Structural Change and Aggregate Employment

Fluctuations in China and the US

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Abstract

One salient feature of business cycles in developed countries is that the aggregate employment is highly procyclical. In China, however, the correlation of the cyclical components of aggregate employment and output is close to zero. In this paper, we document three new stylized facts: (1) the business cycle properties of employment at sector level (agriculture and non-agriculture) in China are very similar to those in the US; (2) employments in the agricultural and non-agricultural sectors are negatively correlated in both China and the US; and (3) for both economies, the agriculture’s share of employment is negatively correlated with the real GDP per work in both sectors. These facts suggest that difference in sector composition could be an important reason for the difference in aggregate employment fluctuations between the two economies. We then construct a simple two-sector growth model with productivity shocks and non-homothetic preferences and show that the model can simultaneously account for the secular trend in labor reallocation away from agriculture and employment fluctuations at sector level and in the aggregate for both China and the US.

JEL Classification: E24, E32, O41

Keywords: Structural Change, Non-homothetic Preferences, Labor Reallocation, Aggregate Fluctuations

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1 Introduction

One salient feature of business cycles in developed countries is that the aggregate employment is strongly procyclical. This is not the case in China. The correlation of the cyclical components of aggregate employment and output is close to zero. Relative to output, the volatility of aggregate employment is also very low. These puzzling facts about aggregate employment fluctuations in China are present even after we carefully correct for some well-known measurement problems in the official employment series. Some take this as a sign that there are unique institutional constraints that limit the employment variability in China. While it is true that there may be strong employment rigidity in the state-owned enterprises, the labour market for the non-state sector in China is quite flexible – maybe even more flexible than many developed economies due to minimum regulations on hiring and firing by non-state firms. Since the non-state sector employment is usually the margin at which the aggregate employment adjusts over the business cycles, the institutional constraints on state-sector employment cannot explain the puzzle

In this paper, we argue that the key to understanding aggregate employment fluctuations in China is its economic structure. We document three new stylized facts for the period from 1978 to 2010. First, the business cycle properties of employment at sector level (agriculture and non-agriculture) in China are very similar to those in the US. In particular, the correlation of the cyclical components of non-agricultural employment and non-agricultural GDP are close to 90 percent in both countries. Second, employments in the agricultural and non-agricultural sectors are negatively correlated in both China and the US. And third, for both countries, the agriculture’s share of employment is negatively correlated with the real GDP per worker in both sectors. These similarities between China and the US at sector level suggest that the key difference between the two economies is the size of the agricultural sector. Between 1978 and 2010, the agriculture’s share of total employment averaged around 50% in China, but was always less than 3% in the US. Therefore, the labour reallocation between the two sectors could have an important dampening effect on aggregate employment fluctuations in China, but negligible effect in the US. To investigate this possibility, we construct a two-sector growth
model with productivity shocks and non-homothetic preferences, and calibrate it so that the model can account for the secular trend in labour reallocation from agriculture to non-agriculture in both China and the US. We then examine the model’s implications for the labour market dynamics at the business cycle frequency. We find that our calibrated model can indeed account for the employment fluctuations at sector level and in the aggregate for both China and the US. In particular, our model implies low employment-output correlation for China and, at the same time, high employment-output correlation for the US.

The model we use in the paper is a standard two-sector growth model, but with non-homothetic preferences proposed by Comin, Lashkari and Mestieri (2015) to account for the third stylized fact we discussed above: For both China and the US, the agriculture’s share of employment is negatively correlated with real income per worker in both agricultural and non-agricultural sectors. This fact suggests the importance of income effect in determining labour reallocation between sectors – even for a high income country like the US.

Our paper is related to two strands of literature. There is a rapidly growing literature on structural change. See e.g., Caselli and Coleman (2001), Kongsamult, Rebelo and Xie (2001), Ngai and Pissarides (2007), Acemoglu and Guerrrieri (2008), and Herrendorf, Rogerson and Valentinyi (2014) for an excellent survey. Most of the studies in this literature focus on understanding the sources of structural change, our paper builds on this literature and studies the business cycle implications of structural change. In particular, both Boppart (2014) and Comin, Lashkari and Mestieri (2015) emphasize the importance of income effect in understanding the secular trend of labour reallocation from agriculture to manufacturing and services. We show in this paper that income effect is also important for understanding aggregate employment fluctuations at the business cycle frequency. Our paper is also related to the literature on business cycles in China. Brandt and Zhu (2000) is one of the first papers studying business cycles in China during the reform period. Their focus, however, is on understanding the relationship between GDP growth and inflation over the business cycles in the 1980s and early 1990s. Chang et al. (2016) is a more recent study of business cycles in China, and their focus is on understanding the weak correlation between investment and consumption in China.
since the late 1990s. Neither of these studies examine the relationship between aggregate employment and output. He, Chong and Shi (2009) carry out an exercise of business cycle accounting for China in the spirit of Chari, Kehoe and McGrattan (2007). They find that most of the fluctuations in aggregate employment are accounted for by variations in a labour wedge, highlighting the inability of standard one-sector business cycle models in accounting for the employment fluctuations in China. Our paper shows that a standard two-sector model with non-homothetic preferences can account for the aggregate employment fluctuations without introducing a time-varying labour wedge.

There are two studies that are closely related to our paper. Da-Rocha and Restuccia (2006) is the first paper that documents the low correlation between aggregate employment and output in countries with a large agriculture sector. They use a two-sector real business cycle model to examine the role of labour reallocation in accounting for the cyclical behaviour of aggregate employment. To focus on the cyclical fluctuations, they assume that each country is fluctuating around a steady state with a constant employment share of agriculture. Since structural change - the secular decline of the agriculture’s share of employment - is a very prominent phenomenon in China during the period we study, we think it is important to have a unified model that can account for both the secular trend of structural change and the aggregate employment fluctuations around the trend. In an independent study, Storeslettern, Tan, Zhao and Zilibotti (2017) also use a two-sector model to account for both the structural change and aggregate employment fluctuations in China. Their model, however, is very different from ours. They emphasize capital deepening rather than income effect as a driving force for structural change and labour reallocation between the agricultural and non-agricultural sectors. We think their study and ours are complementary.

