Corporate Debt Structure, Precautionary Savings, and Investment Dynamics

Jasmine Xiao
University of Notre Dame

December 15, 2017

Abstract

This paper documents two facts on the Great Recession. First, public firms that switched from bank finance to bond finance actually experienced a slower recovery in investment, despite having no shortage of credit compared to those that did not switch. Second, their debt substitution was accompanied by a substantial increase in cash holdings. As firms substitute toward bonds when bank lending is impaired, they lose the ability to restructure debt to avoid default. Debt substitution thus strengthens firms’ precautionary incentive to simultaneously increase cash holdings at the expense of investment, as they optimally trade-off growth against self-insurance. Model simulations suggest that this “precautionary savings” channel can account for a substantial fraction of the decline in aggregate investment in the recent recession. I show that embedding balance sheet adjustment in a business-cycle model improves the model’s amplification, and helps to disentangle shocks to credit demand from shocks to credit supply.

Keywords: corporate debt; corporate saving; investment; costly external finance; firm heterogeneity; financial frictions; firm dynamics; general equilibrium

JEL Classification: E22, E32, E44, G31, G33

∗yxiao2@nd.edu. I am extremely grateful to Giancarlo Corsetti for his invaluable guidance and support. I am also indebted to Vasco Carvalho for many insightful discussions. Finally, I thank Rüdiger Bachmann, Francisco Buera, Nicolas Crouzet, Jeroen Dalderop, Fiorella De Fiore, John Haltiwanger, Olivier Jeanne, Joseph Kaboski, Sebnem Kalemli-Ozcan, Arvind Krishnamurthy, Hamish Low, Pontus Rendahl, John Shea, Ilya Streibulaev, and seminar participants at Bank of England, Cambridge, ECB, Federal Reserve Board, HEC Montréal, IMF, Mannheim, Maryland, Notre Dame, Reserve Bank of India, Stanford GSB, Stockholm School of Economics, Tilburg, Toulouse, UBC Sauder, Virginia, and Warwick for useful comments at various stages of this project. Financial support from The Cambridge-INET Institute is gratefully acknowledged.
1 Introduction

The 2007–2009 recession and anemic recovery have reinvigorated the study of financial frictions and their impact on macroeconomic fluctuations. This fundamental issue motivates an extensive theoretical literature, whose central goal is to understand the propagation mechanisms. Much of the literature assumes that firms borrow from a single financial intermediary, and overlooks the substitutability between different types of debt instruments. However, recent empirical contributions by Becker and Ivashina (2014), and Adrian, Colla and Shin (2012) suggest that firms with access to public debt markets actively substitute corporate bonds for bank loans when credit conditions tighten: the surge in bond financing during the recent crisis could make up approximately 70% of the total decline in bank lending.\(^1\) Such evidence raises questions on the mechanism through which financial frictions affect real activities, if it is not via a contraction in the total quantity of credit. It also points out the importance of understanding the potential role of debt substitution in the propagation of aggregate shocks over time.

In this paper, I first provide new evidence that firms which substituted bond issues for bank loans during the Great Recession have been hoarding higher fractions of assets as cash and experienced a slower recovery in investment than firms which did not substitute, using firm-level data on the debt structure of public firms between 2006 and 2015. This is a surprising result: in principle, these firms should have been less affected by adverse shocks to bank credit supply, as they did not suffer from a large decline in total leverage. The fact that their investment was more affected suggests that debt substitution may play a complex and previously unexplored role in the propagation and amplification of aggregate shocks. To identify and understand the mechanisms that explain this empirical pattern, I build a quantitative general equilibrium model of firm dynamics, that jointly endogenizes the composition of borrowing, as well as the allocation of assets between savings and capital investment. The model captures two channels in the firms’ response to an unanticipated increase in the bank lending cost. The first is the traditional “financial constraint” channel, whereby firms that relied heavily on bank loans react to the shock by deleveraging and reducing investment. The second channel reflects the fact that, following an unanticipated increase in the bank lending cost, firms adjust the composition of their balance sheets at two margins: on the liability side, they substitute bank loans with bond issues; on the asset side, given leverage, they reallocate assets from productive capital to cash holdings. These decisions jointly gives rise to a novel “precautionary savings” channel that is quantitatively relevant especially through the balance sheet adjustment of firms with intermediate size and default risk, as they are the most likely to switch from a mixed-debt to a bond-only financial regime following a negative bank credit supply shock. Key for the mechanism is the assumption that bonds are more difficult to restructure than loans. Switching to a bond-only debt structure thus exposes firms to a higher risk of default, to which firms respond by increasing their cash holdings.

\(^1\)In absolute terms, aggregate bank loans dropped by USD 577 billions from 2008Q3 to 2010Q1, and aggregate bond issues increased by USD 389 billions in the same period. Data is from the Flow of Funds; see Appendix A.1 for a complete list of data sources and details for construction.
for self-insurance at the expense of investment. By explaining why the large quantities of bond
issues have been saved rather than invested, the model suggests one potential reason for the slow
recovery from the Great Recession.

The model is characterized by three key features. First, firms that are heterogeneous in the
risk of default need to raise debt in order to finance investment, but are subject to agency costs
associated with default (see Cooley and Quadrini (2001); Gilchrist, Sim and Zakrajšek (2014);
Khan and Thomas (2013); among others). Second, there are two types of financial intermediaries:
market lenders and bank lenders. Crucially, they differ in their ability to deal with firms in financial
distress, as in Crouzet (2015). Third, while firms experience sequentially two idiosyncratic
shocks in a period, they can only reoptimize their choice of assets, but not liabilities, after the first
productivity shock. Subsequently, a demand shock is realized after production and determines
the profitability of a firm and its default decision.2 To the best of my knowledge, this is the first
model to generate simultaneous borrowing and saving by risk-neutral firms with limited liability.
The latter is difficult to generate in this class of models and requires more than simply adding an
additional state variable to an otherwise standard firm investment model, as the risk-free return
on saving is weakly dominated by the cost of risky debt, which prices in the probability of firm
defaults. Therefore, firms that maximize expected payoffs have little incentive to borrow for the
purpose of savings, if liability and asset decisions were simultaneously determined. To overcome
this, I introduce a sequence of shocks to separate the decisions on the liability side from those on
the asset side, such that firms choose the optimal asset allocation for a given liability structure in
the middle of a period; by backward induction, firms rationally expect the optimal asset alloca-
tion later on and choose the optimal liability structure at the beginning of a period. This model
thus provides the first unified framework to study the macroeconomic implications of balance
sheet adjustments on both the asset- and liability-side.

The mechanism of the model hinges upon the following trade-offs faced by the firms, which
imply that their balance sheet adjustment in response to aggregate shocks can have important
implications on future macroeconomic dynamics. In choosing the optimal debt structure, firms
trade-off the ability to restructure bank debt in financial distress, with the lower intermediation
costs offered by markets in normal times. In choosing the optimal portfolio of assets, firms face
a trade-off between investing more and getting higher profits in the future—conditional on re-
ceiving a favorable demand shock and not defaulting—and holding more cash, which implies
that returns have a lower variance and firms have a higher chance of survival. As a result of this
combination of trade-offs, for a comparable leverage, replacing bank debt with market debt ex-
poses firms to larger default risks, thus incentivizing them to reallocate assets from capital to cash

2This assumption essentially divides one period into subperiods, such as in Christiano and Eichenbaum (1995);
and De Fiore and Uhlig (2011, 2015). The assumption that firms can adjust their cash holdings more easily than debt
is adopted in other similar studies such as Acharya, Davydenko and Strebulaev (2012). Dynamic capital structure
theories (e.g. Fischer, Heinkel and Zechner (1989); Hackathorn, Miao and Morellec (2006)) show that waiting times to
restructure debt optimally could be substantial, even for small transaction costs, an implication supported by recent
empirical evidence (e.g. Leary and Roberts (2005)). I provide further evidence in Appendix B.1 to show that cash is
indeed significantly more variable than both equity and debt.
holdings. The model prediction matches well with the empirical stylized fact on the robustly positive correlation between cash holdings and corporate bond spreads, shown in Acharya, Davydenko and Streubalaev (2012). Moreover, the results imply that the precautionary motives for saving could be of first-order importance even for public firms with relatively good ratings, and suggest that when the less risky firms switch to bond financing, there could be a convergence in firm dynamics among firms of different default risks. Indeed, the “precautionary savings” channel, through which aggregate shocks affect macroeconomic outcomes, plays a crucial role in explaining the convergence in investment dynamics among the financially unconstrained and constrained firms, which is observed in the data since the 2007-09 financial crisis and at odds with the intuition suggested by the traditional “financial constraint” channel.

