depresses the interest in general equilibrium, hence relaxing the conditions for the existence of rational bubbles. Having market power, firms limit output and investment relative to the social optimum. As a result, both the demand for credit and the interest rate may be sufficiently depressed so that rational asset bubbles become possible even when capital accumulation is dynamically efficient.\(^7\) This is a novel insight. In standard models that incorporate rational bubbles, low interest rates are achieved through two main channels: dynamic inefficiency and financial frictions. In the first one, low interest rates are the result of excessive savings and/or unproductive investment technologies (as for instance in Samuelson [1958] and Tirole [1985]). In the second, they are the consequence of financial market imperfections (such as limited pledgeability) that constrain firms’ borrowing capacity and hence their demand for credit (as in Farhi and Tirole [2011] and Martin and Ventura [2011, 2012, 2016]).\(^8\) In this paper, low interest rates are the result of a low demand for credit, as it happens in the presence of credit market imperfections. However, here the mechanism is quite different: at the given interest rate, firms do not borrow more not because they cannot do so, but rather because they do not want to do so. As we shall see, this difference will have important implications.

I also show that asset bubbles may have a pro-competitive effect and be expansionary. For instance, if the followers can create bubbly firms and overvaluation is proportional to revenues or market shares, they may be willing to produce even when incurring in an operational loss. This may force the leader to set a lower limit price, in order to keep his monopoly position. Therefore, even if attached to unproductive firms, bubbles can nevertheless induce an efficient reaction on the

\[^{7}\text{From a theoretical standpoint, it is well known that rational asset bubbles can only emerge in economies where the interest rate is depressed, i.e. lower than the growth rate. The argument is straightforward. On the one hand, for rational bubbles to exist they must offer a return that is not lower than the interest rate. On the other hand, bubbles cannot grow faster than the economy (otherwise they can be ruled out with simple backward induction arguments).}\]

\[^{8}\text{When there are externalities to capital accumulation (as it happens in models of learning-by-doing) the fact that the interest rate is below the growth rate is not an indication of dynamic inefficiency since there may be a wedge between the private and the social return to capital. Bubbles will be possible when the private return to capital is below the growth rate. However, as long as social return to capital is greater than this last, the economy will be dynamically efficient. This is what happens in the models of Grossman and Yanagawa [1993], King and Ferguson [1993] and Olivier [2000].}\]
part of incumbents. In this paper I emphasize this new channel: by providing a return that mimics an entry or production subsidy, asset bubbles may reduce entry barriers and incumbents’ market power. For this mechanism to work, it is however crucial that the followers can benefit from the formation of bubbles or pyramid schemes. If only the leaders can create overvalued firms, they will typically not expand: they just need to set a limit price to prevent the entry of the followers. As we shall see the assumptions we make about the distribution of bubbles across firms will be a crucial aspect of the model.  

What is perhaps interesting is that asset bubbles can have economic consequences even if they do not materialize: the very belief that the followers can create overvalued firms (if they produce) will be sufficient to force the leader to expand. This is a situation of a latent bubble: bubbles or pyramid schemes can emerge if new firms enter in the market; however, it may be in the best interest of incumbents to blockade the entry of new competitors, as in the example of GE described above. The model will therefore provide a theory for bubble-driven business cycles even when firm values do not deviate from fundamentals.

Even if leading to output expansions, bubbles may nevertheless generate several inefficiencies. By being proportional to revenues or market shares, overvaluation may force firms to expand output beyond the social optimum (therefore incurring income losses) in order to maximize their market value. The model therefore offers a simple rationale for the prevalence of income losses among technological and internet firms at the peak of the dotcom bubble in the years of 1999 and 2000.

Some extensions are considered. First, in line with the recent literature on rational asset bubbles, I introduce financial frictions by means of a limited pledgeability problem. I show that financial frictions have a stronger impact on the least productive firms, therefore reinforcing the leaders’ market power (section 4). In such case, bubbles may provide the followers with additional collateral (even if not proportional to revenues or market shares) thereby forcing the leaders to expand. Second, I introduce endogenous product variety and put competition in a dynamic perspective. By reducing market power in old industries, asset bubbles can foster the development of new industries and growth. However, if they increase competition in new sectors bubbles may hinder economic growth (section 5).

I also show that the main results of this paper are robust to different formulations of product market frictions. In the benchmark model, imperfect competition stems from the fact that only one firm can make use of the best available technology. However, I show in an appendix that the main results go through if all firms are equally productive but there are fixed production cost (which creates increasing returns to scale). In such case, it is still possible to construct rational bubble equilibria in economies that are dynamically efficient. Furthermore, by providing an entry subsidy asset bubbles also promote competition and force operating firms to expand (appendix A).

The rest of the paper is organized as follows. Section 2 describes the benchmark economy with no bubbles. It shows that bubbly equilibria will be possible even when the economy is dynamically efficient. Section 3 looks at different bubbly equilibria and explores its implications in terms of the market structure, investment and output. Section 4 introduces
financial frictions and compares the results to the ones of previous models. Section 5 introduces endogenous growth and looks at the consequences of asset bubbles from a dynamic perspective. Section 6 reviews some anecdotal evidence from two important stock market overvaluation episodes: the British railway mania of the 1840s and the dotcom bubble of the late 1990s. These episodes are reinterpreted through the lens of the theory developed in this paper. Section 7 concludes. Before proceeding, I offer a brief review of the related literature.

1.1 Related literature

This paper is mostly related to the literature that forms the theory of rational bubbles. Different models have emphasized different aspects of asset bubbles. In very broad terms, we can separate the literature in two categories. On the one hand, there are models that view bubbles as assets whose main role is being a store of value. This is the central tenet of the seminal contribution of Samuelson [1958] who argues that bubbles may complete intergenerational markets and provide for an efficient intertemporal allocation of resources. Tirole [1985] makes the same point in the context of the neoclassical growth model and emphasizing a direct crowding-out effect: bubbles drive resources away from investment. However, in the model of Tirole this effect happens to be welfare-improving as it eliminates inefficient capital accumulation.\(^\text{10}\) Being a store of value, bubbles can also be a liquidity instrument that may help firms overcome financial frictions as in Caballero and Krishnamurthy [2006], Farhi and Tirole [2012], Hirano and Yanagawa [2017], Kocherlakota [2009] or Miao and Wang [2012]. Finally, Ventura [2012] shows that bubbles can increase the return on savings in low productivity countries, thereby eliminating cross-country differences in rates of return and acting as a substitute for capital flows.

A different strand of the literature, on the other hand, has put emphasis on the formation of new bubbles: the formation of a new pyramid scheme always provides some kind of subsidy or return that can have economic consequences. Within this category, we find the model of Olivier [2000] who shows that if attached to R&D firms, bubbles can stimulate the invention of new goods and foster economic growth. More recently, Martin and Ventura [2011, 2012, 2016] argue that the creation of new bubbles may be a source of collateral that allows credit-constrained firms to borrow and invest more. I will provide a theory of how asset bubbles can be expansionary and I will focus on bubble formation. My paper will be closest in spirit to the models of Martin and Ventura [2011, 2012, 2016], though there will be important differences and conclusions. Here, the focus will be on frictions in product markets, not in financial markets. I will also consider the limited pledgeability problem and discuss how it may exacerbate imperfect competition in final goods markets.

In this paper I will allow firm overvaluation to depend on revenues or market shares (consistent with systematic and anecdotal evidence about valuation models). I should however stress that the possibility that overvaluation is a function of fundamentals has already been admitted by the rational bubbles literature. For instance, Froot and Obstfeld [1991] constructed a simple asset pricing model with rational bubbles that are a function of dividends. However, this is perhaps\(^\text{10}\) Some authors have later pointed out that in the presence of externalities to capital accumulation, such crowding-out effect could be growth-impairing and welfare-reducing. This is the main message of the endogenous growth models of Grossman and Yanagawa [1993] and King and Ferguson [1993].
the first paper to explicitly study the economic implications of such hypothesis.

By establishing a connection between product market competition and the interest rate, the model can also shed light on recent macroeconomic trends. There are signs suggesting that market power has been increasing in the US since the 1980s. Using a sample of publicly listed firms, De Loecker and Eeckhout [2017] document a substantial increase on average markups from 1980 to now. Such increase on average markups has been accompanied by a decline in business dynamism, particularly evident in a secular decline in the start-up rate and a greater concentration of activity and employment in larger and older firms (Decker et al. [2014]). All these trends have coincided with a persistent decline in real interest rates, which have even become negative in recent years. Even though there may be multiple forces contributing to the interest rate decline, the model presented in this paper suggests that it can be linked to the increase in market power.

This paper is also related to the vast literature studying the cyclical properties of markups which includes contributions by Rotemberg and Woodford [1992] and Chevalier and Scharfstein [1996]. Finally, this paper speaks to the literature describing firm and investor behavior during the British railway mania of the 1840s (Campbell and Turner [2010, 2012, 2015], Odlyzko [2010]) and during dotcom bubble of the late 1990s (such as Brunnemeier and Nagel [2004], Griffin et al. [2011] and Campello and Graham [2013]). It is important to mention that some authors have proposed fundamental-based explanations for the dotcom bubble of the late 1990s. For instance, Pastor and Veronesi [2009] argue that the level attained by the Nasdaq index at its peak in the year 2000 could be explained on the basis of high uncertainty about the newly formed internet sector. Under such view, investors were initially uncertain about the long-run profitability of the internet sector and revised their expectations downwards as profits declined in 2000/2001. The paper presents a quite different view: the low profitability levels reached by internet companies in 2000/2001, rather than the realization of a stochastic process, were an endogenous reaction to high stock prices. This view seems to receive support from anecdotal evidence reviewed in section 6.

2 The Benchmark Economy with no Bubbles

The model is built upon the popular overlapping generations model by Diamond [1965]. There is an economy populated by two overlapping generations. Members of the first generation will be referred to as the young, and members of the second as the old. Within each generation, there will be two types agents: the workers and the entrepreneurs. Each generation-type will have measure one. All agents are born with no wealth and maximize old-age consumption:

\[ U_{i,t} = C_{i,t+1} \]

where \( U_{i,t} \) is the welfare at time \( t \) of a young agent \( i \) and \( C_{i,t+1} \) is his consumption when old. Throughout, I assume there is no uncertainty (hence the absence of an expectation operator).\(^{11}\)

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\(^{11}\)Uncertainty can be easily introduced in the current framework. However, it will bring no major insight as it shall become clear.
Workers supply inelastically one unit of labor when young and get a wage \( w_t \). The wage will naturally be saved. Workers will have to choose between two savings schemes. On the one hand, they have access to a storage technology with gross return \( r < 1 \). On the other hand, they can buy financial securities promising a state-contingent gross return \( R_{t+1} \). Storage must be seen as an inefficient investment opportunity that may nevertheless be used in equilibrium. The supply of funds in the credit market will therefore be given by

\[
F_{t}^{S} = \begin{cases} 
  w_t & \text{if } R_{t+1} > r \\
  [0, w_t] & \text{if } R_{t+1} = r
\end{cases}
\]  

When they retire, workers will run a firm in the final goods sector, which operates under perfect competition. They will hire \( L_{t+1} \) units of labor (supplied by young workers) and buy capital goods \( K_{t+1} \) to produce \( Y_{t+1} \) units of the final good according to a Cobb-Douglas technology

\[
Y_{t+1} = L_{t+1}^{1-\alpha} \cdot K_{t+1}^{\alpha}
\]  

Capital fully depreciates in production. It is be a CES composite of different intermediate inputs, which have measure one:

\[
K_{t+1} = \left( \int_{0}^{1} x_i^{\rho} \, di \right)^{\frac{1}{\rho}}
\]  

where \( x_{i,t+1} \) is the quantity of intermediate \( i \in [0,1] \), \( 0 < \rho < 1 \) and \( \sigma \equiv \frac{1}{1-\rho} \) is the elasticity of substitution among inputs. The parameter \( \rho \) can be seen as a measure of substitution among inputs. When \( \rho \) is low, differentiation is high and firms can possibly have high market power.\(^{12}\) The final good will be used as the numeraire.

Since young workers supply one unit of labor inelastically and they have measure one, the labor market clears at \( L_{t+1} = 1 \). Factor markets are competitive and factors are paid their marginal product

\[
w_{t+1} = (1 - \alpha) \cdot Y_{t+1}
\]  

\[
q_{t+1} = \alpha \cdot K_{t+1}^{\alpha-1}
\]  

\[
p_{i,t+1} = q_{t+1} \cdot \left( \frac{K_{t+1}}{x_{i,t+1}} \right)^{1-\rho}
\]  

where \( w_{t+1} \) is the wage rate, \( q_{t+1} \) is the rental rate and \( p_{i,t+1} \) is the price of intermediate input \( i \in [0,1] \).