Before presenting our model, we first discuss in detail the data and facts about employment fluctuations in China and the US.
2 Data and Facts

For the US, we directly use the annual sector-level data on real GDP and employment from the Groningen Growth and Development Centre’s 10-Sector Database (Timmer, de Vries and de Vries (2015)), and aggregate the nine sectors outside agriculture into one non-agricultural sector. For China, the 10-Sector Database uses the official employment series from China’s National Bureau of Statistics (NBS) that are published in the annual China Statistical Yearbook. However, as pointed out by Brandt and Zhu (2010), there are two serious problems with the NBS’ employment series that need to be dealt with. We discuss next how we deal with these problems and construct revised annual employment series for China.

First, there is a discrete upward jump in total employment in 1990. This jump is due to a change in the official definition of employment after 1990 census that broadened the coverage of the series. The NBS publishes the employment data using the new definition for the years since 1990, but still reports the employment data using the old definition for the years prior to 1990. Brandt and Zhu (2010) use the 1982 census data to adjust the employment data for the years before 1990 so that the entire employment series has a consistent coverage. The official and the revised employment series are plotted in the left panel of Figure 1. The second problem of the NBS employment series is the overestimation of agricultural employment. Brandt and Zhu (2010) find that the official agricultural employment series can be closely approximated by the Total Rural Employment minus the Employment of the Township and Village Enterprises (TVEs). This series clearly overestimates agricultural employment because non-agricultural workers in rural private enterprises and rural individual enterprises (those that employ less than eight employees) are counted as agricultural workers. To better account for employment in agriculture, we follow Brandt and Zhu (2010) and construct agricultural employment series as the total rural employment minus rural employments in TVEs, private enterprises and individual enterprises. The official and the revised agricultural employment series are plotted in the right panel of Figure 1. Note that this revised agricultural employment series still has the same problem as the official total employment series for the years prior to 1990. To generate a consistent agricultural employment
series for the entire period, for each year we first use the revised agricultural employment and the official total employment to calculate the share of employment in agriculture; we then calculate the final revised agricultural employment as the product of the share and the revised total employment; and finally we calculate the revised non-agricultural employment as the difference between the revised total employment and the revised agricultural employment. Figure 2 plots the revised agricultural and non-agricultural employments and the agriculture’s share of total employment using the revised data series.

2.1 Employment Fluctuations at the Aggregate Level

Given the revised employment data for China, we now examine the business cycle properties of aggregate employment in China and compare them to those in the US. Our sample runs from 1978 to 2010 for both countries. We present the business cycle properties by calculating several statistics from the hp-filtered time series. Table 1 reports the relative standard deviation of aggregate employment to aggregate output and the correlation of aggregate employment with aggregate output in both China and the US. All variables are normalized by the size of population.

From Table 1, we observe two interesting stylized facts of aggregate employment fluctuations in China:
1. The magnitude of fluctuations in aggregate employment is much lower than that of aggregate output in China. This is in stark contrast with the well known fact for the US economy, see Cooley and Prescott (1995), where aggregate employment fluctuates almost as much as the aggregate output.

2. Aggregate employment is acyclical in China: data shows correlation of employment with output close to zero. This is also very different from the established business cycle fact that the US employment is strongly pro-cyclical.

Figure 3 plots the cyclical movements of aggregate employment and aggregate output for the two countries which confirms our observations.

Table 1: Aggregate Moments

<table>
<thead>
<tr>
<th></th>
<th>China</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma (L) / \sigma (Y)$</td>
<td>0.11</td>
<td>0.70</td>
</tr>
<tr>
<td>$\rho (L,Y)$</td>
<td>0.09</td>
<td>0.87</td>
</tr>
</tbody>
</table>
### 2.2 Employment Fluctuations at Sector Level

In order to further understand the aggregate facts for China, we now look at the business cycle moments at sector level in China and compare them to the moments in the US. We will show that China and the US are actually very similar at sector level. The data is divided into agricultural and non-agricultural sectors and hp-filtered. Panel (A) and (B) in Table 2 show the employment fluctuations in the two sectors, where subscript $a$ stands for agriculture and $na$ stands for non-agriculture. Panel (C) documents the labor reallocation between sectors by calculating the correlation of employments in the two sectors and the correlations between the agriculture’s share of employment and sector-level labour productivities $A_i$, which is measured as the real GDP per worker in sector $i$, for $i = a, na$. Three new stylized facts emerge from the moments presented in Table 2:

1. Employment fluctuations in each sector are very similar between China and the US. For example, in the non-agricultural sector, the relative magnitude of employment fluctuation is comparable to that of the US. Moreover, non-agriculture employment in China is highly pro-cyclical. In the agricultural sector, the employment in China also has non-trivial volatility while the correlations between employment and output is low in both China and the US.

2. The correlation between employment in agricultural sector and non-agricultural
sector is negative in both China and the US, as shown in the second column in Table 2 and Figure 4. This negative correlation suggests a potential important role of labour reallocation between sectors in dampening aggregate employment fluctuations. Of course, the degree to which fluctuations of aggregate employment are dampened depends on the relative size of the agricultural sector. Between 1978 and 2010, the agriculture’s share of total employment averaged around 50% in China, but was always less than 3% in the US. Therefore, the labour reallocation between the two sectors could have an important dampening effect on aggregate employment fluctuations in China, but negligible effect in the US.

3. For both China and the US, the agriculture’s share of total employment is negatively correlated with labour productivities in both agricultural and non-agricultural sectors. This observation suggests that income effect, that is, the agricultural good has lower income elasticity than the non-agricultural good, is an important factor for labour reallocation between sectors. Comin, Lashkari and Mestieri (2015) emphasize the importance of income effect in understanding the secular trend of labour reallocation from agriculture to manufacturing and services. Our fact suggests that income effect is also important for labour reallocation at the business cycle frequency.
Table 2: Sector Moments

<table>
<thead>
<tr>
<th></th>
<th>China</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) Non-Agriculture Sector</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma(L_{na}) / \sigma(Y_{na})$</td>
<td>0.75</td>
<td>0.71</td>
</tr>
<tr>
<td>$\rho(L_{na}, Y_{na})$</td>
<td>0.88</td>
<td>0.87</td>
</tr>
<tr>
<td>(B) Agriculture Sector</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma(L_a) / \sigma(Y_a)$</td>
<td>0.70</td>
<td>0.33</td>
</tr>
<tr>
<td>$\rho(L_a, Y_a)$</td>
<td>0.24</td>
<td>-0.05</td>
</tr>
<tr>
<td>(C) Cross Sector</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho(L_a, L_{na})$</td>
<td>-0.83</td>
<td>-0.23</td>
</tr>
<tr>
<td>$\rho(L_a, A_a)$</td>
<td>-0.44</td>
<td>-0.33</td>
</tr>
<tr>
<td>$\rho(L_a, A_{na})$</td>
<td>-0.35</td>
<td>-0.42</td>
</tr>
</tbody>
</table>

Motivated by these new stylized facts, we now present our two-sector model with non-homothetic preferences that we will use to quantitatively account for labour market dynamics in both the long-run and short-run.