The model provides a useful framework to study the transmission of aggregate shocks and the macroeconomic implications of balance sheet adjustment. In response to a financial shock that reduces the effective supply of bank credit, firms reduce their borrowing from banks, increase their cash holdings, and scale down investment. This is the traditional “financial constraint” channel, and is particularly relevant to firms of high default risks that relied heavily on bank loans before the shock. This channel can account for about 60% of the decline in output in the first two years of the crisis, but only less than half of the decline in the following five years. The remaining response of output is accounted for by a novel “precautionary savings” channel that is particularly significant for firms which respond to the shock by switching from a mixed-debt to a bond-only debt regime, since the latter strips away the possibility to restructure debt in times of financial distress, in the absence of bank lenders. Qualitatively, these predictions match well with the micro-level evidence on the cross-sectional changes in cash holdings and firm growth since the Great Recession. Quantitatively, this “precautionary savings” channel plays a significant role in the contraction of aggregate investment and output. The fraction of firms employing a bond-only debt structure more than triples after the shock, with each of these firms reallocating a significant fraction of assets towards cash reserves.

It is worth emphasizing that the results of this paper by no means dispute the idea that capital markets could act as a “spare tyre” to traditional, bank-based intermediation at times when the latter is impaired (Greenspan (1999)). In fact, in line with the findings by Kashyap, Lamont and Stein (1994) and De Fiore and Uhlig (2015), this paper lends supports to the hypothesis that firms’ ability to substitute among alternative debt instruments is important to shield the economy from adverse real effects of a financial crisis. In a counterfactual exercise where substitution towards bond financing is not allowed, the decline in aggregate output is significantly higher than in the baseline scenario. What this paper emphasizes, however, is that imperfect substitutability between bank and bond debt can significantly reduce the effectiveness of the “spare tyre”. In particular, the model predicts that substitution towards bond financing has adverse effects for the investment by firms of intermediate default risk. Hence the results suggest that policies supporting the corporate bond market with the goal of sustaining firms growth during a financial crisis may have some unintended implications for investment, unless complemented by further measures, such as investment tax credit, to contain firms’ cash hoarding incentives.
Lastly, I examine through the lens of the model whether financial frictions manifest themselves through shocks to the demand for credit or to its supply in the Great Recession. Specifically, I compare the transmission of a financial shock with that of an uncertainty shock, where firms face time-varying idiosyncratic technology uncertainty, in the spirit of, for instance, Bloom (2009); Bloom, Bond and Van Reenen (2007); Bachmann and Bayer (2013); and Gilchrist, Sim and Zakrajsk (2014). The aggregate responses of total leverage and investment to an uncertainty shock are in line with the existing literature: by widening the credit spreads, unanticipated increases in uncertainty lead to a large decline in investment. Nonetheless, a closer look at the debt structure and heterogeneity in firm dynamics reveals two results that are at odds with the data. First, instead of retiring bank loans whilst increasing bonds—as shown in the data—all firms increase the fraction of their bank debt following an unanticipated increase in volatility, as they value the flexibility associated with bank debt more when uncertainty is high. Second, an increase in aggregate uncertainty induces the smaller firms with higher default probabilities to increase their cash holdings much more than the larger firms. This is the opposite of the empirical evidence. These counterfactual results suggest that shocks to the supply of intermediated credit are the key driver of financial frictions, echoing the findings by Adrian, Colla and Shin (2012) and Kashyap, Stein and Wilcox (1993). Therefore, introducing corporate balance sheet adjustment to an otherwise standard business cycle model can serve as one way to disentangle shocks to credit demand from shocks to credit supply.

Related Literature This paper relates to a number of existing literatures. First, I contribute to a growing literature on the macroeconomic implications of debt heterogeneity, which have been addressed by relatively few papers thus far, but have received increasing attention since the 2007-09 financial crisis. In a model of procyclical bank leverage and the co-existence of bank loans and bonds, Adrian, Colla and Shin (2012) argue that the impact on real activity comes from the sharp increase in risk premiums, rather than contraction in the total quantity of debt. De Fiore and Uhlig (2011, 2015) build an asymmetric information model of bond and bank borrowing, and provide a model-based assessment of the changes in corporate debt composition in the U.S. during the Great Recession, following an increase in firm-level uncertainty and in the intermediation costs of banks. Closest to this paper is Crouzet (2015), who examines the aggregate implication of debt substitution by focusing on the adjustment on the liability-side. I argue that it is important to consider adjustments on both sides of firms’ balance sheets for three reasons. First, Crouzet (2015) predicts that holding firms’ debt structure constant would generate a milder recession – in other words, the substitution to bond financing can amplify rather than dampen aggregate shocks. I show that by introducing adjustment on the asset side, this is no longer the case. This is because holding safe assets can lower the debt financing cost for a given leverage, and hence can relax the borrowing constraint of the firm when it substitutes towards bond financing. Second, the general equilibrium framework shows how a credit crunch in the banking sector can generate a recession with low interest rates, due to an increase in precautionary savings by the relatively large firms that have substituted into bond financing. In addition, the model can generate a per-
sistent response in output that exceeds the degree of persistence of the financial shock, as internal financing using cash is not a perfect substitute for debt financing: whilst newly issued debt can be costlessly transformed into capital, liquid assets cannot. Therefore, on the one hand, increasing liquid assets in a firm’s portfolio reduces the likelihood of firm default and thus lowers the cost of borrowing today; on the other hand, it slows down the adjustment of capital towards its productive capacity. Thus, turning off cash holdings by firms produces the counterfactual results of a larger decline in total debt during the crisis but a faster recovery in output thereafter.

This paper also contributes to an important literature on the role of financial frictions in the propagation of aggregate shocks, following the seminal contributions of Bernanke and Gertler (1989); Bernanke, Gertler and Gilchrist (1999); Kiyotaki and Moore (1997). In particular, this paper studies the role of financial frictions in a model of firm dynamics, and one key friction is limited liability, as in Cooley and Quadrini (2001), Clementi and Hopenhayn (2006), and Hennessy and Whited (2007), among other models of firm dynamics. In particular, one key result of this paper—that a shock to the availability of bank credit can, on its own, generate a large and prolonged recession, by disrupting the allocation of capital further from that implied by firm productivities—echoes the findings in Khan and Thomas (2013) in a firm dynamics model with debt finance. Moreover, there has been a growing literature on the substitution between debt and equity finance, such as Jermann and Quadrini (2012a); Covas and Den Haan (2012); and Begenau and Salomao (2016) in a firm dynamics model. Nevertheless, these papers only allow for one type of debt and hence do not address the implication of debt substitution. The novel empirical evidence presented in this paper poses a challenge to many of these models in explaining the Great Recession and the slow recovery thereafter, as it suggests that a significant fraction of the contraction in output cannot be explained by the decline in total debt, thus pointing to the importance of modelling debt substitution and studying the propagation of aggregate shocks through firms’ balance sheet adjustment—a gap that this paper aims to fulfil.

Finally, the facts presented in the paper also speak to a stream of papers in corporate finance on the determinants of the rise in U.S. corporate cash holdings. On the empirical side, several explanations have been put forth such as, for example, precautionary motives in the face of uncertainty (Bates, Kahle and Stulz (2009)), rising intangible capital and the share of R&D-intensive firms in the U.S. (Falato, Kadyrzhanova and Sim (2013); Begenau and Palazzo (2016)). These papers focus on explaining the secular trend in U.S. corporate cash holdings over the last decades, whereas this paper focuses on explaining firm heterogeneity in their cash holding behaviors since the recent 2007-2009 recession. On the theory side, the motivation for cash holding in much of the literature arises from the existence of external finance costs (Riddick and Whited (2009); Bolton, Chen and Wang (2011); Gamba and Triantis (2008)). The simultaneous existence of cash and debt is featured in Gamba and Triantis (2008) and Acharya, Almeida and Campello (2007), with both arguing that cash is not the same as negative debt. I contribute to this literature by introducing a model framework whereby firms simultaneously save and borrow using only short-term debt that firms can default upon in equilibrium. Moreover, by introducing an endogenous debt structure choice, I illustrate its implications for the allocation of assets between savings and capital
expenditures. To this end, the paper suggests that firms’ allocation of external finance between productive capital and liquid assets in turn depends on the properties of the external finance raised by firms—an idea that is also featured in Eisfeldt and Muir (2016), who focus on the cost of external funds, whereas the focus here is on the type of funds.