Intermediate inputs will be produced by the young entrepreneurs, who will be indexed by \( j \in [0,1] \). The production of intermediates uses the final good as its only input. To motivate the existence of a credit market, I assume it needs to be invested one period in advance. Moreover, I will assume constant returns to scale so that entrepreneur \( j \in [0,1] \) produces

\(^{12}\)As \( \rho \to 1 \), capital becomes linear in intermediates; in this case, they will be perfect substitutes. As \( \rho \to 0 \), capital becomes a Cobb-Douglas aggregate of intermediates; in such case, production requires a strictly positive amount of each variety and the degree of differentiation is very high.
intermediate \( i \in [0, 1] \) according to

\[ x_{i,t+1}^j = \pi_{i,t}^j \cdot m_{i,t}^j \tag{7} \]

where \( x_{i,t+1}^j \) is the output of the intermediate good, \( m_{i,t}^j \) the quantity of the final good used and \( \pi_{i,t+1}^j \) is the productivity of entrepreneur \( j \in [0, 1] \) in the production of intermediate \( i \in [0, 1] \). The assumptions about the distribution of productivity types will be a crucial element of the model. I will assume that

\[
\pi_{i,t}^j = \begin{cases} 
1 & \text{if } j = i \\
\pi < 1 & \text{if } j \neq i
\end{cases}
\tag{8}
\]

Therefore, each variety \( i \in [0, 1] \) can be produced either with productivity \( \pi_{i,t}^j = 1 \) by entrepreneur \( j = i \) or with productivity \( \pi < 1 \) by all the others. I will refer to entrepreneur \( j = i \) as the leader of industry \( i \) and to all other entrepreneurs \( j \neq i \) as the followers. Note that every entrepreneur is a leader in one industry and a follower in all the other markets. However, the important aspect of (8) is that for every input variety there is only one individual with access to the best technology. This is a crucial assumption, since it creates scope for imperfect competition and market power. Since they are born with no wealth, young entrepreneurs need to raise funds to invest. In order to do so, they sell financial securities in the credit market. Clearly, in equilibrium all financial securities must promise the same return \( R_{t+1} \).

### 2.1 Industry Equilibrium

Before solving for the general equilibrium of the economy, let us look at each individual industry separately. If the leader were granted a monopoly, he would solve

\[
\max_{x_{i,t+1}} \left[ q_{t+1} \cdot \left( \frac{K_{t+1}}{x_{i,t+1}} \right)^{1-\rho} - R_{t+1} \right] \cdot x_{i,t+1}
\]

where I used the fact that \( p_{i,t+1} = q_{t+1} \cdot \left( \frac{K_{t+1}}{x_{i,t+1}} \right)^{1-\rho} \). The solution to this problem yields

\[
x_{i,t+1} = \left( q_{t+1} \cdot \frac{\rho}{R_{t+1}} \right)^{\frac{1}{\rho-1}} K_{t+1}
\]

and

\[
p_{i,t+1} = \frac{1}{\rho} \cdot \frac{R_{t+1}}{\text{markup}} \cdot \frac{R_{t+1}}{\text{marginal cost}}
\]

This a well-known result: given an elasticity of substitution equal to \( \frac{1}{1-\rho} \), the monopoly price consists of markup \( \frac{1}{\rho} \) over the marginal cost. However, as long as \( \pi > \rho \) we have \( p_{i,t+1} > R_{t+1}/\pi \), implying that the followers would be willing to enter in the industry. This forces the leader to set a limit price equal to the followers’ marginal cost \( R_{t+1}/\pi \) and to produce
a quantity greater than the monopoly level. In this case we observe

\[ x_{i,t+1} = \left( q_{t+1} \cdot \frac{\pi}{R_{t+1}} \right)^{\frac{1}{1-\rho}} K_{t+1} \]  \hspace{1cm} (9)

\[ p_{i,t+1} = \frac{R_{t+1}}{\pi} \]  \hspace{1cm} (10)

To sum up, as long as \( \pi > \rho \) the leader will charge a price-cost markup below his desired level to prevent the entry of the followers. This markup is equal to \( \frac{1}{\pi} \), implying a profit share on revenues equal to \( 1 - \pi \).

Throughout this paper, I will focus on the case in which \( \pi > \rho \) to ensure that the leader always faces competition from the followers. This happens either when the productivity gap is not very large (\( \pi \) is close to one) or when the desired markup is high (\( \rho \) is low). Equations (9) and (10) determine the equilibrium quantity and price of industry \( i \in [0, 1] \) given the capital stock and the interest rate of the economy. We shall now see how these aggregate variables are determined.

### 2.2 General Equilibrium

To determine the aggregate capital stock, we can start by combining equations (3), (5) and (9) to find that

\[ K_{t+1} = \left( \frac{\pi \cdot \alpha}{R_{t+1}} \right)^{\frac{1}{1-\rho}} \]  \hspace{1cm} (11)

This equation establishes a negative relationship between the capital stock and the interest rate. Since there are diminishing returns to capital, a higher capital stock implies a lower rental rate and a lower interest rate through (10). Let us now determine \( K_{t+1} \) and \( R_{t+1} \). Equilibrium in the credit market requires that \( K_{t+1} = F_{t}^{S} \), where \( F_{t}^{S} \) is the amount of credit supplied by young workers at time \( t \). We can combine this condition with equations (1), (4) and (11) to find the law of motion of this economy

\[ K_{t+1} = \min \left\{ (1 - \alpha) \cdot K_{t}^{\rho}, \left( \frac{\pi \cdot \alpha}{r} \right)^{\frac{1}{1-\rho}} \right\} \]  \hspace{1cm} (12)

\[ R_{t+1} = \pi \cdot \alpha \cdot K_{t+1}^{\alpha-1} \]  \hspace{1cm} (13)

Equation (12) just says that when savings are low, the storage technology is not used. In this case all labor income can be invested with a high return (i.e. greater than the return on storage \( r \)). If savings are sufficiently high, then only a fraction will be converted into capital so that the rental rate does not fall short of \( r \). Equation (13) determines the interest rate as a function of the capital stock; it also implies that (gross) interest payments represent a fraction \( \pi \cdot \alpha \) of aggregate output. To understand this result, note that the labor and capital shares are equal to respectively a fraction \( 1 - \alpha \) and \( \alpha \) of output. Of this last share, a fraction \( 1 - \pi \) represents profits in the intermediate inputs sector, and the remaining fraction \( \pi \) represents interest payment to creditors.

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\(^{13}\)Combining equations (5) and (6), we have \( p_{i,t+1} = \alpha \cdot K_{t+1}^{\alpha-\rho} \cdot x_{i,t+1}^{\rho-1} \).
Note that despite the assumptions about the distribution of productivity types and the market structure, this economy behaves as a standard Solow model. It will converge to a steady-state characterized by

\[ K^* = \min \left\{ (1 - \alpha)^{\frac{1}{1-\alpha}}, \left( \frac{\pi \cdot \alpha}{1 - \alpha} \right)^{\frac{1}{1-\alpha}} \right\} \tag{14} \]

\[ R^* = \max \left\{ \frac{\pi \cdot \alpha}{1 - \alpha}, r \right\} \tag{15} \]

### 2.3 Dynamic Efficiency and the Steady-State Interest Rate

There are a few aspects that are worth discussing. The first one pertains to the effect of imperfect competition on the interest rate. As we can see from equation (13), the interest rate is lower than the social return to investment \( \alpha \cdot K_{t+1}^{\alpha-1} \).

\[ R_{t+1} = \pi \cdot \alpha \cdot K_{t+1}^{\alpha-1} < \alpha \cdot K_{t+1}^{\alpha-1} \]

This wedge is a direct consequence of imperfect competition in the market for intermediates. In each industry, the leader sets a price above his marginal cost of production and restricts investment. This lowers the demand for credit and depresses the interest rate in general equilibrium.

A second aspect has to do with the relationship between the interest rate and capital overaccumulation. In the standard OLG of Diamond [1965], there is overaccumulation of capital (i.e. the economy is dynamically inefficient) if and only if the interest rate is below the economy’s growth rate. This results implies that rational asset bubbles are possible if and only if capital accumulation is dynamically inefficient.\(^{14}\) However, such equivalence need not be true under the current framework given the presence of imperfect competition. To assess this hypothesis, let us ignore storage for a moment (assume that \( r \to 0 \)), so that in equilibrium all savings are intermediated. In this case, the economy converges to a steady-state characterized by

\[ K^* = (1 - \alpha)^{\frac{1}{1-\alpha}}, \quad Y^* = (1 - \alpha)^{\frac{\alpha}{1-\alpha}} \]

In such steady-state, the interest rate will be equal to

\[ R^* = \frac{\pi \cdot \alpha}{1 - \alpha} \]

Therefore, the interest rate will be below the growth rate (and hence rational bubbles are possible) if and only if

\[ 1 - \alpha > \pi \cdot \alpha \tag{16} \]

\(^{14}\)Recall that in equilibrium asset bubbles grow at the rate of interest. Therefore, they are only possible if the steady-state interest rate is below the long-run growth rate.
This condition says that the steady-state interest rate is lower than one if the savings rate \( s = 1 - \alpha \) is higher than the share of output that accrues to lenders \( \pi \cdot \alpha \). Note that this last share depends positively \( \pi \) which, as we have seen, is inversely related to the degree of market power.

But under what conditions is capital accumulation dynamically efficient (i.e. the economy does not overaccumulate capital)? To answer this question, note that the steady-state capital accumulation is efficient if the marginal product of capital is above its marginal cost of production. Formally this happens if

\[
\frac{\partial Y(K)}{\partial K} \bigg|_{K=K^*} \geq 1
\]

It is easy to verify that such condition is verified as long as

\[
1 - \alpha \leq \alpha
\]  

(17)

This is a well-known result. It says that the economy does not feature capital overaccumulation if the savings rate \( s = 1 - \alpha \) is not higher than the capital share in production \( \alpha \). A comparison between conditions (16) and (17) shows that if \( \pi \) is sufficiently low, the interest rate can be below one even when capital accumulation is dynamically efficient. This simple model therefore provides an environment where rational bubbles are possible even when the economy is dynamically efficient. Figure 2 summarizes these two conditions in the \((\alpha, \pi)\) space. Rational asset bubbles can emerge in both regions I and II. In these two regions we have \( 1 - \alpha > \pi \cdot \alpha \) and the steady-state interest rate is lower than one. In region I, this happens because there is overaccumulation of capital, i.e. \( 1 - \alpha > \alpha \). This steady-state is inefficient as the welfare of all generations could be increased if investment were lower. In region II, we have \( 1 - \alpha < \alpha \) and hence there is no overaccumulation of capital. However, the interest rate is still depressed because the leaders have a large productivity advantage over the followers (\( \pi \) is low) and hence high market power. In this case, the demand for funds is depressed because each individual monopolist restricts investment to enjoy monopoly rents. Putting it differently, the leaders are only willing to absorb the existing level of savings if the cost of borrowing is sufficiently low.

The above discussion may appear somehow puzzling. I have argued that even if the economy is dynamically efficient, the interest rate can be low because imperfect competition depresses the demand for funds. Nevertheless, the two above conditions only impose a joint restriction on \( \alpha \) (the capital share in the aggregate production function) and \( \pi \) (the relative efficiency of the leader). Indeed, they say nothing about the degree of product substitutability, captured by \( \rho \). However, this parameter plays an important role: all the above equations were derived under the condition that \( \pi > \rho \). This condition forces the leader to expand output beyond their optimal level in order to prevent the entry of the followers. If this condition was not satisfied, the presence of the followers would make no difference within this setup without bubbles: each leader would be a monopolist.\(^{15}\) To sum up, for bubbles to be possible when there is no overinvestment, if \( \pi \leq \rho \) and \( r \to 0 \) (storage is never used), the economy would converge to the same steady state as before: \( K^* = (1 - \alpha)^{\frac{1}{1-\alpha}} \). Therefore, the condition for investment to be efficient would still be the same, namely that \( \alpha \geq \frac{1}{2} \). However, now we would observe \( R^* = \rho \cdot q^* = \rho \frac{\alpha}{1-\alpha} \).

\(^{15}\)If \( \pi \leq \rho \) and \( r \to 0 \) (storage is never used), the economy would converge to the same steady state as before: \( K^* = (1 - \alpha)^{\frac{1}{1-\alpha}} \). Therefore, the condition for investment to be efficient would still be the same, namely that \( \alpha \geq \frac{1}{2} \). However, now we would observe \( R^* = \rho \cdot q^* = \rho \frac{\alpha}{1-\alpha} \).
the productivity gap must be sufficiently high. Given the condition that \( \pi > \rho \), this requires \( \rho \) (the degree of product substitutability) to be low.