3 The Model

There are two sectors indexed by $i = a$ and $na$, representing agriculture and non-agriculture, respectively. Each sector produces a consumption good with a linear technology using labour as the only input:

$$Y_{it} = A_{it} N_{it}, \ i = a, na,$$

where $Y_{it}$, $A_{it}$ and $N_{it}$ are the output, labour productivity and employment in sector $i$, respectively. There is a stand-in representative household who has preferences over a composite consumption good $C_t$ and working time $L_t$. The preferences are represented by a standard utility function in the business cycle literature, the GHH utility function that was first introduced by Greenwood, Hercowitz and Huffman.
Here $\sigma > 0$ is the inverse of the Frisch labour supply elasticity and $B_t$ is a time-varying labour supply parameter that is used to capture the demographic factors (e.g., age structure and gender composition of the labour force) that affect average household’s labour supply decisions. Following Comin, Lashkari and Mestieri (2015), the composite consumption $C_t$ is defined implicitly by the following equation:

\[
(\varphi_a)^{1/\varepsilon} C_t^{\mu_a - 1} c_{at}^{\varepsilon - 1} + (\varphi_{na})^{1/\varepsilon} C_t^{\mu_{na} - 1} c_{nat}^{\varepsilon - 1} = 1,
\]

where $\varphi_a$, $\varphi_{na}$, $\mu_a$, $\mu_{na}$ and $\varepsilon$ are all positive constants. The parameter $\varphi_i$ represents the household’s preference weight on consumption good in sector $i$ ($\varphi_a + \varphi_{na} = 1$), $\mu_i$ measures the income elasticity of consumption good $i$ and $\varepsilon$ is the elasticity of substitution between the two consumption goods. The implicit utility function is a generalization of the standard CES utility function by allowing for potentially different income elasticities for the two consumption goods. If $\mu_a = \mu_{na} = 2 - \varepsilon$, then the utility function is reduced to the standard CES utility function. If $\mu_a < \mu_{na}$, the income elasticity is smaller for the agricultural good than for the non-agricultural good. As a result, relative demand for the agricultural good declines with income. Note that for the implicit utility function for $C_t$ to be monotonic and concave, we need both $\mu_a$ and $\mu_{na}$ to be less than one if $\varepsilon > 1$ and greater than one if $\varepsilon < 1$. (See the footnote 12 of Comin, Lashkari and Mestieri (2015)).

### 3.1 Social Planner’s Problem

Since we assume that there is no friction nor externality in the economy, the competitive allocation is the same as the social optimal allocation, which is the solution to the following social planner’s problem:

\[
\max_{c_{at}, c_{nat}, L_{at}, L_{nat}, C_t} \left\{ N_t \left[ C_t - \frac{B_t}{1 + \sigma} L_t^{1+\sigma} \right] \right\}
\]

subject to

\[
(\varphi_a)^{1/\varepsilon} C_t^{\mu_a - 1} c_{at}^{\varepsilon - 1} + (\varphi_{na})^{1/\varepsilon} C_t^{\mu_{na} - 1} c_{nat}^{\varepsilon - 1} = 1,
\]
Here, $N_i$ is the population size and $L_{it} = N_{it}/N_i$ is the ratio of employment in sector $i$ to total population ($i \in \{a, na\}$). In the Appendix A, we show that the optimal consumption, $c_{at}$ and $c_{nat}$, and working time $L_t$ satisfy the following equations:

$$c_{at} = A_{at} L_{at},$$  \hspace{1cm} (2) \\
$$c_{nat} = A_{nat} L_{nat},$$  \hspace{1cm} (3) \\
$$L_{at} + L_{nat} = L_t.$$  \hspace{1cm} (4) \\

$$L_t = \left[ \frac{(1-\epsilon) \left( \frac{e}{\epsilon-1} \left( \frac{\mu_a^{\mu_a-1}}{A_{at}^{\mu_a-1}} + \frac{\mu_{na}^{\mu_{na}-1}}{A_{nat}^{\mu_{na}-1}} \right) \right) - \frac{1}{\epsilon}}{B_t \left( (\mu_a - 1) \frac{e}{\epsilon-1} \left( \frac{\mu_a^{\mu_a-2}}{A_{at}^{\mu_a-2}} + (\mu_{na} - 1) \frac{e}{\epsilon-1} \left( \frac{\mu_{na}^{\mu_{na}-2}}{A_{nat}^{\mu_{na}-2}} \right) \right) \right)} \right]^{\frac{1}{\epsilon}}.$$  \hspace{1cm} (7) \\

\subsection*{3.2 Equilibrium Employment, Consumption and Output}

From the goods market clearing conditions, (2) and (3), and equations (5) and (6), we have,

$$L_{at} = \frac{e \left( e-1 \right) \left( \frac{e}{\epsilon-1} \left( \frac{\mu_a^{\mu_a-1}}{A_{at}^{\mu_a-1}} + \frac{\mu_{na}^{\mu_{na}-1}}{A_{nat}^{\mu_{na}-1}} \right) \right) - \frac{1}{\epsilon}}{B_t \left( (\mu_a - 1) \frac{e}{\epsilon-1} \left( \frac{\mu_a^{\mu_a-2}}{A_{at}^{\mu_a-2}} + (\mu_{na} - 1) \frac{e}{\epsilon-1} \left( \frac{\mu_{na}^{\mu_{na}-2}}{A_{nat}^{\mu_{na}-2}} \right) \right) \right)}.$$  \hspace{1cm} (8) \\

$$L_{nat} = \frac{e \left( e-1 \right) \left( \frac{e}{\epsilon-1} \left( \frac{\mu_a^{\mu_a-1}}{A_{at}^{\mu_a-1}} + \frac{\mu_{na}^{\mu_{na}-1}}{A_{nat}^{\mu_{na}-1}} \right) \right) - \frac{1}{\epsilon}}{B_t \left( (\mu_a - 1) \frac{e}{\epsilon-1} \left( \frac{\mu_a^{\mu_a-2}}{A_{at}^{\mu_a-2}} + (\mu_{na} - 1) \frac{e}{\epsilon-1} \left( \frac{\mu_{na}^{\mu_{na}-2}}{A_{nat}^{\mu_{na}-2}} \right) \right) \right)}.$$  \hspace{1cm} (9) \\