I begin the remainder of the paper by documenting in section 2 the main empirical regularities on which the paper is based. In section 3, I develop a model framework to investigate the economic mechanisms that could explain the empirical patterns observed in the data. In section 4, I assess quantitatively the role of these economic mechanisms in explaining the documented empirical facts. Section 5 concludes.

2 Evidence from Micro-level Data

In this section I summarize a novel set of stylized facts on corporate debt choices, cash holdings and investment since the Financial Crisis of 2007-09. To this end, I retrieve firm-level data from Compustat and Capital IQ to assemble a panel of public firms with Standard & Poor’s ratings and debt capital structure data from 2006Q1 to 2015Q4. I document that firms with higher fractions of market debt have higher cash to asset ratios, and are less likely to use the cash for investment and growth.

2.1 Sample Description and Characteristics

The sample consists of non-financial (SIC codes 6000-6999) and non-utility (SIC codes 4900-4949) firms incorporated in the U.S. that lie in the intersection of the Compustat and Capital IQ database on debt structure. For a firm to be included in the analysis, I require the firm-quarter observations in Compustat to (1) have positive total assets (203,033 observations); (2) have data available on debt structure from Capital IQ (67,908 observations); (3) have Standard & Poor’s ratings (21,759 observations). In order to capture any firm heterogeneity in cash holdings, debt structures and investment dynamics, I split the sample into investment-grade issuers (‘BBB-’ and higher) and speculative-grade issuers. Furthermore, I remove the 25 largest cash holders from the sample of investment-grade firms, as there is substantial evidence that their cash-versus-debt dynamics are significantly different from the remaining investment-grade firms; moreover, the mechanism proposed in this paper is likely to be irrelevant to these firms, since they were almost exclusively bond financed even before the crisis. The final sample comprises 21,402 firm-quarter

A report released by S&P Global Ratings on 20 May 2016 shows that the top 25 U.S. nonfinancial corporations now control just over half of the total amount of cash held by all nonfinancial U.S. corporations, an increase from just 38% five years ago, and that “such extreme wealth of a handful of U.S. corporations is masking a liquidity problem—the worst in a decade—for the vast majority of companies.” These firms include: Apple, Microsoft, Alphabet, Cisco Systems, Oracle, Pfizer, Johnson & Johnson, Amgen, Intel, Qualcomm, Merck & Co., Gilead Sciences, Ford Motor, General Motors, Coca-Cola, Amazon, Medtronic, EMC, Procter & Gambler, Schlumberger, FCA US, Boeing, PepsiCo, Chevron, and The Priceline Group. The empirical evidence presented in this section are robust when these 25 firms are included in the sample, and the evidence is available upon request.
observations involving 938 unique firms.4 To remove outliers, all firm characteristic variables are winsorized at the 1\textsuperscript{st} and 99\textsuperscript{th} percentiles (Bates, Kahle and Stulz (2009)). Appendix A.1 provides a detailed description of the firm characteristic variables used in the analysis.

The obvious disadvantage of using Compustat data is that it only focuses on the behavior of publicly traded firms, which only account for about one-third of employment in the United States (Davis, Haltiwanger, Jarmin and Miranda (2006)). As the focus here is on the impact of switching from bank debt to market debt on firms’ asset allocations, and market debt is mostly issued by public firms with credit ratings, arguably this is the relevant sample for the purpose of this study. The Compustat-Capital IQ sample of firms with available data on debt structure constitute over 72\% of the universe of U.S. firms by assets and cash holdings, and 65\% by total capital expenditures during the sample period 2006-2015.

2.2 Stylized Facts

Figures 1 and 2 illustrate the main point of this paper: that the debt structure of a firm affects its asset allocation, whereby higher fractions of market debt motivate firms to hold proportionally more cash, and invest less in capital.

Figure 1 plots, for investment-grade and speculative-grade firms, the quarterly averages across firms of the key metrics related to the hypothesis, including: fraction of market debt per firm (a), cash holdings to book assets (b), debt to book assets (c), and cash to debt (d). I begin by highlighting the different degrees of substitution from bank debt to market debt between the two groups of firms in panel (a). Investment grade firms have largely replaced bank debt with market debt since 2009, and the fraction of market debt remains high even after the financial crisis. On the contrary, such substitution has been much more moderate for the speculative grade firms. Panel (b) illustrates the divergence in cash hoarding behaviors between the two groups of firms. Investment grade firms have been holding an increasing proportion of their assets as cash since 2009, such that even though the speculative grade firms had higher cash-to-asset ratio than the investment grade firms before the crisis, the latter rapidly increased their cash holdings and overtook by 2011, with a 5 percentage point lead by 2015. As the decline in bank debt was largely replaced by a significant increase in market debt, investment grade firms did not suffer from any steep decline in leverage compared to their speculative grade counterparts, both during the crisis and in the recovery period, as shown in panel (c). In fact, the investment grade firms have increased their leverage ratios by almost 10 percentage points from 2008 to 2014. Nonetheless, panel (d) suggests that they have been saving a high proportion of the new funds available as cash, and that the rate of saving is increasing at a faster rate than the rate at which its leverage is growing.

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4Out of the 938 firms, 880 of them belonged to either the investment-grade subsample or the speculative-grade subsample throughout the period 2006Q1-2015Q4, whereas 58 firms (6.2\%) have switched between the two groups at least once. I classify these firms according to the length of time period that they had a certain rating, e.g. if a firm had an investment grade for more than half of the sample, then it is considered an investment grade firm. The results on the differences between the investment-grade and speculative-grade firms do not change if I use only the 880 firms that consistently belonged to the same ratings group in the sample.
Figure 1: Firm Heterogeneity in Debt Composition and Firms’ Balance Sheet Policies

(a) Market debt fraction

(b) Cash to asset

(c) Leverage

(d) Cash to debt

Note: The figure highlights the differences in balance sheet policies between investment (‘BBB-’ and above) and speculative grade firms for debt structure (a), cash to asset (b), leverage (c), and cash to debt (d). The sample includes all Compustat firm-quarter observations from 2006Q1 to 2015Q4 with positive values for the book value of total assets, and data available on debt structure from Capital IQ, for firms with Standard & Poor’s ratings incorporated in the United States. Financial firms (SIC code 6000-6999), utilities (SIC 4900-4949) and the largest 25 cash holders are excluded from the sample, yielding a panel of 21,402 firm-quarter observations for 938 unique firms. To remove seasonality in financing activities, all panels report the raw series (dashed lines) and its smoothed version (solid lines) as a moving average straddling the current term with two lagged and two forward terms. Variable definitions are provided in Appendix A.1.
Moreover, even though the two groups of firms had similar cash to debt ratios at the onset of the financial crisis, the investment grade firms had an over 15 percentage point lead by 2014.\textsuperscript{5}

Next, I document the empirical regularities that pertain to the real side decisions of firms. To illustrate the different rates of recovery in investment and sales between the investment grade and speculative grade firms, Figure 2 plots, for each group of firms, the quarterly averages across firms of the metrics related investment dynamics: total investment (the sum of capital expenditures and R&D expenditures) to book assets (a), and sales to book assets (b). As indicated by both measures, the speculative grade firms have recovered faster than the investment grade firms since 2009, suggesting that the latter group’s large sum of new market debt issues and cash holdings since the onset of the crisis (see Figure 1) have not been channelled into productive uses to boost investment and growth. Specifically, although the two types of firms had similar investment rates before the crisis and suffered from similar magnitudes of a decline in investment during the crisis, the speculative grade firms recovered much more quickly and their investment was back to the pre-crisis level by 2012, leading the speculative grade firms by a percentage point. Turning to sales, despite having a lead of more than 5 percentage points in the pre-crisis

\textsuperscript{5}Unlike the diverging patterns of debt financing and cash holdings, the two groups of firms had similar patterns of equity financing during and after the crisis, as illustrated in Appendix A.3. This suggests that one should search elsewhere for explanation of their divergence in cash holdings and investment dynamics.
period, the investment grade firms suffered from a larger decline in sales during 2008-09, and
have shown no trend of recovery since.\textsuperscript{6}

Importantly, the pattern of heterogeneity in firms’ balance sheet policies remains clear if one
splits the sample by firms’ total assets, instead of by credit ratings, as shown in Figure A.1. Fur-
thermore, I show in appendix A.4 that the descriptive stylized facts presented in this section are
supported by panel evidence, which highlights the positive (negative) correlation, for a given
leverage, between the fraction of bond financing and cash holdings (investment).