All the above results were derived under the assumption that \( r \rightarrow 0 \) so that storage was never used in equilibrium. However, as it can be seen from (14), storage will be used if

\[
r > \frac{\pi \cdot \alpha}{1 - \alpha}
\]

Therefore, as long as

\[
\frac{1}{2} < \alpha < \frac{r}{\pi + r}
\] (18)

we will observe underinvestment: storage is used even if the aggregate return to investment is above one. This happens in region II.1 of Figure 2. As it should now be clear, for bubbles to be possible there does not need to be underinvestment (i.e. storage does not need to be used). In the absence of underinvestment, all savings will be intermediated and asset bubbles (if they exist) will always be contractionary. I will assume that (18) holds, so that asset bubbles can be expansionary. As we shall see in the next section, if fostering competition bubbles can lead to an expansion in the output of each intermediate variety. The results will however depend on the distribution of bubbles across firm types.

Therefore, \( R^* < 1 \Leftrightarrow \rho < \frac{1-\alpha}{\alpha} \). As we can see, the condition for the existence of bubbles now depends directly on the degree of market competition: \( \rho \) needs to be sufficiently low.
3 The Bubbly Economy

Before proceeding it is perhaps useful to clarify the concept of bubble creation. Every young entrepreneur \( j \in [0,1] \) needs to raise funds in order to invest. To do so, he must sell financial securities in the credit market, promising a gross return \( R_{t+1} \). Let \( d_{t+1}^j \) be the dividends these securities pay at time \( t+1 \). Their fundamental price at time \( t \) is defined as

\[
f_t^j := \frac{d_{t+1}^j}{R_{t+1}}
\]  

Is this price we should observe? The answer is no, as there may be a bubble component attached to it. That is, the price may be equal to

\[
v_t^j = \frac{d_{t+1}^j}{R_{t+1}} + \frac{b_{t+1}^j}{R_{t+1}}
\]  

where \( b_{t+1}^j \geq 0 \) is the bubble component, as of time \( t+1 \), attached to the securities issued by entrepreneur \( j \) at time \( t \). Throughout this paper, I will refer to \( b_{t+1}^j \) as the new bubbles created by entrepreneur \( j \). As we will see, the way \( b_{t+1}^j \) is determined will be a crucial aspect of the model. For the time being, I must only stress that \( b_{t+1}^j > 0 \) can be consistent with a perfect information and rational expectations equilibrium. Even if the securities issued by entrepreneur \( j \) at time \( t \) pay a single dividend \( d_{t+1}^j \), any saver at time \( t+1 \) would be willing to pay \( d_{t+1}^j + b_{t+1}^j \) if he can resell them by \( R_{t+2} \cdot b_{t+1}^j \) in the subsequent period.

3.1 Industry Equilibrium

Let us focus on an arbitrary industry \( i \in [0,1] \) and consider three different processes for the creation of bubbles.

3.1.1 Constant Bubbles at the Firm Level

Assume that entrepreneur \( j \in [0,1] \) operating in industry \( i \in [0,1] \) can create a firm with a fixed bubble component \( b_{i,t+1}^j > 0 \). How would such bubble affect the industry equilibrium? As it should be clear, the equilibrium would be exactly the same as before. The leader has no incentive to expand his output beyond the one given by equation (9): this is the output level that maximizes his profits given the competition he faces from the followers. On the other hand, the followers do not invest their resources as long as the leader produces the quantity that guarantees \( p_{i,t+1} = \frac{R_{t+1}}{\pi} \).
Therefore, if every entrepreneur gets a constant bubble that is not linked to any variable he can control (for instance output), there will be absolutely no effect in the industry equilibrium. This would be not true in the presence of financial frictions (section 4) or when there are fixed production costs (appendix A).

### 3.1.2 Constant Bubbles at the Industry Level

Assume that instead of being created by each individual entrepreneur, bubbles are created at the industry level. In particular, assume there is a bubble with size $b_{i,t+1} > 0$ being created in industry $i$ at time $t+1$ and that each entrepreneur gets a fraction that is equal to his market share. To understand this bubble, think for instance of the British railway industry in 1845 or in some internet or high-tech industry in 1999. According to this process, investors’ total demand for shares in this such industry exceeds its fundamental value by a fixed amount $b_{i,t+1}$. Furthermore, this industry-aggregate bubble is distributed across firms according to their market shares, so that bigger firms get a larger share in the bubble. This assumption is not unrealistic: as discussed above, the valuation of firms is often based on multiples of revenues or market shares. This bubble process could indeed provide a rationale for such valuation methods.

Under these conditions, will the leader still produce the quantity given by (9)? The answer is no. To see this, note that for any industry output level $x_{i,t+1}$ such that

$$\frac{R_{t+1}}{\pi} < p_{i,t+1} + \frac{1}{x_{i,t+1}} \cdot b_{i,t+1}$$

the followers can profitably enter! The reason that their marginal cost of production is still $R_{t+1}/\pi$, but they now get an additional return of $(1/x_{i,t+1}) b_{i,t+1}$ per each unit that they sell. Therefore, to prevent the entry of the followers, the leader must set a limit price lower than the followers’ marginal cost so that the above condition holds with equality. In this case, output is implicitly defined by

$$\frac{R_{t+1}}{\pi} \cdot x_{i,t+1} = \alpha \cdot x_{i,t+1}^\rho \cdot K_t^{\alpha - \rho} + b_{i,t+1}$$

(21)

where equations (5) and (6) have been used. It can be easily checked from (21) that $x_{i,t+1}$ is increasing in $b_{i,t+1}$ (see appendix B). This bubble process therefore fosters competition in the industry and forces the leader to expand. He will do so up to the point that prevents the followers from entering in the market. Figure 3 shows the output and price of good $i$ as a function of the industry bubble $b_{i,t+1}$. Naturally, as $b_{i,t+1}$ rises total output increases and the price decreases. But as the new bubble gets too large, $p_{i,t+1}$ will fall short of the leader’s marginal cost of production $R_{t+1}$ and profits become negative. Therefore, this bubble process may lead to a situation of excessive investment and corporate losses as in some of the bubbly episodes described in section 6.
Parameters: $\alpha = 0.5$, $\rho = 0.5$, $\pi = 0.8$ and $r = 0.95$

Figure 3: Industry-level Bubble

3.1.3 Multiplicative Bubbles

In the example described above, competition for a fixed industry bubble forces the leader to expand. The total size of the industry bubble was independent of output. Suppose however that instead of taking a fixed size, each firm can create a new bubble in proportion to revenues. In particular, assume that entrepreneur $j \in [0,1]$ in industry $i \in [0,1]$ can create a bubble with size

$$b_{i,t+1}^j = \theta_{i}^j \cdot p_{i,t+1} \cdot x_{i,t+1}^j$$

with $\theta_{i}^j \geq 0$. This process is not unreasonable given that valuation is often based on multiples of revenues (as discussed before). The constant $\theta_{i}^j$ is allowed to differ across types. This is done mainly for theoretical clarity, because bubbles may have different effects depending on who creates them.

Let us start by looking at an equilibrium with $\theta_1^L > 0$ and $\theta_1^F = 0$ (i.e. only the leader can create bubbles). In this case, if he had no competitors, the leader would produce

$$x_{i,t+1}^L = \left( (1 + \theta_1^L) \frac{\rho \cdot \alpha}{R_{t+1}} \right)^{\frac{1}{\rho}} K_{t+1}^{\frac{\alpha - \rho}{\rho}}$$

Note however that if $(1 + \theta_1^L) \cdot \rho \leq \pi$, this value is lower than the quantity in (9), implying that the followers can profitably enter. Therefore, when $(1 + \theta_1^L) \cdot \rho \leq \pi$ the bubble will have no effect in terms of economic activity. The reason is that the leader always needs to produce at least the quantity in (9) to keep the followers out of the market. The bubble only
leads to an increase in output when \((1 + \theta_L^t) \cdot \rho > \pi\). In this case, the leader produces the quantity given by (22). This is shown in the left panels of Figure 4.

Let us now look at an equilibrium with \(\theta_L^t = 0\) and \(\theta_F^t > 0\). In this case, the followers will enter in the market if

\[
p_{i,t+1} > p_{i,t+1}^r (\theta_F^t) = \frac{1}{1 + \theta_F^t} \frac{R_{t+1}}{\pi}
\]

Therefore, to keep the followers out of the market the leader must produce a sufficiently large quantity such that \(p_{i,t+1} = p_{i,t+1}^r (\theta_F^t)\). Of course, he can only do so as long as he does not incur in a loss, which happens if \((1 + \theta_F^t) \pi \leq 1\). Note that in this case the followers will not produce and no bubble will appear! This is a situation in which there is a latent bubble: if the followers were to produce, a bubble would materialize. However, the leader optimality expands production to the point in which it is not profitable for the followers to enter. This example therefore provides a theory for sentiment-driven business cycles even when prices do not depart from fundamentals. Finally note that if \((1 + \theta_L^t) \pi > 1\) and \(\theta_L^t = 0\), the followers produce and dethrone the leader.

\[
\theta_L > 0 \text{ and } \theta_F = 0 \\
\theta_L = 0 \text{ and } \theta_F > 0
\]

Parameters: \(\alpha = 0.5, \rho = 0.7, \pi = 0.8\) and \(r = 0.9\)

Figure 4: Multiplicative Bubble: leader versus followers
3.2 General Equilibrium

I will look at symmetric equilibria in which all industries are subject to identical bubble processes

\[ b_{i,t+1}^z = b_{i,t+1}^z, \forall i \text{ and } z \in \{L,F\} \]

In this case, all industries produce the same output which will be a negative function of the interest rate

\[ K_{t+1} = x_{i,t+1} = f(R_{t+1}), \forall i \]

The exact functional form \( f(\cdot) \) will depend on the assumptions about the creation of new bubbles. If only the leaders produce, equilibrium in the credit market requires that

\[ R_{t+1} = \max \{ f^{-1}[(1 - \alpha) \cdot K_t^\alpha - B_t], r \} \]

To understand the previous equation note that when all savings are intermediated

\[ f(R_{t+1}) + \underbrace{B_t}_{\text{investment}} = \underbrace{(1 - \alpha) \cdot K_t^\alpha}_{\text{bubble}} + \underbrace{B_t}_{\text{savings}} \]

Therefore, as long as \( R_{t+1} = r \), bubbles crowd out storage and will not be contractionary. However, if \( R_{t+1} > r \) bubbles will crowd out investment and will be contractionary. To conclude, I must characterize the bubble dynamics. In equilibrium, the return on existing bubbles must equal the interest rate. There will also be new bubbles being created in every period. Therefore, we observe the following law of motion

\[ B_{t+1} = R_{t+1} \cdot B_t + \int_{j \in [0,1]} \int_{i \in [0,1]} b_{i,t+1}^j \]

The remaining of this section characterizes the steady-state of this economy under the different bubble processes considered above.

3.2.1 Constant Bubbles at the Firm Level

If each individual entrepreneur is able to create a bubble with size \( b_i^j = b \), aggregate investment and output will not change when as long as \( R^* = r \). In such case, the bubble just absorbs resources from storage and the economy will still converge to \( K^* = \left( \frac{\alpha}{r} \right)^{\frac{1}{1-\alpha}} \) as in the bubbleless equilibrium. However, the bubble introduces an efficient intergenerational allocation of resources, as it crowds out investment from the low-return storage technology. As a result, aggregate consumption increases even when output remains constant (see Figure 5).

When the bubble gets too large, storage is no longer used and \( R^* > r \). In this case, the bubble diverts away resources
from investment and leads to a contraction of output. The steady-state capital stock is implicitly defined by

\[ K + \frac{b}{1 - \pi \cdot \alpha \cdot K^{\alpha - 1}} = (1 - \alpha) \cdot K^\alpha \]

\[ K + \frac{b}{1 - \pi \cdot \alpha \cdot K^{\alpha - 1}} = (1 - \alpha) \cdot K^\alpha \]

\[ \alpha \cdot K^{\alpha - 1} = \frac{R}{\pi} - \frac{b}{K} \]  

(23)

It is easy to verify that the bubble is expansionary as long as \( R_{t+1} = r \) (see proof in appendix B). As before, if \( R_{t+1} > r \) storage stops being used and the bubble becomes contractionary. A steady-state of this economy as a function of \( b \) is represented in Figure 6. As we can see, there is an expansionary region when \( b \) is small.

\[ \alpha \cdot K^{\alpha - 1} = \frac{R}{\pi} - \frac{b}{K} \]  

(23)
3.2.3 Multiplicative Bubbles

First let us focus on equilibria with $\theta^L = 0$ and $\theta^F > 0$: only the followers create bubbles. In this case

$$K_{t+1} = x_{i,t+1} = \left[ \frac{\alpha \cdot \pi (1 + \theta^F)}{R_{t+1}} \right]^{\frac{1}{1-\alpha}}$$

If $\theta^F$ is sufficiently small, expectations about the formation of bubbles by the followers force the leaders to expand. This results in additional capital formation and a contraction of storage. This happens as long as

$$1 + \theta^F \leq \frac{1 - \alpha}{\alpha} \frac{r}{\pi}$$

Once storage stops being used, the interest rate increases with $\theta^F$ but the capital stock does not decline. This happens because we are always in a situation of a latent bubble: as $\theta^F$ rises, the demand for funds increases; however, since no bubble materializes, the supply of funds is fixed and all the adjustment occurs through the interest rate. Under this version of the model, economies with identical levels of capital stock and no bubbles can nevertheless have different interest rates due to pure expectations about the appearance of new bubbles. A steady-state of this economy is represented in Figure 7.