Hence the aggregate employment to population ratio is

$$L_t = L_{at} + L_{nat} = \left( \frac{e \left( e-1 \right) \left( \frac{e}{\epsilon-1} \left( \frac{\mu_a^{\mu_a-1}}{A_{at}^{\mu_a-1}} + \frac{\mu_{na}^{\mu_{na}-1}}{A_{nat}^{\mu_{na}-1}} \right) \right) - \frac{1}{\epsilon}}{B_t \left( (\mu_a - 1) \frac{e}{\epsilon-1} \left( \frac{\mu_a^{\mu_a-2}}{A_{at}^{\mu_a-2}} + (\mu_{na} - 1) \frac{e}{\epsilon-1} \left( \frac{\mu_{na}^{\mu_{na}-2}}{A_{nat}^{\mu_{na}-2}} \right) \right) \right)} \right)^{\frac{1}{\epsilon}}.$$  \hspace{1cm} (10)
and the sector employment shares are

\[
l_at \equiv \frac{L_at}{L_t} = \frac{\varphi_a A_{at}^{\varepsilon-1} C_t^{\mu_a - 1}}{\varphi_a A_{at}^{\varepsilon-1} C_t^{\mu_a - 1} + \varphi na A_{nat}^{\varepsilon-1} C_t^{\mu_{na} - 1}},
\]

(11)

\[
l_{nat} \equiv \frac{L_{nat}}{L_t} = \frac{\varphi na A_{nat}^{\varepsilon-1} C_t^{\mu_{na} - 1}}{\varphi a A_{at}^{\varepsilon-1} C_t^{\mu_a - 1} + \varphi na A_{nat}^{\varepsilon-1} C_t^{\mu_{na} - 1}}.
\]

(12)

Equation (11) can also be written as

\[
l_at \equiv \frac{L_at}{L_t} = \frac{\varphi_a \left( \frac{A_{at}}{A_{nat}} \right)^{\varepsilon-1} \lambda^{C_t^{\mu_a - \mu_{na}}}}{1 + \frac{\varphi_n \left( \frac{A_{at}}{A_{nat}} \right)^{\varepsilon-1} \lambda^{C_t^{\mu_a - \mu_{na}}}}{C_t^{\mu_{na} - \mu_{na}}}},
\]

(13)

which shows that the agriculture’s share of employment is affected by two factors, the relative productivity of agriculture \(A_{at}/A_{nat}\) and the aggregate consumption per capita \(C_t\). The first factor represents the substitution effect and the second factor the income effect. The substitution effect depends on whether \(\varepsilon\) is smaller or larger than one. If \(\varepsilon\) is less than one, the agriculture’s share of employment is a decreasing function of the agricultural sector’s productivity and an increasing function of the non-agricultural sector’s productivity. The opposite is true if \(\varepsilon\) is greater than one. Therefore the substitution effects of the two sector’s labour productivity on the agricultural sector’s employment is in the opposite directions as long as the value of \(\varepsilon\) is not equal to one, in which case there is no substitution effect. However, in Table 3 we have documented that the cyclical component of the agriculture’s share of employment is negatively correlated with the cyclical components of real labour productivity in both sectors, suggesting that the second factor, income effect, is also important for labour reallocation at the business cycle frequency. If \(\mu_a < \mu_{na}\), then the agriculture’s share of employment is a decreasing function of the aggregate consumption. Since labour productivities in the agricultural and non-agricultural sectors both have positive impact on the aggregate consumption, they both have a negative effect on the agriculture’s share of employment.

Equation (10) and (7) can be combined to yield the following equation for the
Equilibrium value of aggregate consumption $C_t$.

$$
\left[ \frac{(1 - \varepsilon) \left( \varphi_a A_{at}^{\varepsilon - 1} C_{it}^{\mu_{at} - 1} + \varphi_{na} A_{nat}^{\varepsilon - 1} C_{it}^{\mu_{nat} - 1} \right) \varepsilon^t}{B_t \left( (\mu_a - 1) \varphi_a A_{at}^{\varepsilon - 1} C_{it}^{\mu_{at} - 2} + (\mu_{na} - 1) \varphi_{na} A_{nat}^{\varepsilon - 1} C_{it}^{\mu_{nat} - 2} \right)} \right]^{\frac{1}{\sigma}}
$$

Equations (8), (9) and (14) can be used to solve for the equilibrium employment and output in the two sectors as follows. Given the preference parameters and the real labour productivities in the two sectors, $A_{at}$ and $A_{nat}$, equation (14) can be used to solve for $C_t$. Given $C_t$, equations (8) and (9) can be used to solve for $L_{at}$ and $L_{nat}$. Finally, GDP per capita in the two sectors can be calculated as $Y_{at} = A_{at} L_{at}$ and $Y_{nat} = A_{nat} L_{nat}$ and, when the labour productivity levels are normalized so that the relative price of agriculture at $t = 0$ is 1, the aggregate GDP per capita is simply $Y_t = Y_{at} + Y_{nat}$. Next, we discuss how we calibrate the values of the preference parameters.

## 4 Calibration

We set $\sigma = 0.6$ so that the Frisch elasticity of labour supply is 1.7, a value used by Greenwood, Hercowitz, and Huffman (1988) and many others in the business cycle literature. For the rest of the model parameters, we calibrate their values so that when there is no productivity shock, the model matches the long-run trends of structural change and aggregate employment rate in China over the sample period of 1978 to 2010.

We use the hp-filter to filter out the trends of the agricultural employment to population ratio, the total employment to population ratio and the labour productivities in the two sectors. Because of the non-homothetic preferences, the agriculture’s share of employment is affected by the relative productivity between the two sectors as well as the level of productivities. Given that the productivity is an index and its level depends on the normalization, we treat the initial productivity level
of agriculture sector, $A_0$, as a free parameter and choose its value along with the preference parameters $\varphi_a, \varepsilon, \mu_a, \mu_{na}$ to match the trend of the agriculture’s share of employment.