3 Structural Model of Firm Dynamics

In this section I develop a quantitative general equilibrium model with heterogeneous firms that
optimize the composition and amount of borrowing on the liability side, and the portfolio alloca-
tion between savings and investment on the asset side. The main departure from the majority of
existing firm dynamics models with financial frictions (e.g. Khan and Thomas (2013); Gilchrist,
Sim and Zakrajšek (2014)) is twofold: first, debt financing in this model can take two forms:
bank debt and market debt; second, firms simultaneously borrow and hold cash, even though
the return on cash is less than or equal to the interest rate on risky debt.

The model has a continuum of identical households, a continuum of heterogeneous inter-
mediate goods firms, final goods firms, and two types of financial intermediaries. Households
solve a standard consumption-savings problem, and are the owners of all firms. The final goods
firms are competitive and converts intermediate goods into a final good. Crucially, this technol-
ogy is subject to a demand shock, which affects the relative demand of the final goods firms for
different types of intermediate goods. As the intermediate goods firms can only borrow state-
uncontingent debt, they cannot insure away the fluctuations in demand that they face.

The intermediate goods firms—that face heterogeneous productivity and demand—are the
key agents in the model. They produce using decreasing returns-to-scale production technology,
which guarantees that they have a finite optimal scale of operation. However, there are three
sources of financial frictions that prevent them from investing to their optimal scale. First, firms
have limited liability and can default on their debt obligations, and if they do, they exit the mar-
ket. However, liquidation is inefficient: it involves deadweight losses, so debt finance commands
an external finance premium. Second, unlike assets, liabilities cannot be reoptimized after the
productivity shock is realized. Third, I assume that firms can only issue equity at entry, but not
thereafter, in order to focus solely on the impact of debt substitution.\textsuperscript{7} These frictions not only

\textsuperscript{6}One may argue that the different patterns in sales-to-asset indicate that the investment grade firms may be facing
weaker demands than speculative grade firms if, for instance, the former have a disproportionally high percentage of
industries that have suffered from a greater decline in demand than other industries during the recession. Nevertheless,
this is unlikely as Appendix A.2 shows that the distribution of industries in each sample is roughly similar.

\textsuperscript{7}The assumption that firms cannot issue equity \textit{at all} is not necessary for the main results to hold, but allows this
paper to focus on debt composition. A version of the model that allows firms to issue new shares after entry can be
found in appendix D.4.
restrict a firm’s ability to obtain external finance to invest to its optimal scale, but also incentivize them to transfer resources from capital accumulation to savings, for a given leverage.

Within this framework, I analyze the role of firms’ balance sheet adjustment in the propagation of aggregate shocks. More specifically, I focus on two sources of aggregate fluctuations. Disturbances from the first source increase the costs of bank intermediation, and hence capture “financial” shocks by affecting the availability of credit to firms. The study of financial shocks have received increasing attention since the Great Recession (see, for example, Christiano, Motto and Rostagno (2010); Del Negro, Eggertsson, Ferrero and Kiyotaki (2011); Jermann and Quadrini (2012)). In these studies, financial shocks are typically modelled as shocks to the liquidation value of capital. As this study distinguishes between bank debt and market debt, and evidence from the crisis points overwhelmingly to a shock in the supply of intermediated credit by banks (e.g. Adrian, Colla and Shin (2012)), the departure from Jermann and Quadrini (2012) and other studies of financial shocks is that I model a financial shock that asymmetrically affects the supply of bank debt and not market debt, rather than a shock to the liquidation value of capital that affects the supply of both types of debt.

The second type of disturbances alter the dispersion of the idiosyncratic technology shock across all firms and hence capture (technology) “uncertainty” shocks in the aggregate sense, as in Bloom (2009); Bloom, Bond and Van Reenen (2007); Bachmann and Bayer (2013); Gilchrist, Sim and Zakrajšek (2014)). In the presence of irreversibilities and nonconvex capital adjustment costs, uncertainty and financial shocks have real consequences for macroeconomic outcomes, regardless of the structure of the financial markets. Distortions in financial markets can significantly amplify the initial impact of each shock on aggregate investment.

3.1 Overview of Firms’ Problem

Timing Figure 3 summarizes the timing of each intermediate goods firm’s problem. At the beginning of each period, all shocks pertaining to the production and borrowing decisions—including the level of idiosyncratic uncertainty (σ), the relative supply of bank credit (γ∗), and the level of idiosyncratic technology (z)—are realized. The volatility level σ determines the distribution of z′(σ) in the next period, so from the agents’ perspective, an increase in σ represents “news” regarding the distribution of profits tomorrow (Bloom (2009); Gilchrist, Sim and Zakrajšek (2014)). Before production, firms can re-optimize their allocation of assets (k, a), given the predetermined level and composition of debt (b, m), in order to maximize their expected profits by taking into account of the realized financial and productivity shocks. This motivates the first key assumption of the model:

Assumption 1. (Portfolio adjustment) The portfolio of assets (cash holdings versus capital) can be adjusted after the realization of productivity and financial shocks, but the portfolio of liabilities cannot be.

In other words, upon observing the productivity shock, the firm can either invest some or its cash on hand in capital for production later this period, or liquidate its capital—subject to an
adjustment cost (discussed below)—and retain within the firm as cash reserve and carry over to the next period. In other words, there must be some financing frictions in the debt markets at the asset reallocation stage. One implication of this assumption is that variations in cash holdings is larger than variations in leverage ratios. To test this conjecture, I use the Compustat dataset between 2006 and 2015 as described in Section 2, focusing on non-financial firms with non-trivial debt amounts (book leverage above 5%). I find that the coefficient of variation (standard deviation divided by the mean) for cash is consistently higher than the correlation of variation for debt, across all definitions of debt and all quartiles of firms, and the differences are significant at the 1% level.8

The firm then produces output using the re-optimized amount of capital \( \hat{k} \). After production, idiosyncratic demand shocks (\( \psi \)) are realized, which are shocks to the profit functions of the intermediate goods firms. At the debt settlement stage, the firm can either repay both types of debt, restructure bank debt, or default, in which case it exits endogenously. To capture the pattern that firms grow slowly because of the lack of internal funds, I impose an exogenous exit rate (see, for example, Khan and Thomas (2013)): Firms reach the end of their life cycle and exogenously exit the economy after debt settlement with probability \( 1 - \eta \). Firms that receive this exit signal leave the economy immediately and pay any remaining profits as dividends to the households. This assumption prevents that in the model all firms become financially unconstrained and also allows the model to exhibit a life cycle for firms. The exiting firms are replaced by new firms at the beginning of the next period, whose initial state will be discussed in Section 3.4. Finally, upon surviving the exit shock, firms choose the amount of capital \( k' \), cash holdings \( a' \), bank debt \( b' \), and market debt \( m' \) that they want to take into next period.

To streamline notation, I define the idiosyncratic state of a firm as \( s = [z, \hat{k}, x, \psi] \), including a firm’s idiosyncratic productivity (\( z \)), end-of-period capital (\( \hat{k} \)), net liquid asset position (\( x \), to be defined below), and idiosyncratic demand shock (\( \psi \)). Moreover, I define the aggregate state

8The detailed results are reported in Table B.1 of Appendix B.1. Market debt has the lowest coefficient of variation. Intuitively, this is because market debt cannot be renegotiated due to high bargaining costs; for example, it might be held by dispersed bondholders prone to coordination problems. As discussed below, although it is easier to renegotiate bank debt than market debt, bank debt also has a lower coefficient of variation than cash, due to, for example, screening and monitoring of borrowers in need of debt restructuring by banks.
of the economy as \( s = [\sigma, \gamma, \mu] \), where \( \mu \) is the distribution of the firms across the idiosyncratic state \( s \). Following the Arellano, Bai and Kehoe (2012), it is convenient to record the idiosyncratic demand shock in the beginning-of-period aggregate state, even though an individual firm’s \( \psi \) is not realized until the middle of the period. This approach is permissible, as there is a continuum of firms of each type \((z, k, x)\) at the beginning of the period, so the fraction of these firms that will experience each level of \( \psi \) is known. Moreover, the evolution of the firm distribution \( \mu \) is determined in part by the actions of continuing firms and in part by the potential entrants, as discussed in Section 3.5.