It can be shown that in this symmetric equilibrium the followers will never produce even if $\theta^L = 0$. This happens because when $\alpha \geq 0.5$ (investment is efficient), the minimum bubble that allows the followers to enter requires an interest
rate greater than one.\textsuperscript{16}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure7}
\caption{Multiplicative Bubble on the Followers}
\end{figure}

Now let us look at equilibria with $\theta^L > 0$ and $\theta^F = 0$. In this case only the leaders create bubbles and

\[
K = \max \left\{ \left[ \left( 1 + \theta^L \right) \frac{\alpha \cdot \rho}{R} \right]^{\frac{1}{\gamma - \alpha} \left( \frac{\alpha \cdot \pi}{R} \right)^{\frac{1}{\gamma - \alpha}}}, \left( \frac{\alpha \cdot \pi}{R} \right)^{\frac{1}{\gamma - \alpha}} \right\}
\]

Recall that the bubble is expansionary only when $\left( 1 + \theta^L \right) \cdot \rho > \pi$. The minimum (steady-state) expansionary bubble is

\[
B = \frac{1}{1 - R} \cdot \frac{\pi - \rho}{\rho} \cdot \alpha K^\alpha
\]

For this bubble to be possible when storage is built, we need

\[
K + \frac{1}{1 - R} \cdot \frac{\pi - \rho}{\rho} \cdot \alpha K^\alpha \leq (1 - \alpha) K^\alpha
\]

As we can see from this equation, bubbles can be expansionary only when $\rho$ is high enough (i.e. when firms have low

\textsuperscript{16}When no storage is used, equilibrium in the credit market requires

\[R = \frac{\alpha}{1 - \alpha} \left( 1 + \theta^F \right) \pi\]

Given that $\alpha \geq 0.5$, $(1 + \theta^F) \pi \geq 1$ requires $R \geq 1$ which precludes the appearance of bubbles.
market power). This is represented in Figure 8.

Figure 8: Multiplicative Bubble on the Leaders

4 Financial Frictions

Most papers analyzing the macroeconomic consequences of asset bubbles have focused on credit market imperfections. I have however taken a different perspective and focused on the workings of product markets. In this section, I will introduce financial frictions and discuss how it can affect product market competition. Following the literature, I will consider a limited pledgeability problem and assume that the institutional framework is such that borrowers can only pledge a fraction \( \phi \in (0, 1) \) of revenues. However, they can pledge entirely all the bubbles they can create. Furthermore, I shall assume that the credit market is segmented across industries, so that an entrepreneur cannot collateralize his borrowing in one industry against his revenues (or bubbles) in other industries.\(^{17}\) Let \( g_{i,t}^j \) denote the funds raised by entrepreneur \( j \) at time \( t \) in industry \( i \). Then, we must observe the following credit constraint

\[
R_{i,t+1} \cdot g_{i,t}^j \leq \phi \cdot p_{i,t+1} \cdot x_{i,t+1}^j + b_{i,t+1}^j
\]

\(^{17}\)This last assumption, being not unrealistic, renders the analysis simple and clear. It could naturally be relaxed, but at the expense of extra complexity and no interesting insight.
As we shall see, an interesting interplay between financial frictions and imperfect competition will emerge: financial frictions may affect disproportionately more the followers and hence exacerbate the lack of competition in product markets. Indeed, absent the formation of bubbles and if the pledgeability parameter $\phi$ is not too low (see conditions below), only the followers will face a binding credit constraint.

In this setup, bubbles will play a new role as they can serve as a source of collateral and hence allow constrained entrepreneurs to increase their borrowing. This is the channel considered in several recent models, such as Martin and Ventura [2011, 2012, 2016] and Tang [2017]. However, even when they simply provide collateral (and not the sort of production subsidy considered in the previous section) bubbles may still have a pro-competitive effect. This is what happens if the followers are constrained, but the leader is unconstrained. In such case, bubbles will relax the constraint faced by the followers. However, as the followers can borrow and invest more, the unconstrained leader will be forced to expand.

### 4.1 Moderate Financial Frictions: $\phi \cdot \pi > \rho$

As before, let us focus on an arbitrary industry $i \in [0,1]$. Before introducing bubble creation, I will characterize the industry equilibrium with no bubbles.

#### 4.1.1 Bubbleless Equilibrium

Let us start by assuming that in industry $i$ the leader was granted a monopoly. In this case, if the constraint in (24) does not bind, he produces

$$x_{i,t+1} = \left( q_{t+1} \cdot \frac{\rho}{R_{t+1}} \right)^{\frac{1}{1-\rho}} K_{t+1}$$

If the leader cannot invest such amount, this is because the constraint binds.$^{18}$ In such case, he can only invest

$$x_{i,t+1} = \left( q_{t+1} \cdot \frac{\phi}{R_{t+1}} \right)^{\frac{1}{1-\rho}} K_{t+1}$$

A direct comparison of these two quantities implies that as long as $\phi > \rho$, the leader is unconstrained if facing no competition. However, as long as $\phi \cdot \pi > \rho$ (which I will assume throughout), we have

$$p_{i,t+1} = q_{t+1} \cdot \left( \frac{K_{t+1}}{x_{i,t+1}} \right)^{1-\rho} = \frac{R_{t+1}}{\rho} > \frac{R_{t+1}}{\phi \cdot \pi}$$

$^{18}$In this case we have $R_{t+1} \cdot x_{i,t+1}^L = \phi \cdot p_{i,t+1} \cdot x_{i,t+1}^L$. 

24
implying that the followers are able and willing to enter in the industry. This forces the leader to expand and produce the quantity that guarantees 

\[ p_{i,t+1} = \frac{R_{i+1}}{\phi \cdot \pi} \] 

Therefore, in the absence of bubbles, we have that

\[ x_{i,t+1} = \left( \frac{\alpha \cdot \pi \cdot \phi}{R_{i+1}} \right)^{\frac{1-\rho}{K_{i+1}}} \] (25)

\[ p_{i,t+1} = \frac{R_{i+1}}{\phi \cdot \pi} \] (26)

These two equations characterize the industry equilibrium when \( \phi \cdot \pi > \rho \) and there are no bubbles. These are the natural counterparts of equations (9) and (10), which are obtained as the limiting case when \( \phi \to 1 \). As we can see, financial frictions exacerbate the degree of imperfect competition: given \( \phi \cdot \pi > \rho \), the lower is \( \phi \), the higher the price charged by the leader. To sum up, as long as \( \phi \cdot \pi > \rho \), the leader will be unconstrained. Still, he will need to expand beyond the desired monopoly quantity to prevent the entry of the followers.

### 4.1.2 Constant Bubbles at the Firm Level

Since I am interested in studying the role of bubbles as a source of collateral, I will just focus on constant-firm level bubbles. Assume that all entrepreneurs can create a bubble with constant size \( b_{i,t+1} > 0 \). How would this bubble affect the industry equilibrium? Clearly, as long as \( p_{i,t+1} > \frac{R_{i+1}}{\pi} \) the followers want to invest. We must therefore ask under what conditions the price is strictly above the followers’ marginal cost of production. First note that as long as \( \phi \geq \pi \), the leader can always keep the followers out of the market (I will assume this condition holds). This is because the financial friction is not extremely severe so that the leader can borrow and invest the amount that guarantees 

\[ p_{i,t+1} = \frac{R_{i+1}}{\phi \cdot \pi} \] 

\[ x_{i,t+1} = \left( \frac{\alpha \cdot \pi \cdot \phi}{R_{i+1}} \right)^{\frac{1-\rho}{K_{i+1}}} \] (25)

in order to guarantee \( p_{i,t+1} = \frac{R_{i+1}}{\phi \cdot \pi} \). In this case, the followers just create empty firms to appropriate the bubble creation rents: these however force the leader to increase production. On the other hand, when \( b_{i,t+1} \) is sufficiently small the leader will prefer to produce a quantity lower than \( \bar{x}_{i,t+1} \) and accommodate the entry of the followers. In this case, we observe \( p_{i,t+1} > \frac{R_{i+1}}{\pi} \) and the followers face a binding credit constraint. The characterization of the solution can be found in appendix C.

Figure 9 shows some equilibrium variables as a function of \( b_{i,t+1} \). When \( b_{i,t+1} \) is sufficiently small, the leader lets the followers enter. As \( b_{i,t+1} \) increases, the profits of the leader decrease, as the industry price falls. When \( b_{i,t+1} \) gets too

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19It is to see that the followers will enter as long as \( p_{i,t+1} > \frac{R_{i+1}}{\phi \cdot \pi} \).

20The alternative processes considered in section 3 (industry bubbles and multiplicative bubbles) will not provide any major insight in this context. If credit constraints are binding, bubbles will still serve as a source of collateral.
large, the leader will prefer to raise his output to $\bar{x}_{i,t+1}$ and keep the followers out of the market. When choosing whether to accommodate the entry of the followers, the leader faces a trade-off. On the one hand, by letting the followers enter, he will have a lower market share, but may charge a high price. If he decides to impede the entry of the followers, he can instead have a high market share (indeed a monopoly) but with a low price. This is shown in the second and fourth panels of Figure 9.

In the example depicted in Figure 9, the leader always expands in reaction to the creation of bubbles (by the followers). However, this does not need to be always the case. As shown in Figure 10, if $\phi$ is large enough and $b_{i,t+1}$ is low, the leader may accommodate the entry of the followers and reduce output when a bubble appears. This happens because when $\phi$ is high, even a small bubble allows the constrained followers to produce a large quantity. In such case, bubbles lead to a market inefficiency: the productive firm contracts, and the unproductive constrained firms expand. However, given that $\phi \geq \pi$, as $b_{i,t+1}$ gets too large the leader will prefer to produce the quantity that guarantees $p_{i,t+1} = R_{t+1}/\pi$ in order to keep the whole market.

Figure 9: Constant Bubble with Financial Frictions
4.2 General Equilibrium

4.2.1 Bubbleless Economy

We can combine equations (3) and (25) to find that

\[ K_{t+1} = \left( \frac{\alpha \cdot \pi \cdot \phi}{R_{t+1}} \right)^{\frac{1}{1-\alpha}} \]  
\[ (27) \]

This economy will have the following law of motion

\[ K_{t+1} = \min \left\{ (1 - \alpha) \cdot K_t^\alpha, \left( \frac{\alpha \cdot \pi \cdot \phi}{r} \right)^{\frac{1}{1-\alpha}} \right\} \]
\[ (28) \]

\[ R_{t+1} = \phi \cdot \alpha \cdot \pi \cdot K_{t+1}^{\alpha-1} \]
\[ (29) \]

and will converge to a steady-state given by

\[ K^* = \min \left\{ (1 - \alpha)^{\frac{1}{1-\alpha}}, \left( \frac{\alpha \cdot \pi \cdot \phi}{r} \right)^{\frac{1}{\alpha}} \right\} \]
\[ (30) \]
\[ R^* = \max \left\{ \frac{\phi \cdot \alpha \cdot \pi}{1 - \alpha}, r \right\} \]  \hspace{1cm} (31)

As it can be seen from equations (29) and (31), the presence of financial frictions puts additional downward pressure on the interest rate. When storage is not used in the steady-state, the condition for the existence of rational bubbles is

\[ \alpha < \frac{1}{1 + \phi \cdot \pi} \]  \hspace{1cm} (32)

It can be easily checked that the condition for dynamic efficiency is the same as before, namely

\[ \alpha \geq \frac{1}{2} \]  \hspace{1cm} (33)

Therefore, when \( \phi \) and \( \pi \) are not simultaneously too high, rational asset bubbles will still be possible even when capital accumulation is dynamically efficient.

Finally, note that storage will be used in the steady-state if

\[ r > \frac{\phi \cdot \alpha \cdot \pi}{1 - \alpha} \]

As before, I will focus on a parameter space under which the economy is dynamically efficient, but storage is built (so that there is underinvestment). This happens if

\[ \frac{1}{2} < \alpha < \frac{r}{\phi \cdot \pi + r} \]  \hspace{1cm} (34)

### 4.2.2 Constant Bubble at the Firm Level

Assume that in every industry firms can create a bubble with size \( b_i = b \geq 0 \quad \forall i \in [0, 1] \) that is not linked to the industry’s output. Appendix C describes how the equilibrium is determined. When \( b \) is low, there is a symmetric equilibrium in which all industries accommodate the entry of the followers. On the other hand, when \( b \) is high, the leaders impede the entry of the followers in all industries. As it can be seen in Figure 11, such symmetric equilibrium does not exist for intermediate values of \( b \). In this case, the followers are able to enter in a fraction \( \mu \in (0, 1) \) of the industries. In such industries, the leaders have a low market share, but charge a high price. In the industries in which the followers are unable to enter, the price is low but the leaders have a high market share (indeed they have the whole market). Figure 11 also represents the equilibrium output of intermediates and aggregate output as a function of \( b \).