To be specific, let $\bar{x}_t$ denote the hp-filtered trend component of any variable $x_t$. First, for any $t = 0, 1, ..., T$, given the trend aggregate employment rate $L_t$ and trend labour productivities $\bar{A}_a$ and $\bar{A}_{na}$ in the data, we can solve the trend aggregate consumption $\bar{C}_t(A_0, \varphi_a, \varepsilon, \mu_a, \mu_{na})$ as an implicit function of the parameters $A_0, \varphi_a, \varepsilon, \mu_a, \mu_{na}$ from equation (10),

$$L_t = \left( \varphi_a \left( \frac{A_0}{\bar{A}_a A_{a0}} \right) \varepsilon^{-1} \bar{C}_t^{\mu_a-1} + \varphi_{na} \left( \frac{A_0}{\bar{A}_{a0} \bar{A}_{nat}} \right) \varepsilon^{-1} \bar{C}_{t}^{\mu_{na}-1} \right)^{\frac{1}{1-\varepsilon}}.$$

Then, from (11), we can write the trend of agriculture’s share of employment as

$$l_{at}(A_0, \varphi_a, \varepsilon, \mu_a, \mu_{na}) = \varphi_a \left( \frac{A_0}{\bar{A}_a A_{a0}} \right) \varepsilon^{-1} \bar{C}_t^{\mu_a-1} \left( A_0, \varphi_a, \varepsilon, \mu_a, \mu_{na} \right) \varphi_{na} \left( \frac{A_0}{\bar{A}_{a0} \bar{A}_{nat}} \right) \varepsilon^{-1} \bar{C}_{t}^{\mu_{na}-1} \left( A_0, \varphi_a, \varepsilon, \mu_a, \mu_{na} \right)$$

Finally, we choose $(A_0, \varphi_a, \varepsilon, \mu_a, \mu_{na})$ to minimize the following loss function (i.e., non-linear least square):

$$\sum_{t=0}^{T} \left\{ l_{at}(A_0, \varphi_a, \varepsilon, \mu_a, \mu_{na}) - I_{at} \right\}^2.$$

In our model, $B_t$ in the utility function is exogenously given to capture the long-run changes in aggregate employment rate due to demographic factors. For any given value $\sigma$ and the calibrated values of $(A_0, \varphi_a, \varepsilon, \mu_a, \mu_{na})$, we choose $B_t$ so that the model implied trend of aggregate employment rate matches that in the data.
That is, we choose \( B_t \) to solve the following equation

\[
\mathcal{L}_t = \left[ \frac{(1 - \varepsilon) \left( \varphi_a \left( \frac{A_0}{A_{a0}} \overline{A}_{at} \right) \varepsilon^{-1} C_t^{\mu_a - 1} + \varphi_{na} \left( \frac{A_0}{A_{a0}} \overline{A}_{nat} \right) \varepsilon^{-1} C_t^{\mu_{na} - 1} \right)^{\frac{\varepsilon}{\varepsilon - 1}}}{B_t \left( (\mu_a - 1) \varphi_a \left( \frac{A_0}{A_{a0}} \overline{A}_{at} \right) \varepsilon^{-1} C_t^{\mu_a - 1} + (\mu_{na} - 1) \varphi_{na} \left( \frac{A_0}{A_{a0}} \overline{A}_{nat} \right) \varepsilon^{-1} C_t^{\mu_{na} - 1} \right)} \right]^{\frac{1}{\bar{\sigma}}}
\]

Table 3 summarizes the calibration results. The calibrated value of the elasticity of substitution (\( \varepsilon \)) is less than one, implying that the substitution effect is such that the agriculture’s share of employment is negatively related to the agriculture’s relative productivity. This is consistent with the theoretical assumption of Ngai and Pissarides (2007) and the finding of Herrendorf, Rogerson and Valentinyi (2013). The calibrated value of \( \mu_a \) is less than that of \( \mu_{na} \), implying that the income effect is such that the agriculture’s share of employment is negatively related to the labour productivities in both sectors.

Table 3: Benchmark Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varphi_a )</td>
<td>average employment share of agriculture</td>
<td>0.11</td>
</tr>
<tr>
<td>( \varepsilon )</td>
<td>elasticity of substitution between two goods</td>
<td>0.47</td>
</tr>
<tr>
<td>( \mu_a )</td>
<td>income elasticity of agricultural good</td>
<td>1.57</td>
</tr>
<tr>
<td>( \mu_{na} )</td>
<td>income elasticity of non-agricultural good</td>
<td>3.76</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>inverse of Frisch elasticity of labour supply</td>
<td>0.6</td>
</tr>
<tr>
<td>( A_0 )</td>
<td>initial productivity level</td>
<td>0.026</td>
</tr>
</tbody>
</table>

Figure 5 displays the trends of the agriculture’s share of employment and aggregate employment rate from both the model and the data. Our calibrated model matches perfectly the trend of aggregate employment rate by construction, and matches the trend of the agriculture’s share of employment very well. We next turn to the cyclical properties of our calibrated model when there are shocks to productivities in the two sectors.
5 Benchmark Results

Table 4 presents the results of simulating our calibrated model. We feed the model with the actual labour productivities series from the data, $A_{at}$ and $A_{nat}$, $t = 0, 1, \ldots, T$. We filter the simulated time series from the model using the hp-filter in the same way as we filter the data series to extract the cyclical components of the time series. The first and second columns of Table 4 present the business cycle statistics calculated from China’s data and the simulated time series from the model. Panel A shows the relative standard deviations of the aggregate employment to output and the correlation between the aggregate employment and output, Panel B the sector level correlations and relative standard deviations, and Panel C the correlation between sector employment and the correlation of the agriculture’s share of employment with sector labor productivities. Figure 6 compares the simulated employment from model with the data for both sectors.