**Production** The intermediate goods firms produce the output \((y)\) using a production technology that has decreasing returns to capital \((\hat{k})\). Production is subject to idiosyncratic technology shock \((z)\), and also requires a payment of fixed operating costs that are proportional to firm size as measured by its existing capital stock, with the proportionality factor denoted by \( F_0 > 0 \). Formally, these assumptions are summarized by a production function:

\[
y = z \hat{k}^\alpha - F_0 \hat{k}; \quad 0 < \alpha < 1,
\]

where \( \alpha \) is the degree of decreasing returns in production. The idiosyncratic technology shock \( z \) evolves according to an \( N \)-state Markov chain with time-varying volatility. I assume a Markov chain with \( N \) states, and let \( p_{i,j} \) denote the transition probability of moving from state \( i \) in the current period to state \( j \) in the subsequent period. Importantly, the Markov chain of the idiosyncratic technology shock is constructed in such that: (1) its conditional mean is not affected by fluctuations in volatility; and (2) its conditional variance, however, is a linear function of the realization of the time-varying volatility process, given by:

\[
\log \sigma'_z = (1 - \rho) \log \sigma + \rho \sigma + \log \epsilon'_\sigma; \quad \log \epsilon'_\sigma \sim N(-0.5 \omega^2_{\sigma}, \omega^2_{\sigma}).
\]

Therefore, by construction, the aggregate shock to the uncertainty level does not alter the economy-wide mean productivity level, hence there is no aggregate technology shock. Moreover, a shock to the uncertainty level is an aggregate shock in that all firms have the same uncertainty level.

**Capital Accumulation** Capital adjustment is subject to a combination of convex and non-convex frictions, that are critical to generate a more realistic firm size distribution by inducing slow convergence to the optimal firm size implied by the decreasing returns to scale assumption. Formally, the total costs of capital adjustment for inter-period optimization is given by:

\[
g(k', \hat{k}) = F_{k_0} \hat{k} + \frac{F_{k_1,t}}{2} \left( \frac{k' - (1 - \delta) \hat{k}}{\hat{k}} \right)^2 \hat{k}
\]

where

\[
F_{k_1,t} = p_{k}^+ \times \Xi_{(k' - (1 - \delta) k) > 0} + p_{k}^- \times \left( 1 - \Xi_{(k' - (1 - \delta) k) < 0} \right),
\]

and \( 0 < \delta < 1 \) denotes the depreciation rate. \( \Xi_{(k' - (1 - \delta) k) < 0} \) is an indicator that equals one when the firm dis-invests, and \( 0 \leq p_{k}^- < p_{k}^+ \) captures the costly reversible investment framework of Abel and Eberly (1996).
I assume that firms face the same adjustment cost for intra-period optimization, which is given by:

\[ g(\hat{k}, k) = F_{k_0} k + \frac{F_{k_1,t}}{2} \left( \frac{\hat{k} - k}{k} \right)^2 k, \]  

(4)

where

\[ F_{k_1,t} \equiv \Xi_{(k-k)>0} + p_k \times \left( 1 - \Xi_{(k-k)<0} \right). \]

Findings by Cooper and Haltiwanger (2006) suggest that a model that mixes both convex and non-convex adjustment costs fits the data best. To that end, there are two components of capital adjustment frictions. First, the term \( F_{k,k} \) represents the fixed costs associated with capital expenditures—which are assumed to be proportional to the initial capital stock \( k \) to eliminate any size effect—capture the inherent indivisibility of physical capital and potential increasing returns to both the installation of new capital and restructuring of productive capacity during periods of intensive investment. The second term is a quadratic adjustment cost that is related to the rate of adjustment, such that the cost of investment is higher for more rapid changes. This term is responsible for smoothing investment over time. Moreover, I use asymmetric adjustment costs as in Zhang (2005), and Begenau and Salomao (2016). The irreversibility assumption \( p_k^+ > p_k^- \) implies that investment is more risky because firms cannot react to positive shocks without taking into account that a future negative shock can make it very expensive to become smaller. This assumption also means that firms may have to sit out several negative shocks without immediately choosing to downsize.

**Final Goods Firms** Final goods firms buy the products from intermediate goods firms, and produce the final good \( Y \) via the technology:

\[ Y = \left( \int \psi y(s)^{\frac{1}{\zeta}} \mu(ds) \right)^{\frac{1}{1-\zeta}}, \]

(5)

where \( y \) denotes the intermediate goods produced by a firm with idiosyncratic state \((z, \hat{k}, x, \psi)\), \( \zeta > 1 \) is the elasticity of substitution across goods, and \( \psi \) is the idiosyncratic demand shock, that follows a continuous Markov process:

\[ \log \psi' = \rho_\psi \log \psi + \log \epsilon'_\psi; \quad \log \epsilon'_\psi \sim N(-0.5\sigma^2_\psi, \sigma^2_\psi), \]

(6)

and the distribution is independent of that of the idiosyncratic productivity shock. The final goods firms choose the intermediate goods to solve:

\[ \max_{y(s)} Y - \left( \int \psi y(s)^{\frac{1}{\zeta}} \mu(ds) \right)^{\frac{1}{1-\zeta}}, \]

(7)

\(^9\)In the model, the final goods producer has no value added, and hence this producer is a simple device to aggregate the output of the heterogeneous firms—referred to as intermediate goods firms—into a single value. Equivalently, one can think of these heterogeneous firms as final goods producers, and equation (5) reflects agents’ preferences over these final goods.
subject to (5). This yields the demand $y(s)$ for any good with idiosyncratic state $s = [z, \hat{k}, x, \psi]$: 

$$y(s) = \left( \frac{\psi}{p(s)} \right)^\xi Y,$$  

(8)

with $Y(s) = \left( \int \psi y(s) \frac{\xi - 1}{\mu(ds)} \right)^{\frac{1}{\xi - 1}}$, and $p(s)$ is the price of the good, which is determined after the demand shock $\psi$ is realized. Next I turn to the details of the problem faced by an intermediate goods firm, including the prices of debt ($q^b, q^m$) and the firm’s optimization problem.

### 3.2 Debt Settlement and Pricing

To finance investment projects, firms use a combination of internal and external funds, where the sources of internal funds are operating income and cash holdings ($a_f$), whilst external funds consist of bank debt ($b$) and market debt ($m$). Relative to internal finance, debt finance commands a premium because of the agency costs associated with default. The debt contracts specify the par values of issues ($b', m'$) and the prices ($q^b, q^m$), yielding the total amount of debt financing to the sum of $q^b b'$ and $q^m m'$ in each period. By combining the proceeds from debt issuance with other sources of funds, the firm purchases capital ($k'$) to be used in production, or accumulates safe assets ($a'_f$) to gain financial flexibility. In the subsequent period after observing the realization of shocks, the firm decides whether to fulfill its debt obligations.

**Debt settlement outcomes**  The firm has three options at the debt settlement stage: full repayment, debt restructuring, or liquidation. If the firm fully repays its creditors, it pays the face values of the debt $b'$ and $m'$ to the bank lender and market lender, respectively. If it chooses to restructure its debt, it enters a debt-renegotiation process with the bank lenders. If it defaults, the firm is liquidated and its resources are passed onto creditors, subject to a deadweight loss. The decision chosen by the firm depends on its net worth, after the realization of the demand shock:

$$n' = p'(\psi') y' + p_k^- (1 - \delta) \hat{k}' - F_o \hat{k}' + \hat{a}' - b' - m'$$  

(9)

where $\pi' = p'(\psi') y' + p_k^- (1 - \delta) \hat{k}' - F_o \hat{k}' + \hat{a}'$ represents the sum of the firm’s profit from sales, the market value of undepreciated capital, and the return from savings. Note that the value of capital in place is evaluated at the resale value $p_k^-$, rather than its book value $p_k^+$.