\footnote{For the case \( \mu \in (0, 1) \), the corresponding averages are plotted.}
5 Endogenous Growth

So far, I have focused on a static setup. I assumed there was a constant set of intermediate goods $I = [0, 1]$ at all points in time. Under such framework, shocks to bubble creation could lead to fluctuations in output, but they could never generate sustained, long-run growth. However, it might be interesting to think of a set-up with endogenous product variety and determine the conditions under which the emergence of bubbles can boost or hinder growth. In order to do so, I will assume that entrepreneurs must choose between two different occupations: the production of existing intermediate varieties or the invention of new ones. This extension will offer new insights. Bubbles in existing sectors, if increasing competition and lowering their profitability, can foster the creation of new products/industries. However, bubbles appearing in new industries may inhibit their development: this happens if the increase in competition in these industries (which lowers profits) more than offsets the bubble creation return.

5.1 The Model with Increasing Varieties

Consider an economy similar in preferences and technology to the one described in Section 2. The main difference concerns the number of intermediates input varieties. Let $M_{t+1} \geq 1$ be the number of intermediate input varieties that are available
at time $t + 1$. Equation (3) will be replaced by

$$K_{t+1} = \left( \int_0^{M_{t+1}} x_{i,t+1}^d \, di \right)^{1/\delta}$$  \hspace{1cm} (35)$$

I will refer to $I_{t+1} \equiv [0, M_{t+1}]$ as the set of intermediates produced at time $t + 1$. Some of these goods were invented before time $t$: these are the *old* industries $[0, M_t]$. Other varieties are invented at time $t$: these are the *new* industries $[M_t, M_{t+1}]$. The assumptions about the distribution of productivity types need also to be generalized. It will be useful to define

$$S^j_t \equiv \left\{ i \in I_t : \pi_{i,t}^j \geq \pi_{l,t} \quad \forall l \in [0,1] \right\}$$

as the set of *old* varieties for which entrepreneur $j \in [0,1]$ is the most efficient producer. I will assume that in all *old* industries $i \in I_t$

$$\pi_{i,t}^j = \begin{cases} 1 & \text{if } i \in S^j_t \\ \pi < 1 & \text{if } i \notin S^j_t \end{cases}$$  \hspace{1cm} (36)$$

Furthermore, I impose that \( i \) $S^j_t \cap S^h_t = \emptyset$ if $j \neq h$ and that $I_t = \bigcup_{j \in [0,1]} S^j_t$. Given these assumptions, in each *old* industry there is one and only one leader. I also require that each set $S^j_t$ has infinitesimal measure, so that no individual entrepreneur can affect aggregate variables.

Entrepreneurs producing *old* varieties receive profits in the industries in which they are leaders. Alternatively, entrepreneurs can engage in innovation. Innovation consists in the invention and production of new varieties. Entrepreneurs differ in their ability to invent new varieties. At time $t$, an innovator $j \in [0,1]$ may discover a number $z^j_t$ of new varieties according to a technology

$$z^j_t = \lambda \cdot M_t \cdot j^{-\delta}, \delta > 0$$  \hspace{1cm} (37)$$

where $\lambda$ reflects the average efficiency of innovation and $M_t$ is the number of varieties invented prior to time $t$. According to this specification, the more advanced the technological frontier is, the easier it is to innovate. Entrepreneurs differ in their innovation capacity: without loss of generality, high index entrepreneurs are assumed to invent new varieties per period. If an entrepreneur decides to innovate, he will be the leader in the new industries that he invents. However, he cannot produce in the old industries $S^j_t$ that he commands: these are taken by the followers. This will be a critical assumption but will offer interesting insights as we will see now.

### 5.2 Equilibrium

Let $n_t$ be the number of entrepreneurs who innovate at time $t$. Given the innovation technology in (37), all entrepreneurs $[0, n_t]$ innovate. The technology frontier evolves according to

$$M_{t+1} = M_t + \int_0^{n_t} \lambda \cdot M_t \cdot j^{-\delta} \, dj$$  \hspace{1cm} (38)$$
All we need to determine is the fraction \( n_t \) of entrepreneurs who innovate. In what follows I will allow for the existence of multiplicative bubbles (this particular process is chosen just for analytical convenience). I will distinguish between bubbles attached to old versus to new industries. As before \( \theta^L \) and \( \theta^F \) will refer to bubbles created by the leaders and the followers in old industries. \( \varphi^L \) and \( \varphi^F \) will refer to bubbles created by the leaders and the followers in new industries.

Note that each entrepreneur is a leader in a measure \( M_t \) of old industries. \( \theta^L \) and \( \theta^F \) will refer to bubbles created by the leaders and the followers in old industries. \( \varphi^L \) and \( \varphi^F \) will refer to bubbles created by the leaders and the followers in new industries. Moreover, if he innovates he will produce in a number \( \lambda \cdot M_t \cdot n_t \cdot \delta \) of new industries. Therefore, in equilibrium we should observe the following indifference condition for the marginal innovator \( n_t \):

\[
\left( \frac{1 + \theta^L}{1 + \theta^F} - 1 \right) \cdot (1 + \theta^F) = \left( \frac{1 + \varphi^L}{1 + \varphi^F} - 1 \right) \cdot (1 + \varphi^F) \cdot \lambda \cdot n_t \cdot \delta
\]

This equation pins down the number of innovators \( n_t \). It says that the marginal innovator \( n_t \) must be indifferent between producing old or new varieties. It is easy to see that the number of innovators increases with both \( \varphi^L \) and \( \varphi^F \), but decreases with both \( \theta^L \) and \( \varphi^F \). \( \varphi^L \) increases the return on new industries for the innovating leaders and stimulates growth. This is the subsidy channel already highlighted by Olivier [2000]. \( \theta^F \) stimulates the creation of new industries through a different channel: by increasing competition in old industries and reducing their profitability, bubbles make new industries relatively more appealing and promote growth. On the other hand, \( \theta^L \) increases the return on existing industries for the leaders, making new ones relatively less appealing and discouraging innovation. Similarly \( \varphi^F \) increases competition in new industries and makes them relatively less attractive for the leaders. Here, the competition channel has the opposite effect: by reducing the profitability of new sectors, bubbles reduce the returns to innovation and growth.

Figure 12 shows the equilibrium number of innovators as a function of \( \theta^L \), \( \theta^F \), \( \varphi^L \) and \( \varphi^F \) (in each case, only one type of bubble exists and storage is built in equilibrium).

Under these assumptions the economy will experience a balanced growth path. The growth rate can be obtained by combining equations (38) and (39):

\[
\frac{M_{t+1}}{M_t} = 1 + \lambda \cdot n_t^{1-\delta}
\]

This extension puts the competition channel that is at the center of the model in a dynamic perspective. As it was shown, asset bubbles can intensify product market competition and reduce monopoly rents. This may bring not only

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22 The ones invented prior to time \( t \).

23 In period \( t + 1 \), the profits a leader can make in an old and a new industry are respectively given by

\[
\left[ \left( 1 + \theta^L \right) \cdot \frac{R_{t+1}}{1 + \theta^F} - R_{t+1} \right] \cdot \left[ \frac{\alpha \cdot \pi \left( 1 + \theta^F \right)}{R_{t+1}} \right] \cdot K_{t+1}^{\alpha \cdot \pi \delta}
\]

\[
\left[ \left( 1 + \varphi^L \right) \cdot \frac{R_{t+1}}{1 + \varphi^F} - R_{t+1} \right] \cdot \left[ \frac{\alpha \cdot \pi \left( 1 + \varphi^F \right)}{R_{t+1}} \right] \cdot K_{t+1}^{\alpha \cdot \pi \delta}
\]

24 Under the assumptions that \( (1 + \theta^L) \cdot \rho \leq \pi \) and \( (1 + \varphi^L) \cdot \rho \leq \pi \), so that the leaders always need to set a limit price.
static gains (stemming from an increase in output), but also dynamic benefits: if they reduce the profitability of existing sectors, bubbles can foster the creation of new ones. However, under this dynamic perspective the competition channel also poses a risk: if they reduce the profitability of new sectors, bubbles will hinder their development.

![Graphs showing Old Industry and New Industry growth rates](image)

Figure 12: Endogenous Growth

### 6 Competition in Famous Bubbly Episodes

Stock market boom/bust episodes are recurring phenomena in financial history. Famous examples include the Mississippi and the South Sea bubbles of 1720, the British *railway mania* of the 1840s or more recently the *dotcom bubble* of the late 1990s. In this section, I provide a brief description of two of these episodes - the British *railway mania* of the 1840s and the *dotcom* bubble of the late 1990s - and discuss how they can be reinterpreted in light of the theory developed above.
6.1 The British Railway *Mania* of the 1840s

![British Railway Share Price Index](image)

Figure 13: British Railway Share Price Index, 1843-1850
(Source: Campbell and Turner [2015])

The mid 1840s was a period of fast economic growth in Britain: favorable weather conditions (resulting in abundant harvests), together with historically low interest rates made Britain’s GDP grew at an average of rate of 4.6% between 1843 and 1845. It was within this environment that a collective enthusiasm about railways emerged. Contrarily to the majority of other countries, where the construction of railway lines was essentially a public investment, the expansion of the British railway system was financed by private companies and individuals. This widespread excitement attracted many new investors to the stock market and triggered a boom in stock prices: between January of 1843 and October of 1845, the share prices of railway companies increased by more than 100% (Campbell and Turner [2010]).

At the same time, investment shot up: total investment in new railway lines authorized by the British Parliament rose by an average of £4 million per year prior to 1843, to £60 million in 1845 and £132 million in 1846 (Haacke [2004]). Even though not all investments granted Parliament authorization would ever materialize, total capital formation by railway companies reached £30 million in 1846 and £44 million in 1847, which represented 5.2% and 7.3% of the British GDP respectively. By comparison, during the *dotcom* bubble of the late 1990s, total US investment in technological industries

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25 Individual investors financing railway projects around this time include famous scientists, intellectuals and politicians such as Charles Darwin, Charles Babbage, John Stuart Mill or Benjamin Disraeli (Odlyzko [2010]).

26 Despite being private investment, the construction of new railway lines required Parliament authorization. This happened because they often involved processes of land expropriation (Odlyzko [2010]).
reached a maximum of 2.8% of GDP in the year 2000.\footnote{Data is from the Bureau of Economic Analysis. The industries considered include Computer and Electronic Products, Publishing Industries, Broadcasting and Telecommunications and Information and Data Processing services.} Given the magnitude of these investments, the British railway mania has been referred to as “arguably the greatest bubble in history”.\footnote{\textit{The Economist}, “The Beauty of Bubbles”, 2008/12/18.}

Such collective enthusiasm would however cease in the middle of the decade. A recession in 1845, associated with failure of the potato crops in Ireland, led many people to fear times of famine and scarcity. At the same time, the escalation of construction costs resulted in substantial calls for capital from railway shareholders.\footnote{\textit{The Economist}, “The Railway Crisis - its Cause and its Cure”, 1848/10/21.} Several projects ended up being less profitable than expected. Many commentators and newspapers (such as the recently founded \textit{The Economist}) also started raising concerns about the potentially negative effects of such large-scale railway investments. As a result, share prices of railway companies started declining and between October 1845 and December 1850 the total stock market capitalization of railway companies decreased by 67\% (Campbell and Turner [2010]).