Overall, the model does a good job in matching both aggregate and sector moments in the data. From Panel A, we see that the model produces a relative volatility of aggregate employment of 0.13, which is very close to 0.11 in the data. The model also generates a correlation of employment and output close to zero, indicating that employment is acyclical. When we look at business cycle facts at sector level from Panel B, the model shows employment in both sector fluctuates a lot, with the rel-
ative volatility of agriculture employment to be 0.82 and non-agriculture employment to be 0.56. The non-agriculture employment is strongly procyclical, as in the data. However, for the agricultural sector, employment is negatively correlated with output in the model and slightly positive in the data. The model-implied negative correlation between employment and output in the agricultural sector obviously implies that the correlation between agricultural employment and agricultural labour productivity is strongly negative. We explain below that this is due to a strong income effect. Panel C shows the labor reallocation between the two sectors. The correlation of employments in the two sectors is -0.83, as in the data, indicating strong reallocation between sectors. Moreover, the model implies that the agriculture’s share of employment is negatively correlated with labor productivity in both sectors. When the labor productivity in agriculture sector increases, relative price of agriculture good falls. Given that agricultural and non-agricultural goods are complements, substitution effect leads to a fall of the agriculture’s share of employment. In addition, higher agricultural labor productivity also raises aggregate consumption. Because $\mu_a < \mu_{na}$, the income effect is such that the agriculture’s share of employment declines. Thus, both substitution and income effects lead to negative correlation between the agriculture’s share of employment and labor productivity. This explains why the model implies a very strong negative correlation of -0.99. When the labor productivity in non-agriculture sector increases, the relative price of agriculture good rises and the substitution effect is such that the agriculture’s share of employment increases. The income effect still leads to a fall of the agriculture’s share of employment. Overall, the income effect dominates, leading to a correlation of -0.26.

In summary, the model economy matches the data for the Chinese economy well. In particular, it generates high employment-output correlation in the non-agricultural sector and low employment-output correlation in the aggregate. It’s implications for labour reallocation between sectors is also broadly consistent the data. In the following section, we show that our model can replicate the business cycle facts in the US as well.
Table 4: Model Moments - China

<table>
<thead>
<tr>
<th></th>
<th>China Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma (L)/\sigma (Y))</td>
<td>0.11</td>
<td>0.13</td>
</tr>
<tr>
<td>(\rho (L,Y))</td>
<td>0.09</td>
<td>-0.06</td>
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</table>

<table>
<thead>
<tr>
<th></th>
<th>China Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma (L_a)/\sigma (Y_a))</td>
<td>0.70</td>
<td>0.82</td>
</tr>
<tr>
<td>(\sigma (L_{na})/\sigma (Y_{na}))</td>
<td>0.75</td>
<td>0.56</td>
</tr>
<tr>
<td>(\rho (L_a,Y_a))</td>
<td>0.24</td>
<td>-0.92</td>
</tr>
<tr>
<td>(\rho (L_{na},Y_{na}))</td>
<td>0.88</td>
<td>0.83</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>China Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\rho (L_a,L_{na}))</td>
<td>-0.83</td>
<td>-0.83</td>
</tr>
<tr>
<td>(\rho (L_a,A_a))</td>
<td>-0.44</td>
<td>-0.99</td>
</tr>
<tr>
<td>(\rho (L_{na},A_{na}))</td>
<td>-0.35</td>
<td>-0.26</td>
</tr>
</tbody>
</table>

Figure 6:
6 Model’s Implications for the US

For the US economy, we keep the preference parameters $\phi_a$, $\varepsilon$, $\mu_a$, and $\mu_{na}$ at the same values as in our benchmark calibration for China. Recall that, from equations (15) and (16), the level of the aggregate consumption matters for the level of the agriculture’s share of employment due to the non-homothetic preferences, and the aggregate consumption depends on the productivity level $A_0$. Therefore, we recalibrate $A_0$ to match the level of the agriculture’s of employment in the US economy.

The left panel of Figure 7 shows a striking result that with the same preference parameters, our model can match well the structural change trend in the US: the agriculture’s share of employment declines from 2.9% in 1978 to 1.3% in 2010. We also keep the value of $\sigma$ the same as in the benchmark calibration for China and recalibrate $B_t$, $t = 0, 1, \ldots, T$, to match the trend of aggregate employment rate in the US.

Table 5 compares the simulated model moments with the US data and shows that our model does a good job in replicating the US business cycles. From Panel A, we see that the model can generate highly procyclical aggregate employment: the correlation between aggregate employment and aggregate output is 0.86. However, the model produces a relative employment volatility than is lower than the data. This problem is common for standard real business cycle model, as pointed
out by Cooley and Prescott (1995), that without additional labor market frictions these models have difficulty in generating sizable employment variations. Panel B and C illustrate the sector level correlations and labor reallocations across sectors. It is worth emphasizing that the model is able to produce a negative correlation between the two sectors’ employments and negative correlations of the agriculture’s of employment with labor productivities in both sectors.

<table>
<thead>
<tr>
<th>Table 5: Model Moments - US</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>(A) Aggregate</td>
</tr>
<tr>
<td>(\sigma(L)/\sigma(Y))</td>
</tr>
<tr>
<td>(\rho(L,Y))</td>
</tr>
<tr>
<td>(B) Within Sector</td>
</tr>
<tr>
<td>(\sigma(L_a)/\sigma(Y_a))</td>
</tr>
<tr>
<td>(\sigma(L_{na})/\sigma(Y_{na}))</td>
</tr>
<tr>
<td>(\rho(L_a,Y_a))</td>
</tr>
<tr>
<td>(\rho(L_{na},Y_{na}))</td>
</tr>
<tr>
<td>(C) Cross Sector</td>
</tr>
<tr>
<td>(\rho(L_a,L_{na}))</td>
</tr>
<tr>
<td>(\rho(t_a,A_a))</td>
</tr>
<tr>
<td>(\rho(t_{na},A_{na}))</td>
</tr>
</tbody>
</table>

Overall, despite being highly stylized, our model economy with the same preference parameters can match well the data from both China and US. We show that our model with the highlighted mechanism is important in understanding the business cycle fluctuations in both US and China.

7 Sensitivity Analysis

In this section we carry out some sensitivity analysis.
Aggregate labour supply parameter $B_t$. Since the long run change of aggregate employment rate is determined by demographic factors, which are not the interest of our paper, we used exogenous $B_t$ to match the trend of aggregate employment rate in our calibration. We now test the robustness of our model by imposing a constant $B_t$. In particular, we use $B$ to match the long run average of aggregate employment rate, which is 0.55 for China from 1978 to 2010. This gives a value of 0.45 for $B$. Table 6 shows the simulation results, where the second column is our benchmark result and the third column is the robustness check when $B$ is a constant. By comparing the second and third column of the table, we see that holding $B_t$ constant barely affects the business cycle properties of our model. All the conclusions from our benchmark model carry through.