The firm can only fully repay its liabilities if the price of the good $p(\psi)$ is high enough such that its resources exceed its total liabilities, i.e. $\pi' \geq b' + m'$. Otherwise it can renegotiate with its lenders to restructure its debt in order to avoid liquidation. The crucial distinction between banks and market lenders lies with their ability to participate in the renegotiation process, which constitutes a key assumption:

**Assumption 2.** (Debt flexibility)  *Only bank debt can be restructured; market debt cannot.*
Bolton and Scharfstein (1996) provide a microfoundation to this assumption, by noting that as the ownership of market debt tends to be more dispersed than ownership of bank debt, market creditors face a free-rider problem and have little individual incentive to participate in debt renegotiations. In addition, as banks build a closer relationship with firms than dispersed investors, they have an informational advantage by assessing and monitoring information about firms—as noted by Rajan (1992); Boot, Greenbaum and Thakor (1993); Chemmanur and Fulghieri (1994)—and hence they are more precisely aware of the going concern value of the firm and can offer greater contractual flexibility.\(^\text{10}\)

In modelling the restructuring process on the liability-side, I follow Crouzet (2015) on two fronts. First, the restructuring process is modelled as a two-stage Nash bargaining game between the firm and the bank: the firm first makes an offer \(b_R'\) to the bank, which is a new amount of repayment instead of the promised amount \(b'\); the bank can choose to accept or reject the offer, and in case the offer is rejected, liquidation occurs. Second, I assume that bank lenders are more senior than market lenders in the priority structure, as empirical evidence documented by Rauh and Sufi (2010) shows that bank debt tends to be placed on top of firms’ priority structures or secured against assets.\(^\text{11}\) This implies that the firm can either default completely – such that it cannot pay either lender – or in part, whereby it repays the more senior bank lender but defaults on the less senior market lender. As a result of these two assumptions, Crouzet (2015) shows that there are two sets of possible equilibria, such that restructuring may occur in one (R-contract) and never occurs in the other (NR-contract), and the latter arises when the stake of the flexible creditors, \(b'_R\), is too small for restructuring to bring about sufficient gains for the firm to avoid default on market debt. In the case of a NR-contract, no restructuring ever occurs, and bankruptcy losses cannot be avoided when the firm’s operating profits \(\pi'\) falls below the threshold at which the firm prefers declaring bankruptcy over repayment \(b' + m'\). In the case of an R-contract, the flexibility of bank debt sometimes allows the firm to make good on its payments on market debt, and restructuring can be the best option for the firm.\(^\text{12}\)

Since restructuring does not always save the firm from liquidation if its realized net worth \(n'\) is sufficiently low, lenders charge the firm a liquidation risk premium in equilibrium. This arises because of the key assumption that the transfer of the firm’s resources to creditors involves a deadweight loss:

\(^{10}\)There is also substantial support in the data for the assumption that banks are more flexible in distress than markets. Gilson, Kose and Lang (1990) show that firms are more likely to restructure their debt privately if they owe more of their debt to banks; Denis and Milhov (2003), using a sample of 1560 new debt financings, show that firms with lower credit quality tend to borrow from banks, as bank debt offers greater flexibility of renegotiation in default. The assumption of bank flexibility maintained in this model thus captures the consequences of differences in creditor concentration between classes of debt, for firms’ ability to successfully restructure debt contracts.

\(^{11}\)In other closely related papers, such as Hackbarth, Hennessy and Leland (2007), bank debt seniority is the optimal priority structure from the perspective of the firm. Moreover, in other models of debt structure where the role of banks is to provide ex-ante monitoring of firms’ projects, the optimality of bank seniority is also a feature of other models of debt structure (e.g. Besanko and Kanatas (1996); De Marzo and Fishman (2007)), the optimality of bank debt is also an important feature as it increases the return of banks to monitoring.

\(^{12}\)See Appendix C for the debt settlement outcomes in each type of contracts, including the definition of \(b'_R\).
**Assumption 3.** (Deadweight loss in default) The liquidation value of the firm is given by $\chi \pi'$, where $0 \leq \chi < 1$.

This is a common assumption in many models in which the underlying financial friction is limited liability. As in Townsend (1979), the bankruptcy costs reflect a loss of resources expanded by creditors to prevent managers of a defaulting firm from behaving opportunistically.

Financial intermediaries receive deposits from households and firms, and use them to extend credit to firms. Asset prices are forward-looking and each lender faces perfect competition, so their expected total profits are driven down to zero in each period. Assuming that lenders cannot cross-subsidize firms, lenders must earn zero profit in expectation on each lending. Both bank lenders and market lenders face identical cost of deposits $q^a(s)$, but different intermediation costs.  

**Assumption 4.** (Financial intermediation costs) The cost of intermediation per unit of lending is $\gamma^b$ for bank lenders, and $\gamma^m$ for market lenders. Define the wedge between the intermediation costs as $\gamma^* = \gamma^b - \gamma^m$, and $\gamma^*$ follows a continuous Markov process:

$$\log \gamma^* = (1 - \rho_{\gamma}) \log \bar{\gamma}^* + \rho_{\gamma} \log \gamma^* + \epsilon_{\gamma}; \quad \log \epsilon_{\gamma} \sim N(-0.5\sigma_{\gamma}^2, \sigma_{\gamma}^2),$$

so the wedge between bank- and market-specific intermediation costs is always strictly positive: $\gamma^b > \gamma^m$.

The lending wedge serves two purposes. First, it is key to motivate a trade-off between bank debt and market debt in the model: whilst firms with higher default risks find the option of-fered by banks to renegotiate more valuable, this type of credit is also associated with a higher marginal cost than market debt, due to the more costly bank-specific activities such as screening and borrowing, as banks spend resources to acquire information and arrange financing accordingly (see, for example, Houston and James (1996), or Mester, Nakamura and Renault (2007)).

Second, it is a proxy for the relative supply of bank credit in this economy.

---

13Since the financial intermediaries are perfectly competitive, they earn zero profit from total lending in expectations, i.e.:

$$\int (q^b(b, \hat{k}, \hat{a}_f, z_{-1}, \psi_{-1}; s) + \gamma^b)b' \mu(dz, d\psi, dk, dx) = q^a(s) a_b(s),$$

and

$$\int (q^m(b, \hat{k}, \hat{a}_f, z_{-1}, \psi_{-1}; s) + \gamma^m)m' \mu(dz, d\psi, dk, dx) = q^a(s) a_m(s).$$

Assuming that lenders do not cross-subsidize firms, they also earn zero profit in expectation from lending to each firm, i.e.:

$$(q^b(b, \hat{k}, \hat{a}_f, z_{-1}, \psi_{-1}; s) + \gamma^b)b' = q^a(s)b',$$

and

$$(q^m(b, \hat{k}, \hat{a}_f, z_{-1}, \psi_{-1}; s) + \gamma^m)m' = q^a(s)m'.$$

---

14Financial intermediation costs consist of all non-interest costs that intermediaries undertake to operate. The assumption that financial intermediation is costly is not controversial. Philippon (2015) provides recent and comprehensive evidence that overall intermediation costs in the U.S. financial sector have averaged approximately 2% between 1870 and 2012.
Debt pricing under NR-contract If \( \frac{m'}{1 - \chi} > \frac{b'}{\chi} \), the firm repays its liabilities in full if \( \pi' \geq b' + m' \); partially defaults (i.e. repays the more senior bank debt but defaults on market debt) if \( \frac{b'}{\chi} \leq \pi' < b' + m' \); and defaults on both types of debt if \( \pi' < \frac{b'}{\chi} \). Hence one can define a pair of thresholds for the demand shock \((\psi''_{NR}, \tilde{\psi}'_{NR})\)—conditional on tomorrow’s aggregate state \(s'\) and the individual state \((k', b'_R, m', a'_f, z', \psi')\)—that are the inverse functions of \((\tilde{p}'_{NR}, \bar{p}'_{NR})\), such that a firm defaults fully in the next period if \( \psi' < \bar{\psi}'_{NR} \), and defaults partially if \( \frac{\psi'}{\psi''_{NR}} \leq \psi' < \bar{\psi}'_{NR} \):

\[
\begin{align*}
\bar{\psi}'_{NR} &= \bar{p}'_{NR}(y) \\
\frac{\psi'}{\psi''_{NR}} &= \tilde{p}'_{NR}(y),
\end{align*}
\]

where

\[
\begin{align*}
\bar{p}'_{NR}(b', m', k', a'_f, z'_j(\sigma)) &= \frac{b' + m' + F_0k' - p^{-1}(1 - \delta)k' - \hat{a}'_f}{z'_j(\sigma)k'^{\alpha}}, \\
\tilde{p}'_{NR}(b', k', a'_f, z'_j(\sigma)) &= \frac{b' + F_0k' - p^{-1}(1 - \delta)k' - \hat{a}'_f}{z'_j(\sigma)k'^{\alpha}}.
\end{align*}
\]