The deteriorating performance of railway companies was ultimately related to an environment dominated by intense competition and, in some cases, overinvestment. The collective euphoria about railways and the demand for railway investments were so high, that “the amounts of capital being committed to the industry made competition ever fiercer and business plans ever rosier.”\footnote{\textit{The Economist}, “The Beauty of Bubbles”, 2008/12/18.} Not only new lines were open in relatively unprofitable regions (serving sparsely populated areas) but there were also obvious examples of duplication of railway lines.\footnote{It is important to note that political environment in Britain at this time was highly favorable to a private market for railways. This contrasted with other countries were governments subsidized the construction of railway line or regulated tariffs (Martin [1849, p.26]). Furthermore, there was a widespread agreement about the necessity of promoting competition between railway companies to prevent monopolies.} Situations of line duplication were described (and sometimes harshly criticized) by many contemporaneous authors. One example, which is described in Cotterill [1849], is the railway line that connected Shrewsbury to Stafford, which opened in 1849 and was in operation until 1966. It was ran by The Shropshire Union Railways and Canal Company, founded in 1846:

\begin{quote}
“The Shropshire Union Railway is another instance of the baneful principle [of competition]. It is a line from Shrewsbury to Stafford, joining the Trent Valley; and there being no intermediate traffic, the expenditure of 6 or 700,000\$ to effect this junction, appears prima facie to be lavish; because, if the Shrewsbury people wish to go to London, there is the Shrewsbury and Birmingham Railway, accommodating at the same time an immense intervening population. If the Shrewsbury people are desirous of moving north, the Shrewsbury and Chester, a line long since in operation, would give ample accommodation. The Shropshire Union to Stafford would therefore appear to be unnecessary and useless. But it is the fruit of competition.”
\end{quote}

Another example involving the duplication of railway lines was the connection between Birmingham and Wolverhampton,
described in Martin [1849, p.37]. In 1846, the two cities were already connected by the Grand Junction Railway (and by water through the Birmingham Canal). Still, two other companies - the London and North Western Railway and the Great Western Company - were granted authorization to build two additional lines between the two cities:

“Three years ago, the district between Birmingham and Wolverhampton possessed a double communication for its traffic (...) by means of the Birmingham Canal and the Grand Junction Railway, each connecting the two towns. Additional Railway accommodation was, however, supposed to be desirable, and two Companies presented their rival plans to a Committee of the House of Commons for selection. Both Railways are now in the course of formation, traversing a highly valuable and thickly peopled district in parallel lines (at some points nearly touching each other), and each intended to terminate in separate stations in the centres of the two towns. At least four millions of money will thus be unprofitably sunk, in order that three lines of railway and one canal may afford a redundant accommodation to a tract some fourteen miles in length.”

This example makes the author conclude that “Monopoly has an ill sound: but, unless it can be proved to be incapable of regulation, we must prefer even monopoly to competition run mad.”

It is interesting to note that some of the duplication of lines was undertaken by established companies, which expanded in order to prevent the appearance of new competitors. For instance, in their study of competition during the railway mania, Campbell and Turner [2015] found that the fraction of lines which enjoyed absolute monopoly fell from 72% in 1843 to 32% in 1850. However, when focusing on competition from other companies, the authors found that 85% of the routes had a complete monopoly in 1843, but this fell to only 66% by 1850. The idea that incumbent railway companies over-expanded and in some cases duplicated their own lines just to deter the entry of new rivals is corroborated by contemporaneous observers. For instance, an article published in 1848 in The Economist notes that the London and North Western Company (one of the most important railway companies of that time) had investments in the order of £7 million “still to be expended on lines, few if any of which had been undertaken with reference to their own merits, but for the purpose (perhaps not an unjustifiable one) of averting threatened opposition”. Cotterill [1849, p.33] also refers, in a highly critical vein, that the North Staffordshire Railway built two parallel lines in the Churnet Valley to impede the appearance of other companies

“[the North Staffordshire Railway] instead of one trunk line running from Manchester to the south, it has two, viz. from Macclesfield to Colwich on the Trent Valley, and from Macclesfield to Burton, joining the Midlands; this is doubly misjudging, two lines nearly parallel. Both cannot answer, and probably one only will be worked. Competition caused it; it was a competition between the Churnet Valley and the North Staffordshire Company.”

The over-expansion of established railway companies has indeed been a distinctive feature of the British railway mania. As described by Jackman [1916, p.599] in his history of the British railway system:

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“Nothing was more common than to see a company eagerly seeking authority to make a branch which could only bring it loss, but which, it was feared, would cause still greater loss if it fell into the hands of a rival. In some cases the companies ran a greater number of trains than the traffic warranted, or carried traffic, for the time being, at unremunerative rates in order to take it away from their rivals.”

Even though the examples mentioned above may constitute situations of overinvestment, one might still argue that they reflected anti-competitive practices on the part of established companies. According to such view, competition may not have increased as incumbents companies built excess capacity to protect their monopoly power. However, the evidence shows that the revenues and profits (per mile) of incumbent railway companies fell during the mania. Campbell and Turner [2015] report that the average revenue per mile of established companies (i.e. existing in 1843) fell from £3,603 in 1843 to £2,559 in 1850 (by 29%). At the same time, average profits per mile dropped from £1,811 to £1,231 (by 32%).

Despite the lower profitability, and confirming some of the anecdotes described above, incumbent companies expanded their capacity quite dramatically: between 1843 and 1850, the milage operated by the average incumbent company grew from 36 to 153 miles.33

Why did railway companies expand so quickly? What was behind “competition run mad”, to use the words of Martin [1849, p.37]? Although different factors may have contributed to the expansion of the British railway system during the 1840s (such as a political environment highly favorable to free markets and competition),34 these events can be rationalized by the model presented in this paper. As investors perceived railway stocks as good financial assets (whose price was likely to appreciate in the future), vast amounts of money were poured into the British railway industry. Such high demand for railway shares may have then opened the door to the appearance of new companies and lines that were not profitable from an operating point of view. That the mania was a time characterized by positive sentiment and speculation in railway companies is confirmed by several contemporaneous writers. For instance, keeping his critical view on the events, Martin [1849, p.40] observes that

“Men and women, high and low, rich and poor, entered the destructive road of which the gates were so widely opened by the Legislature, in the expectation that all could suddenly become rich; the result to many was, that the rich were impoverished, and persons without a shilling rose on their ruin. Shopkeepers augmented their expenditure by hundreds, brokers and share speculators by thousands; 332 new schemes were brought before the public down to the 30th September, 1845, involving capital to the enormous

33Campbell and Turner [2015] also use a short-path algorithm to find best alternative to each route and find that the number of segments with no (reasonable) substitute falls from 67% to 29% between 1843 to 1850. At the same time, the (median) additional time incurred by taking the best alternative to a particular route falls from 22% to 9%

34It is important to note that the political environment in Britain at this time was highly favorable to a private market for railways. This contrasted with other countries were governments subsidized the construction of railway line or regulated tariffs (Martin [1849, p.26]). Furthermore, there was a widespread agreement about the necessity of promoting competition between railway companies to prevent monopolies. This explains for instance why the British parliament approved many railway schemes that constituted duplication of existing lines.
sum of £270,959,000 of which £23,057,492 would have to be deposited with the Accountant-General before Parliament would receive application for the Acts”

In such an environment, and as the evidence above confirms, incumbent companies were forced to expand and cut profit margins in order to prevent the entry of new competitors. Seen in this way, the expansion of the British railway system may have been commanded (at least in part) by financial market sentiment. The idea that investor sentiment may drive firms’ expansion at the expense of profit margins, and ultimately provide a subsidy to consumers, was a central message of the model presented in this paper. As noted by Jackman [1916, p. 602], “although many of the railways were not profitable to their owners in yielding large financial returns they may still have been beneficial to the public in providing for the necessities and conveniences of traffic”.

6.2 The Dotcom Bubble of the Late 1990s

Another famous stock market boom and crash would take place in the United States one century and a half later. Associated with the appearance of the internet and in a period marked by low interest rates, the Nasdaq index increased by more than 560% between January 1995 and March 2000 (Figure 14). However, as in the British railway mania of the 1840, the widespread enthusiasm about internet companies would also cease. Concerns about the persistently negative profitability of most new internet firms and the fact that some were running out of cash (and hence needed to raise additional funds to finance their operations) marked a turning point in market sentiment. An article published in the Barron’s in March 2000 sounded the alarm: “An exclusive study conducted for Barron’s by the Internet stock evaluation firm Pegasus Research International indicates that at least 51 ‘Net firms will burn through their cash within the next 12
months. This amounts to a quarter of the 207 companies included in our study.” And it added “It’s no secret that most Internet companies continue to be money-burners. Of the companies in the Pegasus survey, 74% had negative cash flows. For many, there seems to be little realistic hope of profits in the near term.” A natural question therefore emerged: “When will the Internet Bubble burst?” The downturn would start in that very same month: between March 2000 and October 2002, the Nasdaq index decreased by 77%.

Behind the poor performance of so many dotcom firms was a search for rapid growth involving aggressive commercial practices - such as extremely low penetration prices, advertisement overspending and excess capacity - and which resulted in low levels of profitability or even extensive losses. For instance, many new companies offered their services at unprofitably low prices or even for free. This was for instance common among delivery companies. Kozmo.com and UrbanFetch were two such examples - they offered one-hour delivery services of books, videos, food and other goods totally for free. Many products would even be sold at a discount, gifts were sometimes included and tips were not accepted. None of them survived the stock market crash in 2000. The online music industry also observed many of these practices, with companies such as CDNow.com, Riffage.com or Napster offering downloads or peer-to-peer sharing of music for free. Another example is the software company SunMicrosystems, which decided to enter in the office suite market (largely dominated by Microsoft Office) with a software that was made available completely for free (this example is reviewed in more detail below). The pressure for growth was in some cases so high that some companies would actually pay customers to use their services. One well-known example is the advertising company AllAdvantage.com (launched in 1999), which made famous the slogan “Get Paid to Surf the Web”. Users of AllAdvantage.com needed to download a viewbar that displayed advertisements at the bottom of their screens and would be paid $0.5 per each hour logged. Furthermore, members could also invite friends (without any limit!) and would receive an additional $0.1 for every hour that person was active. In the first quarter of the year 2000 (which coincided with the peak of the bubble) AllAdvantage.com paid a total of $40 million to its members, leading to a loss of $66 million. It also did not survive the market crash and ceased its operations in that same year. Companies that engaged in similar practices include Spedia, Click-Rebates, Jotter Technologies, Radiofreecash and Adsavers.com (Haacke [2004]).

Some companies also spent huge sums in advertisement and promotional campaigns to catch customers’ attention. For instance, Furniture.com - a company that sold furniture online - spent $33.9 million on sales and marketing in 1999, despite generating only $10.9 million in revenues. Boo.com - an internet startup delivering fashion and sports clothing in 18 different countries - spent $42 million on an advertisement campaign alone. Despite its promotional efforts, the company was always far from meeting its targets: it aimed at creating a website that could handle 100 million web visitors at once; however, it reached no more than 300,000 visits in its final two months. Often regarded as one of the biggest

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35 Jack Willoughby, “Burning Up; Warning: Internet companies are running out of cash - fast”, Barron’s, March 20, 2000


37 See Haacke [2004], p.109

38 See Haacke [2004], p.91 and Razi, Siddiqui and Tarn [2004]
dotcom failures, the company burnt $185 million of capital over its 18 months of life (it went out of business in May 2000).  

These business strategies were often justified by a first-mover advantage type of argument - most internet businesses were understood to be natural monopolies, where only one firm could ultimately survive. Hence the search for rapid growth and the “get big fast” or “get large or get lost” mottos. However, it is important to note that such extreme commercial practices were also incited by financial markets and were possible “as long as these start-ups received money from venture capitalist funds because they could not be supported by normal business economics”.  

As already mentioned in section 1, the fact that valuation metrics were often focused on revenue targets or market shares created incentives for rapid growth at the expense of profits (Hong and Stein [2003] and Aghion and Stein [2008]). Indeed, venture capitalists and company executives explicitly admitted their strategies were influenced by financial market sentiment. For instance, Michael Moritz - founder of Sequoia Capital, a venture capital firm that was an initial funder of Yahoo! - admitted in an interview that “The world was rewarding us for raising $250 million and penalizing [us for] raising $25 million. Daring to be great overweighted being cautious”. In a similar vein, eToys’ founder and CEO Toby Lenk admitted that “It was the whole land-grab mentality. Grow, grow, grow. Grab market share and worry about the rest later. When you’re in that cycle, and less capable people are doing I.P.O.’s, it’s like an arms race. If you turn down the gun and put it on the table, all you’re doing is letting other people pick it up and shoot you. I made the decisions and I take full responsibility. But there were a lot of amazing forces at work.” Like many other dotcoms, eToys would not survive the stock market crash in 2000. Toby Lenk recognizes that the attempt to grow too fast and was one of the main reasons behind the failure of eToys: “We had the capacity for $500 million in revenue but came to a stop at $200 million. That’s hard to survive”.

It is therefore interesting to note that as in the British railway mania 150 years before, the Nasdaq boom of the late 1990s was also associated with rising competitive pressures in product markets, and with situations of excessive investment and low (or even negative) profit margins that became unsustainable once market sentiment reversed. As argued by Varian: “the driving force behind the rise and fall of the Nasdaq was simple competition. [...] in 1999 there was no fundamental scarcity of new business models for dot-coms. The result was an intensely competitive environment, where it has been extremely difficult to make money.”

However, even if lacking market expertise and in many cases investing beyond reasonable levels, many of the new companies posed a competitive threat to incumbents. I next review some examples.