Elasticity of labour supply. The parameter $\sigma$ governs the elasticity of labor supply, which affects directly the aggregate employment volatility. In line with the literature, we choose this parameter to be 0.6 in our benchmark calibration. We now check the sensitivity of our model to this parameter by changing the value of $\sigma$ and recalibrating our model to the same target. In the fourth and fifth columns of Table 6, we report the simulation results from the model for different values of $\sigma$. It can be seen that higher labor elasticity, or lower value of $\sigma$, implies higher aggregate employment volatility. Aggregate employment remains acyclical for different values of $\sigma$. While there is some differences in the results across different value of $\sigma$, the properties of sector-level fluctuations and the labour reallocation between the two sectors of the benchmark model still hold.

Identical income elasticity for both goods. The parameter $\mu_i$ controls the income elasticity of good $i$, which we argue is important in understanding the reallocation of labour across sectors. Our benchmark calibration gives $\mu_a < \mu_{na}$, implying a negative income effect on agriculture employment share. We now suppress the income effect by letting the income elasticity of the two goods be the same. To be specific, we set them to be the average of benchmark values of $\mu_a$ and $\mu_{na}$, i.e., $\mu_a = \mu_{na} = 2.66$. Case 4 in Table 6 displays our simulation results. Panel A shows, without income effect, the model produces a highly procyclical aggregate employment whereas data shows acyclical aggregate employment. Panel C shows that when only the substitution effect is present, the model produces a
positive correlation between agriculture’s share of employment and non-agriculture labor productivity, which is contray to the data. This exercise shows the importance of non-homothetic preference in explaining the employment dynamics.

**CES preferences.** To compare with the standard CES preferences, we now set \( \mu_a = \mu_{na} = 2 - \varepsilon \). We recalibrate the model following the strategy used by Da-Rocha and Restuccia (2004). Specifically, we choose a value of 0.5 for \( \varphi_a \) to match the average of the agriculture’s share of employment, which is 0.5 in the data, a value of 8 for \( \sigma \) to match the relative volatility of aggregate employment to output of 0.11, and finally a value of 0.27 for \( \varepsilon \) to match the relative volatility of agriculture employment to non-agriculture employment of 0.77. Case 5 in Table 6 shows the simulation results. Comparing with the benchmark case, the CES case needs a much lower labour elasticity of 0.1 in order to match the relative volatility of aggregate employment, and it also implies a highly procyclical employment series, which contradicts the data. The strong procyclical property of the aggregate employment seems to be a general feature of the CES preferences. In Appendix B, we presents simulation results with different values for the preference parameters, they all show a strong procyclical employment series.\(^1\) At the sector level, without income effect, the CES model produces wrong sign of correlation between the agriculture’s share of employment and non-agriculture productivity.

### 8 Conclusion

The cyclical behavior of aggregate employment differs significantly between China and the US. This sharp difference at the aggregate level conceals similar behavior of cyclical employment at sector level. We argue that the main difference between China and the US is the size of the agricultural sector. We show that a simple two-sector growth model with productivity shocks and non-homothetic preferences can simultaneously account for the secular trend of structural change and employment

\(^1\)Da-Rocha and Restuccia (2004) also uses the CES preferences, but they introduce an ex-post productivity shock in agriculture that helps to lower the correlation between the aggregate employment and output. In the version of their model without the ex-post agricultural productivity shock, the correlation of employment and output is 0.96.
Table 6: Sensitivity Analysis

<table>
<thead>
<tr>
<th></th>
<th>China Data</th>
<th>Benchmark</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
<th>Case 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) Aggregate</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma(L)/\sigma(Y)$</td>
<td>0.11</td>
<td>0.13</td>
<td>0.12</td>
<td>0.18</td>
<td>0.09</td>
<td>0.13</td>
<td>0.11</td>
</tr>
<tr>
<td>$\rho(L,Y)$</td>
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<td>-0.07</td>
<td>0</td>
<td>-0.07</td>
<td>0.75</td>
<td>1.00</td>
</tr>
<tr>
<td>(B) Within Sector</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma(L_a)/\sigma(Y_a)$</td>
<td>0.70</td>
<td>0.82</td>
<td>0.83</td>
<td>0.89</td>
<td>0.76</td>
<td>0.52</td>
<td>0.39</td>
</tr>
<tr>
<td>$\sigma(L_{na})/\sigma(Y_{na})$</td>
<td>0.75</td>
<td>0.56</td>
<td>0.54</td>
<td>0.52</td>
<td>0.60</td>
<td>0.27</td>
<td>0.63</td>
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<tr>
<td>$\rho(L_a,Y_a)$</td>
<td>0.24</td>
<td>-0.92</td>
<td>-0.92</td>
<td>-0.89</td>
<td>-0.94</td>
<td>-0.52</td>
<td>-0.54</td>
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<tr>
<td>$\rho(L_{na},Y_{na})$</td>
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<td>0.83</td>
<td>0.84</td>
<td>0.85</td>
<td>0.82</td>
<td>0.43</td>
<td>0.59</td>
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<tr>
<td>(C) Cross Sector</td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>$\rho(L_a,L_{na})$</td>
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<td>-0.83</td>
<td>-0.87</td>
<td>-0.76</td>
<td>-0.86</td>
<td>-0.75</td>
<td>-0.94</td>
</tr>
<tr>
<td>$\rho(L_a,A_a)$</td>
<td>-0.44</td>
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<td>-0.99</td>
<td>-0.99</td>
<td>-0.86</td>
<td>-0.86</td>
</tr>
<tr>
<td>$\rho(L_{na},A_{na})$</td>
<td>-0.35</td>
<td>-0.26</td>
<td>-0.27</td>
<td>-0.29</td>
<td>-0.24</td>
<td>0.19</td>
<td>0.20</td>
</tr>
</tbody>
</table>

fluctuations at the business cycle frequency in both China and the US.
Appendix

A Derivation of Formulas

The FOCs of the social planner’s maximization problem with respect to \( L_{at} \) and \( L_{nat} \) are:

\[
\frac{\partial C_t}{\partial c_{at}} A_{at} - B_t L_t^\sigma = 0 \tag{15}
\]

\[
\frac{\partial C_t}{\partial c_{nat}} A_{nat} - B_t L_t^\sigma = 0 \tag{16}
\]