The payoffs to the bank and market lender are \( \bar{\mathcal{R}}'_b, NR \) and \( \bar{\mathcal{R}}'_m, NR \), respectively, such that:

\[
\begin{align*}
\bar{\mathcal{R}}'_b, NR &= \begin{cases} 
 b' & \text{if } \psi' \geq \frac{\psi'}{\psi''_{NR}} \\
\chi \pi' & \text{if } \psi' < \frac{\psi'}{\psi''_{NR}}
\end{cases},
\end{align*}
\]

and

\[
\begin{align*}
\bar{\mathcal{R}}'_m, NR &= \begin{cases} 
 m' & \text{if } \psi' \geq \frac{\psi'}{\psi''_{NR}} \\
\chi \pi' - b' & \text{if } \frac{\psi'}{\psi''_{NR}} \leq \psi' < \frac{\psi'}{\psi''_{NR}} \\
0 & \text{if } \psi' \leq \frac{\psi'}{\psi''_{NR}}
\end{cases}.
\end{align*}
\]

Hence the price of debt on each lending is a weighted average of the discounted returns in default and non-default states tomorrow, minus the cost of intermediation today, thus implying the following debt pricing formulae in a NR-contract, for bank debt \( q_{NR}^b(b', k', a'_f, z_i, \psi; s) \) and market debt \( q_{NR}^m(b', m', k', a'_f, z_i, \psi; s) \), respectively:

\[
\begin{align*}
\tilde{q}_{NR}^b(k', b', a'_f, z_i, \psi; s) + \gamma^b &= E\left\{ \lambda(s, s') \left[ 1 + \sum_{j \in \mathcal{D}_{NR}} p_{i,j} \left[ \frac{\chi \pi'}{b'} - 1 \right] \right] s \right\}, \\
\tilde{q}_{NR}^m(k', b', m', a'_f, z_i, \psi; s) + \gamma^m &= E\left\{ \lambda(s, s') \left[ 1 + \sum_{j \in \mathcal{D}_{NR}} p_{i,j} \left[ \frac{\chi \pi'}{m'} - 1 \right] + \sum_{j \in \mathcal{D}_{NR}} p_{i,j} \left[ -1 \right] \right] s \right\},
\end{align*}
\]

where \( \gamma^b \) and \( \gamma^m \) are the costs of intermediation, \( \lambda(s, s') \) is the stochastic discount factor of the households, and

\[
\begin{align*}
\mathcal{D}_{NR}' &= \left\{ j \mid j \in 1, \ldots, N \text{ and } \psi'_{NR}(k', b'_R, m', a'_f, z'_j(\sigma), \psi; s) \leq \psi' < \tilde{\psi}'_{NR}(k', b'_R, m', a'_f, z'_j(\sigma), \psi; s) \right\} \\
\mathcal{D}_{NR}' &= \left\{ j \mid j \in 1, \ldots, N \text{ and } \psi' < \psi'_{NR}(k', b'_R, m', a'_f, z'_j(\sigma), \psi; s) \right\}
\end{align*}
\]
are, respectively, the sets of states of the demand shocks \( \psi' \), in which the firm will default on market debt only and on both types of debt, with \( \bar{\psi}'_{NR} \) and \( \underline{\psi}'_{NR} \) defined in (11).

**Debt pricing under R-contract** If \( \frac{m'}{b'} \leq \frac{\psi'}{\bar{\psi}'_{R'}} \), one can also define a pair of thresholds for demand \((\bar{\psi}'_{R'}, \underline{\psi}'_{R'})\), that are the inverse functions of \((\bar{\psi}'_{R'}, \underline{\psi}'_{R'})\), such that the firm repays its liabilities in full if \( \psi' \geq \bar{\psi}'_{R'} \) restructures its bank debt while repaying its market debt if \( \underline{\psi}'_{R'} \leq \psi' < \bar{\psi}'_{R'} \) and defaults if \( \psi' < \underline{\psi}'_{R'} \):

\[
\begin{align*}
\bar{\psi}'_{R} &= \bar{\psi}'_{R}^{-1}(y) \\
\underline{\psi}'_{R} &= \underline{\psi}'_{R}^{-1}(y),
\end{align*}
\]

where

\[
\begin{align*}
\bar{\psi}'_{R}(b', \hat{k}', \hat{a}'_f, z'_j(\sigma)) &= \frac{b'}{\hat{x}} + F_x \hat{k}' - p^- (1 - \delta) \hat{k}' - \hat{a}'_f, \\
\underline{\psi}'_{R}(m', \hat{k}', \hat{a}'_f, z'_j(\sigma)) &= \frac{m'}{\hat{x}} + F_x \hat{k}' - p^- (1 - \delta) \hat{k}' - \hat{a}'_f.
\end{align*}
\]

The payoffs to the bank and market lender in an R-contract are \( \bar{R}'_{b,R} \) and \( \bar{R}'_{m,R} \), respectively, such that:

\[
\bar{R}'_{b,R} = \begin{cases} 
  b' & \text{if } \psi' \geq \bar{\psi}'_{R} \\
  \chi \pi' & \text{if } \psi' < \bar{\psi}'_{R},
\end{cases}
\]

and

\[
\bar{R}'_{m,R} = \begin{cases} 
  m' & \text{if } \psi' \geq \underline{\psi}'_{R} \\
  0 & \text{if } \psi' < \underline{\psi}'_{R}.
\end{cases}
\]

Hence, the debt pricing formulae in an R-contract, for bank debt \((q'^{b}_{R}(\hat{k}', b', \hat{a}'_f, z_i, \psi; s))\) and market debt \((q'^{m}_{R}(\hat{k}', m', \hat{a}'_f, z_i, \psi; s))\), respectively:

\[
q'^{b}_{R}(\hat{k}', b', \hat{a}'_f, z_i, \psi; s) + \gamma^b = E \left\{ \lambda(s, s') \left[ 1 + \sum_{j \in \mathcal{D}'_{R}} p_{i,j} \left[ \frac{\chi \pi'}{b'} - 1 \right] \right] s \right\}
\]

and

\[
q'^{m}_{R}(\hat{k}', m', \hat{a}'_f, z_i, \psi; s) + \gamma^m = E \left\{ \lambda(s, s') \left[ 1 + \sum_{j \in \mathcal{D}'_{R}} p_{i,j} \left[ -1 \right] \right] s \right\}
\]

where

\[
\begin{align*}
\mathcal{D}'_{R} &= \left\{ j \mid j \in 1, \ldots, N \text{ and } \psi'_R(\hat{k}', m', \hat{a}'_f, z'_j(\sigma), \psi; s) \leq \psi'_j(\sigma) < \bar{\psi}'_{R}(\hat{k}', b', \hat{a}'_f, z'_j(\sigma), \psi; s) \right\} \\
\mathcal{D}'_{R} &= \left\{ j \mid j \in 1, \ldots, N \text{ and } \psi'_j(\sigma) < \underline{\psi}'_{R}(\hat{k}', m', \hat{a}'_f, z'_j(\sigma), \psi; s) \right\}.
\end{align*}
\]

are, respectively, the sets of the idiosyncratic demand shock \( \psi' \), in which the firm will restructure and default on their debt, with \( \bar{\psi}'_{R} \) and \( \underline{\psi}'_{R} \) defined in (16).
3.3 Optimization of the Intermediate Goods’ Firms