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39 Boo.com spent fast and died young but its legacy shaped internet retailing” (05/16/2005), retrieved from https://www.theguardian.com/technology/2005/may/16/media.business

Umar [2004, ch. 13]

40 Haacke [2004], p.109

41 See Haacke [2004], p.108


Sun Microsystems and Microsoft  One significant example in this category is the one involving Sun Microsystems and Microsoft, which is described in Varian [2003]. Back in 1999 when the dotcom bubble was about to reach its peak, Sun Microsystems decided to enter in the office suite market, which was largely dominated by Microsoft Office. It decided to launch a new office suite called StarOffice and to make it available for free. Besides releasing the software at zero price, Sun Microsystems also promised to make its source code, file formats, and protocols free. This move was seen at that time as a clear attack on Microsoft dominant position in the market: “Many in the industry view Sun’s move as a direct assault on Microsoft’s second most lucrative monopoly”. However, Sun would be severely hit by the stock market crash (its stock price plunged from $63.4 in 8/31/2000 to $3.28 in 11/12/2002), which critically compromised the development of StarOffice.

The threats posed by companies such as Sun Microsystems were recognized by Microsoft in its annual reports. For instance, the 2000 report states that “Rapid change, uncertainty due to new and emerging technologies, and fierce competition characterize the software industry, which means that Microsoft’s market position is always at risk. “Open source” software [...] are current examples of the rapid pace of change and intensifying competition. [...] Competing operating systems, platforms, and products may gain popularity with customers, computer manufacturers, and developers, reducing Microsoft’s future revenue.” [Annual Report, 2000, p. 16]

Microsoft also anticipated the necessity to reduce the price of some of its products: “The competitive factors described above may require Microsoft to lower product prices to meet competition, reducing the Company’s net income” [Annual Report, 2000, p. 17]; and to increase its R&D expenditure significantly “It is anticipated that investments in research and development will increase over historical spending levels [...] Significant revenue from these product opportunities may not be achieved for a number of years, if at all.” [Annual Report, 2000, p. 16]

eToys and Toys“R”Us  The retail market for toys experienced considerable action in the late 1990s. Several firms such as eToys, Toymart, Toytime and Red Rocket appeared as online toy retailers, but went bankrupt in the years 2000 and 2001 as stock prices started declining. The case of eToys was particularly impressive: it was established in 1997, had its IPO in 1999 and in the same year reached a market capitalization of 8 billion dollars! This value was 33% larger than that of the market leader Toys“R”Us, a well-known company, much larger in terms of size and profitability (see Table 1).

<table>
<thead>
<tr>
<th></th>
<th>Market Value</th>
<th>Sales</th>
<th>Earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td>eToys</td>
<td>8 billion $</td>
<td>30 million $</td>
<td>-29 million $</td>
</tr>
<tr>
<td>Toys“R”Us</td>
<td>6 billion $</td>
<td>11,200 million $</td>
<td>376 million $</td>
</tr>
</tbody>
</table>

Table 1: Sales and earnings refer to fiscal year 1998, whereas market value refers to 1999

Despite of their short existence, the newly founded companies posed a serious competitive threat to Toys“R”Us, which was forced to enter in the online market. After a series of unsuccessful experiences with its own website (toysrus.com), it then started a 10-year partnership with Amazon.com in the year 2000. According to the agreement Toys“R”Us was to be

Amazon’s exclusive supplier of toys, games and baby products.

The case of eToys is presented by Shiller [2000] as an example of a clear market inefficiency: it reached a market capitalization greater than the purportedly more efficient firm (Toys"R"Us), but went bankrupt immediately after. But even if one agrees that eToys lacked expertise in the toy market and was a relatively inefficient firm, the above conclusion is still unwarranted. It crucially ignores the fact that Toys"R"Us was forced to enter in the online market (and hence to expand) as a strategic response to the entry of eToys and all the other competitors. Seen in this way, the bubble attached to eToys had a positive side effect: it increased competition and forced the market leader to expand.

**GE and the “Destroy Your Business” strategy**  The strategic reaction of Toys"R"Us was common among many large, well-established corporations. One well-known example is the “Destroy Your Business” program launched by GE’s CEO Jack Welch in 1999. Welch asked all GE’s managers to think of possible ways in which Internet start-ups could challenge the market leaderships of their businesses and to adopt effective strategies that could avoid such scenarios. The process was focused on adopting the necessary innovations before a new dotcom company appeared and took advantage of such opportunity. For instance, GE Plastics (a specialized supplier of plastics, established in 1973 as a division of GE), decided to enter the online market in 1997. As part of the “Destroy Your Business” program, GE Plastics e-commerce manager Gerry Podesta and his team decide to equip their website with new tools and functionalities. They got inspiration from car manufacturers’ websites, which were developing configuration tools that allowed costumers to customize their cars. A similar scheme was then introduced in the website GE Plastics, allowing potential costumers (such as engineers from manufacturing plants) to design their products online, indicating different materials that could be used, their characteristics and cost.

We can also mention the examples of several other GE divisions, such as GE Transportation, GE Power Systems, GE Appliances or GE Medical Systems. GE Medical Systems - a manufacturer of diagnostic imaging systems such as CAT scanners and mammography equipment - launched an platform called iCenter as part of the “Destroy Your Business” initiative. This was an online system designed to monitor GE customers’ equipment, collect data and provide each customer with information on his relative performance and suggestions on how to improve it. GE Appliances also started using the internet to sell its products. Appliances were traditionally sold through retail stores, but GE feared that such model could be challenged with the emergence of new internet retailers (which could give preference to appliances from alternative brands). It then developed a point-of-sale system placed in traditional retail stores where customers could make online orders. Customers could also schedule an appointment to have the items delivered and installed at their convenience. This way, consumers would benefit from the advice of retailers while the goods would be sent directly into their hands (allowing stores to have reduced inventories). In 2000 GE Appliances reported 45% of its sales took place on the internet.\(^45\)

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The “Destroy Your Business” program adopted by GE provides a nice illustration of the mechanisms at work in the model developed in this paper. As hundreds of internet startups emerged and raised vast amounts of money in the stock market, Welch feared that some of GE’s businesses could be challenged if the company did not enter in the online market. This example may be interpreted through the lens of the latent bubble process described in section 3: Welch anticipated that if GE did not move, some other companies could easily raise funds and shake their dominant position in specific markets.

7 Conclusion

Financial history shows that stock market boom/bust episodes are often an industry phenomenon which can be accompanied by significant changes in the market structure. Motivated by this observation, this paper developed a framework to think about the interactions between asset bubbles and product market competition. At the heart of the model is the idea that asset bubbles may sometimes reduce barriers to entry and force firms to expand, to the ultimate benefit of consumers. An interesting aspect of the theory is that asset bubbles may force (productive) market leaders to expand even when they are attached to potential (unproductive) competitors. Indeed, if bubbles can only appear attached to the market leaders they will likely have no effect on the market structure and on economic activity. This conclusion suggests that the economic consequences of asset bubbles will crucially depend on its distribution across firm types. It also helps us think about different questions. For instance, how will a large company react to a bubble on its stock prices? Will Apple lower the price of its iPhones if investors suddenly become excited about the company and its market value doubles? This paper suggests that it will probably not. Instead, Apple will more likely expand and cut its profit margins in the presence of a generalized boom in which potential competitors (perhaps smaller and less innovative) can also get overvalued! In such case, as barriers to entry decrease, Apple may be forced to expand in order to preserve its market share. Although subject to each reader’s own assessment, I believe this view is not totally unreasonable.

The model developed in this paper also gives us a novel perspective on famous stock market overvaluation episodes. For instance, it may explain why British railway companies duplicated some of their own lines during the 1840s mania or why large corporations (such as GE) had incentives to quickly adapt their businesses to the internet in the late 1990s. Furthermore, it provides a simple rationale for the low and negative profitability levels reported by internet firms at the peak of the dotcom bubble. Rather than the mere realization of a negative technology shock (as argued by Pastor and Veronesi [2009]) this paper suggests that such income losses may have been a rational reaction to an environment characterized by high stock prices. This view seems indeed to receive support from the anecdotal evidence reviewed in section 6.

I conclude by pointing some avenues for future research. The first one is about the empirical relationship between stock market overvaluation and competition. The evidence reviewed in section 6 suggests that two important bubble episodes were associated with an environment of rising competition. But is there a systematic relationship between stock market
overvaluation and measures of product market competition (such as markups or profits)? This is an empirical question, which is left for future work. The model built in this paper suggests that such an empirical analysis will necessarily be subject to important caveats, such as the possibility of latent bubbles or the fact that the distribution of overvaluation across incumbents and followers may change over time. Furthermore, overvaluation is unlikely to be independent of market conditions, which may give rise to several confounding elements. For instance, bubbles may be more likely to appear in times of increased consumer demand or in periods when firms can charge higher markups. This may originate a positive association between overvaluation and profits/markups that does not necessarily invalidate the predictions of the model.

A second issue pertains to the relationship between bubbles and moral hazard. A central tenet of this model is that despite being attached to unproductive firms, asset bubbles can nevertheless improve the workings of good markets and be welfare-improving. One may however argue that bubbles can have a contrary effect: overvaluation can subsidize bad projects or firms, which may impair the workings of both product and financial markets. For instance, in the dotcom bubble of the late 1990s we can find many examples of inexperienced firms offering poor services to consumers (such as online retailers failing to make on time deliveries) or even situations of fraud (such as the manipulation of income statements).\(^{46}\) May asset bubbles exacerbate moral hazard problems and have a negative impact on consumers’ or investors’ welfare? I believe these are interesting issues that should be explored in future theoretical work.

Finally, by making a connection between the degree of competition in product markets and the interest rate, this paper may also shed light on recent US macroeconomic trends. The last four decades of US history have been characterized by both a steady decline in real interest rates and an increase in market power, evident from an increase in markups (De Loecker and Eeckhout [2017]) and measures of industry concentration (Autor et al. [2017]). Although there may be different forces contributing to the interest decline, this model suggests that it can be connected to the increase of market power. I believe that a serious assessment of this hypothesis is an important avenue for future research.

\(^{46}\)For instance, the telecommunications company Worldcom used fraudulent accounting techniques to artificially increase its earnings during the dotcom bubble. Examples of fraud could also be found in the South Sea Bubble (Garber [1989]).
References


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A The Model with Fixed Costs

A.1 Industry Equilibrium

Assume that demographics, preferences and the production structure are as described in the baseline model of section 2. There are however two differences

1. All entrepreneurs have the same productivity level $\pi_i = 1 \quad \forall i, j$

2. Production of intermediate input varieties entails a fixed investment cost $c_f$ (in units of the final good).

Under this modified framework there are no technological differences among types. However, the presence of fixed production costs introduces increasing returns to scale and opens the door for imperfect competition. As before, a monopolist would like to charge

$$p_{i,t+1} = \frac{1}{\rho} \cdot R_{t+1}$$

However, if $c_f$ is sufficiently low, some followers could profitably enter at this price (the conditions on $c_f$ will be determined below). In such a case, the leader must set a limit price so that no individual follower can make a profit upon entry. How is such a limit price determined? Let $x$ be the quantity chosen by the operating firm (time subscripts are omitted for simplicity). If a competitor decides to enter, he will chose $\tilde{x}$ in order to maximize

$$\max_{\tilde{x}} \left[ \alpha \cdot K^{\alpha - \rho} (x + \tilde{x})^{\rho - 1} - R \right] \cdot \tilde{x} - R \cdot c_f$$

which implies a best response function $\tilde{x}^* = f(x)$ defined by

$$\alpha \cdot (\rho - 1) \cdot K^{\alpha - \rho} \cdot (x + \tilde{x})^{\rho - 2} \cdot \tilde{x} + \alpha \cdot K^{\alpha - \rho} (x + \tilde{x})^{\rho - 1} - R = 0 \quad (41)$$

The leader must produce a quantity such that no follower can enter profitably.

$$\left\{ \alpha \cdot K^{\alpha - \rho} [x + f(x)]^{\rho - 1} - R \right\} \cdot f(x) - R \cdot c_f = 0 \quad (42)$$

This equation says that, given the output $x$ produced by the leader , if a follower were to enter, he could not make a positive profit.

I will impose parameter restrictions so that $x^M < x$ and the equilibrium quantity is given by the solution of equation (42):

$$x = g(R)$$

Figure 15 shows the combinations of $(\rho, c_f)$ that are associated with limit pricing for a given interest rate $R$, capital stock $K$ and capital share $\alpha$. The limit pricing region corresponds to low values of $\rho$. 

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A.2 General Equilibrium

The equilibrium of this economy can be described by the following three equations

\[
\alpha \cdot (\rho - 1) \cdot x_{t+1}^{\alpha - \rho} \cdot (x_{t+1} + \tilde{x}_{t+1})^{\rho - 2} \cdot \tilde{x}_{t+1} + \alpha \cdot x_{t+1}^{\alpha - \rho} \cdot (x_{t+1} + \tilde{x}_{t+1})^{\rho - 1} - R_{t+1} = 0
\]

\[
\left[\alpha \cdot x_{t+1}^{\alpha - \rho} \cdot (x_{t+1} + \tilde{x}_{t+1})^{\rho - 1} - R_{t+1}\right] \cdot \tilde{x}_{t+1} = R_{t+1} \cdot c_f
\]

\[
x_{t+1} = (1 - \alpha) \cdot x_t^\alpha - c_f
\]

The last equations defines the law of motion of this economy. It fully determines the evolution of the capital stock. Due to the presence of a fixed cost, this economy exhibits two steady-states. Only the second one is stable, so the first one will be disregarded.\(^47\) The first two equations can be used to determine the interest rate \(R_{t+1}\).