From equation (1), we have

\[
(\mu_a - 1) (\varphi_a) \frac{1}{2} c_{at}^{\frac{\mu_a - 1}{\epsilon}} C_t^{\frac{1 - \epsilon}{\epsilon}} \frac{\partial C_t}{\partial c_{at}} + (\mu_{na} - 1) (\varphi_{na}) \frac{1}{2} c_{nat}^{\frac{\mu_{na} - 1}{\epsilon}} C_t^{\frac{1 - \epsilon}{\epsilon}} \frac{\partial C_t}{\partial c_{nat}} + (\epsilon - 1) \left( \varphi_a \right)^{\frac{1}{2}} c_{at}^{\frac{1}{2} \frac{\mu_a - 1}{\epsilon}} = 0,
\]

\[
(\mu_a - 1) (\varphi_a) \frac{1}{2} c_{at}^{\frac{\mu_a - 1}{\epsilon}} C_t^{\frac{1 - \epsilon}{\epsilon}} \frac{\partial C_t}{\partial c_{nat}} + (\mu_{na} - 1) (\varphi_{na}) \frac{1}{2} c_{nat}^{\frac{\mu_{na} - 1}{\epsilon}} C_t^{\frac{1 - \epsilon}{\epsilon}} \frac{\partial C_t}{\partial c_{nat}} + (\epsilon - 1) \left( \varphi_{na} \right)^{\frac{1}{2}} c_{nat}^{\frac{1}{2} \frac{\mu_{na} - 1}{\epsilon}} = 0.
\]

Thus, we have

\[
\frac{\partial C_t}{\partial c_{at}} = \frac{(1 - \epsilon) (\varphi_a)^{\frac{1}{2}} c_{at}^{\frac{1}{2} \frac{\mu_a - 1}{\epsilon}} C_t^{\frac{1 - \epsilon}{\epsilon}}}{D_t}, \tag{17}
\]

\[
\frac{\partial C_t}{\partial c_{nat}} = \frac{(1 - \epsilon) (\varphi_{na})^{\frac{1}{2}} c_{nat}^{\frac{1}{2} \frac{\mu_{na} - 1}{\epsilon}} C_t^{\frac{1 - \epsilon}{\epsilon}}}{D_t}, \tag{18}
\]

where

\[
D_t = (\mu_a - 1) (\varphi_a)^{\frac{1}{2}} c_{at}^{\frac{1}{2} \frac{\mu_a - 1}{\epsilon}} C_t^{\frac{1 - \epsilon}{\epsilon}} + (\mu_{na} - 1) (\varphi_{na})^{\frac{1}{2}} c_{nat}^{\frac{1}{2} \frac{\mu_{na} - 1}{\epsilon}} C_t^{\frac{1 - \epsilon}{\epsilon}}. \tag{19}
\]

Substituting equations (17) and (18) into (15) and (16), respectively, and solving for \( c_{at} \) and \( c_{nat} \), we have the following:

\[
c_{at} = \varphi_a \left( \frac{(1 - \epsilon) A_{at}}{D_t B_t L_t^\sigma} \right)^{\epsilon} C_t^{\frac{\mu_a - 1}{\epsilon}}, \tag{20}
\]

\[
c_{nat} = \varphi_{na} \left( \frac{(1 - \epsilon) A_{nat}}{D_t B_t L_t^\sigma} \right)^{\epsilon} C_t^{\frac{\mu_{na} - 1}{\epsilon}}. \tag{21}
\]

Substituting these two equations into (1) we have

\[
\varphi_a \left( \frac{(1 - \epsilon) A_{at}}{D_t B_t L_t^\sigma} \right)^{\epsilon - 1} C_t^{\frac{\mu_a - 1}{\epsilon}} + \varphi_{na} \left( \frac{(1 - \epsilon) A_{nat}}{D_t B_t L_t^\sigma} \right)^{\epsilon - 1} C_t^{\frac{\mu_{na} - 1}{\epsilon}} = 1.
\]

24
which implies that

\[
\left( \frac{D_t B_t L_t^\sigma}{1 - \epsilon} \right)^{1-\epsilon} \left( \phi a A_{at}^{\epsilon-1} C_t^{\mu_{at}-1} + \phi na A_{nat}^{\epsilon-1} C_t^{\mu_{na}-1} \right) = 1,
\]

\[
\frac{D_t B_t L_t^\sigma}{1 - \epsilon} = \left( \phi a A_{at}^{\epsilon-1} C_t^{\mu_{at}-1} + \phi na A_{nat}^{\epsilon-1} C_t^{\mu_{na}-1} \right)^{\frac{1}{1-\epsilon}}.
\] (22)

Substituting (22) into (20) and (21) and solving for \(c_{at}\) and \(c_{nat}\) yield the solution in equations (5) and (6). Substituting (5) and (6) into (19) and simplifying yields the following:

\[
D_t = \frac{(\mu - 1) \phi a A_{at}^{\epsilon-1} C_t^{\mu_{at}-2} + (\mu - 1) \phi na A_{nat}^{\epsilon-1} C_t^{\mu_{na}-2}}{\phi a A_{at}^{\epsilon-1} C_t^{\mu_{at}-1} + \phi na A_{nat}^{\epsilon-1} C_t^{\mu_{na}-1}}.
\]

From (22), then, we have

\[
L_t = \left[ \frac{(1 - \epsilon) \left( \phi a A_{at}^{\epsilon-1} C_t^{\mu_{at}-1} + \phi na A_{nat}^{\epsilon-1} C_t^{\mu_{na}-1} \right)^{\frac{1}{\epsilon-1}}}{B_t \left( (\mu_{at} - 1) \phi a A_{at}^{\epsilon-1} C_t^{\mu_{at}-2} + (\mu - 1) \phi na A_{nat}^{\epsilon-1} C_t^{\mu_{na}-2} \right)} \right]^{\frac{1}{\sigma}}.
\] (23)

B  Sensitivity Analysis for CES Preferences

In Table 7, we show the simulation results for CES preferences with different combinations of \(\sigma\) and \(\epsilon\).

<table>
<thead>
<tr>
<th>(A) Aggregate</th>
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<tbody>
<tr>
<td>(\sigma (L)/\sigma (Y))</td>
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<tr>
<td>(\epsilon = 0.5)</td>
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<tr>
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<table>
<thead>
<tr>
<th>(B) Within Sector</th>
</tr>
</thead>
<tbody>
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<td>(\sigma (L_{at})/\sigma (Y_{at}))</td>
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<td>(\epsilon = 0.5)</td>
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<tbody>
<tr>
<td>(\rho (L_{at},L_{na}))</td>
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<tr>
<td>(\rho (L_{at},L_{na}))</td>
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<tr>
<td>(\rho (L_{at},A_{at}))</td>
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<tr>
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References