Under what conditions is the steady-state interest rate lower than one? Figure 16 shows the steady-state interest rate for different combinations of \((\alpha, c_f)\).\(^48\) When \(c_f\) is sufficiently high (i.e. it lies above the middle curve \(R = 1\)), the steady-state interest rate is below one. Note that if \(c_f = 0\), the condition is \(\alpha < 0.5\) as in the benchmark model.

---

\(^47\)The stable steady-state is characterized by \(\alpha (1 - \alpha) x_{ss}^{\alpha - 1} < 1\).

\(^48\)Only the stable steady-state is considered.
Under what conditions is steady-state investment dynamically efficient? Steady-state investment is efficient if

\[
\alpha \cdot x^{\alpha - 1} > 1
\]

i.e. if the marginal product of capital is above its marginal cost of production. Note that we can use equation (45) to implicitly define the steady-state capital stock as

\[
(1 - \alpha) \cdot x^{\alpha - 1} = 1 + \frac{c_f}{x}
\]

A sufficient (though not necessary) condition for steady-state investment to be efficient is therefore \(\alpha > 0.5\). In this case

\[
\alpha \cdot x^{\alpha - 1} > (1 - \alpha) \cdot x^{\alpha - 1} = 1 + \frac{c_f}{x} > 1
\]

As shown in Figure 16, it is possible to find values of \(c_f\) such that \(R < 1\) even when \(\alpha > 0.5\). Indeed, if the interest rate is sufficiently low, storage can be used in equilibrium; in such case, the economy exhibits underinvestment.

### A.3 Bubbly Equilibrium

Suppose that entrepreneur can get a firm with size \(b \geq 0\). It is easy to see that in this case a bubble has the effect of a reduction in the entry cost. Therefore, it makes entry easier and reduces monopoly profits. As before, equation (41) still
holds but the no entry condition must be modified to

\[
\left[ \alpha \cdot K^{\alpha - \rho} \left( x + \hat{x} \right)^{\rho - 1} - R \right] \cdot \hat{x} - R \cdot c_f + b = 0
\]  

(46)

Equations (41) and (46) describe the industry equilibrium as a function of \( R \). Naturally, bubbles are expansionary as long as \( b \leq R \cdot c_f \). Figure 17 shows the equilibrium output and operating profits (of the only producer) as a function of \( b \). As one can see, once it gets too large the bubble may sustain negative operating profits in equilibrium: to deter the entrance of potential competitors, the leader needs to expand his output so much that he ends up having an operating loss. However, such loss is “financed” by the bubble he gets and is optimal: if his output were lower, a competitor would enter and would reduce his profits even further.

\[
\Theta (x) = \frac{R}{\pi} - \alpha \cdot x^\rho K^{\alpha - \rho}
\]

\( ^{49} \) Operating profits are revenues \( p \cdot x \) minus total cost \( R \cdot (x + c) \).
C is increasing in $b$. Moreover, we have that

$$\Theta'(x) > 0 \iff \frac{R}{\pi} - \alpha \cdot x^{\rho-1} K^{\alpha-\rho} > 0 \iff \alpha \cdot x^{\rho-1} K^{\alpha-\rho} < \frac{R}{\pi} \iff \pi \cdot x^{\rho-1} K^{\alpha-\rho} < R \iff x > \left( \frac{\pi \cdot x^{\rho-1} K^{\alpha-\rho}}{R} \right)^{\frac{1}{\rho}}$$

Finally, note that

$$x |_{b=0} = \left( \frac{\pi \cdot x^{\rho-1} K^{\alpha-\rho}}{R} \right)^{\frac{1}{\rho}} > \left( \frac{\rho \cdot x^{\rho-1} K^{\alpha-\rho}}{R} \right)^{\frac{1}{\rho}}$$

Therefore, $\Theta$ is increasing in $x$ when $b = 0$. Together with the fact that $\Theta$ is increasing in $b$, this implies that $x$ is monotonically increasing in $b$.

C Constant Bubbles under Financial Frictions

C.1 Industry Equilibrium

If the leader lets the followers produce, he must solve the following problem

$$\max_{x^L} (p - R) \cdot x^L + b^L \quad \text{s.t.} \quad R \cdot \frac{x^F}{\pi} = \phi \cdot x^F + b^F$$

$$p = \alpha \cdot K^{\alpha-\rho} (x^L + x^F)^{\rho-1}$$

We can combine the two constraints to write

$$F(x^L, x^F) : x^F \left[ \frac{R}{\pi} - \phi \cdot x^{\alpha-\rho} (x^L + x^F)^{\rho-1} \right] - b^F = 0$$

This equation defines implicitly $x^F$ as a function of $x^L$: $x^F = f(x^L)$. The leader will solve the unconstrained maximization problem

$$\max_{x^L} \left\{ \alpha \cdot K^{\alpha-\rho} [x^L + f(x^L)]^{\rho-1} - R \right\} \cdot x^L + b^L$$

The first order condition requires that

$$\alpha \cdot K^{\alpha-\rho} [x^L + f(x^L)]^{\rho-1} - R + x^L \left\{ \alpha \cdot K^{\alpha-\rho} (\rho - 1) [x^L + f(x^L)]^{\rho-2} \left[ 1 + f'(x^L) \right] \right\} = 0$$

This equation implicitly defines $x^L$ as a function of $b^F$ through $f(x^L)$. Note that

$$f'(x^L) = -\frac{\partial F/\partial x^L}{\partial F/\partial x^F}$$

$$= -\frac{-x^F (\rho - 1) \phi \cdot x^{\alpha-\rho} (x^L + x^F)^{\rho-2}}{\frac{R}{\pi} - \phi \cdot x^{\alpha-\rho} (x^L + x^F)^{\rho-1} - x^F (\rho - 1) \phi \cdot x^{\alpha-\rho} (x^L + x^F)^{\rho-2}}$$

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If this condition is not satisfied, we have followers out of the market. For this to be the case, have an incentive to deviate and accommodate the entry of the followers. Let will be 

If the followers do not produce, then we have 

C.2.2 Equilibrium with no accommodation of the followers

As long as 

C.2.1 Equilibrium with accommodation of the followers

For this to be the case, no individual leader can have a profitable deviation from this equilibrium. This means that, 

Let 

\[ x^L = \frac{x^F \cdot (\rho - 1) \cdot \alpha \cdot K^{\alpha - \rho} \cdot (x^L + x^F)^{\rho - 2}}{\frac{R}{\pi} - \phi \cdot \alpha \cdot K^{\alpha - \rho} \cdot (x^L + x^F)^{\rho - 1} \left[ 1 + x^F \cdot (\rho - 1) \cdot (x^L + x^F)^{-1} \right]} \]

Let \( x^{L^*} \) denote the solution to this problem. If the followers produce, the leader’s profits will be equal to \( \left\{ \alpha \cdot K^{\alpha - \rho} \cdot [x^{L^*} + f(x^{L^*})]^{\rho - 1} \right\} \). If he instead decides to produce \( \bar{x} = \left( \frac{\alpha \pi}{\rho} \right)^{\frac{1}{\alpha - \rho}} \cdot K^{\frac{\alpha - \rho}{\alpha - 1}} \), the followers will be out of the market and the leader’s profits will be \( \left[ \alpha \cdot K^{\alpha - \rho} \cdot (\bar{x})^{\rho - 1} - R \right] \cdot \bar{x} \). Therefore, the leader will accommodate the entry of the followers entrepreneurs only if

\[ \left\{ \alpha \cdot K^{\alpha - \rho} \cdot [x^{L^*} + f(x^{L^*})]^{\rho - 1} - R \right\} \cdot x^{L^*} > \left[ \alpha \cdot K^{\alpha - \rho} \cdot (\bar{x})^{\rho - 1} - R \right] \cdot \bar{x} \]

C.2 General Equilibrium

In general equilibrium, we have \( x^L + x^F = K \).

C.2.1 Equilibrium with accommodation of the followers

As long as \( b^F \) is sufficiently small, the leaders prefer to accommodate the entry of the followers. In this case, \( x^{L^*} \) can be found by plugging \( K^* = x^{L^*} + f(x^{L^*}) \) in the system of equations above. In this case we have

\[ f(x^L) \left\{ \frac{R}{\pi} - \phi \cdot \alpha \cdot [x^L + f(x^L)]^{\alpha - 1} \right\} - b^F = 0 \]

\[ \alpha \cdot [x^L + f(x^L)]^{\alpha - 1} - R + x^L \left\{ \alpha \cdot (\rho - 1) \cdot [x^L + f(x^L)]^{\alpha - 2} \left[ 1 + f'(x^L) \right] \right\} = 0 \]

\[ f'(x^L) = \frac{x^F \cdot (\rho - 1) \cdot \phi \cdot \alpha \cdot (x^L + x^F)^{\alpha - 2}}{\frac{R}{\pi} - \phi \cdot \alpha \cdot (x^L + x^F)^{\alpha - 1} \left[ 1 + x^F \cdot (\rho - 1) \cdot (x^L + x^F)^{-1} \right]} \]

For this to be the case, no individual leader can have a profitable deviation from this equilibrium. This means that, treating \( K^* \) as given, the leaders cannot prefer to produce \( \bar{x} = \left( \frac{\alpha \pi}{\rho} \right)^{\frac{1}{\alpha - \rho}} \cdot (K^*)^{\frac{\alpha - \rho}{\alpha - 1}} \), for in that case they would drive the followers out of the market. For this to be the case,

\[ \left\{ \alpha \cdot (K^*)^{\alpha - \rho} \cdot [x^{L^*} + f(x^{L^*})]^{\rho - 1} - R \right\} \cdot x^{L^*} > \left[ \alpha \cdot (K^*)^{\alpha - \rho} \cdot (\bar{x})^{\rho - 1} - R \right] \cdot \bar{x} \]

If this condition is not satisfied, we have \( \bar{x} = \left( \frac{\alpha \pi}{\rho} \right)^{\frac{1}{\alpha - \rho}} \cdot K^{\frac{\alpha - \rho}{\alpha - 1}} \) and \( K = x^L = x \) implying that \( x^L = K = \left( \frac{\alpha \pi}{\rho} \right)^{\frac{1}{\alpha - \rho}} \).

C.2.2 Equilibrium with no accommodation of the followers

If the followers do not produce, then we have \( x^L = K = \left( \frac{\alpha \pi}{\rho} \right)^{\frac{1}{\alpha - \rho}} \). For this to be an equilibrium, each leader cannot have an incentive to deviate and accommodate the entry of the followers. Let \( \tilde{x}^L \) be the solution to the constrained
maximization problem in A.1 when \( K = \left( \frac{\alpha \pi}{R} \right)^{\frac{1}{1-\rho}} \). For \( \bar{K} = \left( \frac{\alpha \pi}{R} \right)^{\frac{1}{1-\rho}} \) to be an equilibrium,

\[
\alpha \cdot (\bar{K})^{\alpha-\rho} \left( \hat{x}^L + f(\hat{x}^L) \right)^{\rho-1} - R \leq \alpha \cdot (\bar{K})^{\alpha-1} - R \cdot \bar{K}
\]

C.2.3 Mixed Equilibrium

If a symmetric equilibrium is not possible, then there will be a mixed equilibrium: some industries will accommodate the entry of the followers whereas other will not. In this case, we must observe the following indifference condition

\[
\left\{ \alpha \cdot K^{\alpha-\rho} \left( \hat{x}^L + f(\hat{x}^L) \right)^{\rho-1} - R \right\} \cdot \hat{x}^L = R \left[ \frac{1}{\pi} - 1 \right] \cdot \left( \frac{\alpha \cdot \pi}{R} \right)^{\frac{1}{1-\rho}} K^{\frac{\alpha-\rho}{1-\rho}}
\]

where \( x^L \) is the output of the leader in industries in which there is accommodation and \( \left( \frac{\alpha \pi}{R} \right)^{\frac{1}{1-\rho}} K^{\frac{\alpha-\rho}{1-\rho}} \) is the output in industries in which there is not. \( x^L \) and \( f(x^L) \) are determined by the two equations in A.2.1.

Finally, denote by \( \mu \) the fraction of industries in which there is accommodation. \( K \) must satisfy

\[
K = \left\{ \mu \cdot \left[ x^L + f(x^L) \right]^\rho + (1 - \mu) \cdot \left[ \left( \frac{\alpha \cdot \pi}{R} \right)^{\frac{1}{1-\rho}} K^{\frac{\alpha-\rho}{1-\rho}} \right]^\rho \right\}^{\frac{1}{\rho}}
\]

We therefore have a system of four equations in four unknowns that needs to be solved numerically.