At the end of period $t$, if the firm has survived the exogenous exit shock $1 - \eta$, it chooses the optimal cash ($a_f'$) and investment policies ($k'$), as well as the amount of bank debt ($b'$) and market debt ($m'$) for period $t + 1$. I formulate the firm’s profit maximization problem recursively in this section, starting with the definition of dividend, after debt settlement:

$$d_t = \begin{cases} 
    p(\psi)y(z_{i(\sigma-1)}) - vg(k', \hat{k}) - b - m + \hat{a}_f - q^a a_f' + q^b b' + q^m m', & \text{if firm repays both } b \text{ and } m \\
    p(\psi)y(z_{i(\sigma-1)}) - vg(k', \hat{k}) - b_R - m + \hat{a}_f - q^a a_f' + q^b b' + q^m m', & \text{if firm restructures } b \text{ and repays } m 
\end{cases}$$  \hspace{1cm} (21)

where the subscript $l \in \{NR, R\}$ denotes whether the firm chooses a NR-contract or an R-contract for the next period, which has implications for the prices of debt, as shown in the previous section, and $b_R$ is the restructured amount of bank debt. $y$ and $p(\psi)$ are defined in equations (1) and (8), and $v \in \{0, 1\}$ is the choice variable indicating whether the firm is in the investment inaction ($v = 0$) or action ($v = 1$) regime. Moreover, firms can save at the risk-free rate $q^a$:

$$q^a(s) = E[\lambda(s, s') \mid s].$$  \hspace{1cm} (22)

As there is no tax advantage to debt, in order to motivate firms to take on debt, I posit that firms face a non-negative dividend constraint as in Khan and Thomas (2013):\(^{15}\)

$$d \geq 0.$$  \hspace{1cm} (23)

I define a composite state variable, the net liquid asset position of the firm ($x$):

$$x \equiv \begin{cases} 
    p(\psi)y(z_{i(\sigma-1)}) - F_0 \hat{k} - b - m + \hat{a}_f, & \text{if firm repays both } b \text{ and } m \\
    p(\psi)y(z_{i(\sigma-1)}) - F_0 \hat{k} - b_R - m + \hat{a}_f, & \text{if firm restructures } b \text{ and repays } m 
\end{cases}$$  \hspace{1cm} (24)

so the firm’s dividend (21) can be rewritten as: $d = x - vg(k', \hat{k}) + q^b b' + q^m m' - q^a a_f'$. The firm’s problem can be formulated recursively backwards within each period. As noted in the timeline (Figure 3), let $V_i^1(\hat{k}, x; s)$ denote the value function of the firm at the dividend issuance stage, $V_i^0(k, x; s)$ denote the value function of at the debt settlement stage, and $\hat{V}_i^0(\hat{k}, x; s)$ denote the value function of at the asset reallocation stage. The subscript $i$ denotes the firm’s relative position in the discrete distribution of the idiosyncratic technology level $z$ in the current period.

Asset reallocation stage  
Upon observing the productivity and financial shocks, and given the amount and composition of debt ($b, m$), the firm choose whether or not to reallocate their assets

\(^{15}\)An alternative way to induce firms to be exposed to debt is a working capital requirement. Introducing this requirement to the model does not significantly alter the implications of the model, as shown in Appendix D.3. Nonetheless, a working capital requirement only affects the constrained firms that tend to be small. In a framework with intra-temporal and inter-temporal debt, a working capital requirements can have more significant effects on the economy (e.g. Jermann and Quadrini (2012b)).
(k, a_f), either by purchasing more capital at price p^+ using the cash on hand upon observing a favorable shock, or by liquidating some of its capital at price p^- when a negative shock is realized. Therefore, it solves the following problem at the asset reallocation stage:

\[ \hat{V}^0(k, a_f, \psi, z; s) = \max_{\hat{k}, \hat{a}_f} V^0(\hat{k}, x; s) \]  

subject to:  
\[ \hat{k} + \hat{a}_f + g(\hat{k}, k) \leq k + a_f, \]  

where x is defined in (24) and g(\hat{k}, k) is defined in (4). The constraint implies that the firm cannot issue additional debt during the asset allocation stage. If the firm chooses not to reallocate its assets, it proceeds to production and subsequently debt settlement with value function \( V^0(\hat{k}, x; s) \), with \( \hat{k} = k \), and \( \hat{a}_f = a_f \) in the net liquid asset position x.

**Debt settlement stage**  
At the debt settlement stage, the firm has three options: liquidation, restructuring, and full payment of its liabilities. Let \( V^0_P \) and \( V^0_R \) denote, respectively, the value function of a firm that repays and restructures its liabilities today. By assumption, firm that liquidates exits with its resources being passed to its creditors, so its continuation value in liquidation is 0. Therefore, the firm solves the following discrete choice problem at the debt settlement stage:

\[ V^0(\hat{k}, x; s) = \max \{ V^0_P(\hat{k}, x; s), V^0_R(\hat{k}, x_R; s), 0 \}, \]  

The firm knows that with probability \( 1 - \eta \) that it is not going to survive until the next period and with probability \( \eta \) it survives and has value \( V^1 \) (defined below). Thus, today’s value of the firm—depending on if the firm repays or restructures its liabilities—is either:

\[ V^0_P(\hat{k}, x; s) = (1 - \eta)n + \eta V^1(\hat{k}, x; s), \]  

or

\[ V^0_R(\hat{k}, x_R; s) = (1 - \eta)n_R + \eta V^1(\hat{k}, x; s), \]  

where \( n \) is the realized net worth defined in (9), and \( n_R \) is

\[ n_R = p(\psi)z_i(\sigma - 1)\hat{k}^\alpha - F_o\hat{k} + p^- (1 - \delta)\hat{k} - b_R - m + \hat{a}_f \]

\[ = \pi - b_R - m, \]

where the restructured amount is \( b_R = \chi \pi' \) (see Appendix C).

**Dividend issuance stage**  
Firms that do not default in period t and survive can choose between a NR-contract and an R-contract, with value functions \( V^1_{i,NR} \) and \( V^1_{i,R} \) respectively:

\[ V^1(\hat{k}, x; s) = \max \{ V^1_{i,NR}(\hat{k}, x; s), V^1_{i,R}(\hat{k}, x; s) \}. \]
The optimization problem for the firm that chooses a NR-contract \( \left( \frac{b'}{\chi} < \frac{m'}{1-\chi} \right) \) takes the following form:

\[
V^R_{i,NR}(\hat{k}, x; s) = \max_{v, k', b', m', a'_f} \left\{ d_{NR} + E \left[ \lambda(s, s') \sum_{j=1}^{N} p_{i,j} V^0(\hat{k}', a'_f, \psi', z'; s') \right] | s \right\}
\]  

subject to (1), (3), (13), (14), (21), (22), (23), (26), (28),

and \( s' = \Gamma(s); \ i, j = 1, 2, ..., N, \)

where \( s' = \Gamma(s) \) is the law of motion governing the evolution of the aggregate state vector, which I describe below. For a firm that chooses an R-contract \( \left( \frac{b'}{\chi} \geq \frac{m'}{1-\chi} \right) \), the Bellman equation becomes:

\[
V^R_{i,R}(\hat{k}, x; s) = \max_{v, k', b', m', a'_f} \left\{ d_{R} + E \left[ \lambda(s, s') \sum_{j=1}^{N} p_{i,j} V^0(\hat{k}', a'_f, \psi', z'; s') \right] | s \right\}
\]  

subject to (1), (3), (18), (19), (21), (22), (23), (26), (28), (29),

and \( s' = \Gamma(s); \ i, j = 1, 2, ..., N. \)

The set of state variables is compact because \( k, z \) and \( \psi \) are bounded, and from equation (24), it is straightforward to see that the net liquid asset position \( x \) lies in a closed and bounded interval \([x, \bar{x}]\). The continuation value of the firm is bounded below at zero—the value of the firm upon its default and exit—due to its limited liability.

Continuing firms do not pay dividends to households, unless they assign a zero probability to a binding dividend constraint in the future (Khan and Thomas (2013)).\(^\dagger\) The intuition is as follows. For firms that borrow to finance investment (either \( b' > 0 \) or \( m' > 0 \), or both), since the price of debt is less (or equal to) the stochastic discount factor of firms, debt is on average costly and thus firms are better off by paying back their debt. Moreover, for firms that do not borrow and only save \( (a'_f > 0) \), as firms and households share the same stochastic discount factor, firms are at most indifferent between paying dividends and saving. Firms want to avoid to be in the situation in the future that their dividend constraint might be binding and thus want to save for precautionary reasons and pay zero dividends.

\[^\dagger\]See Appendix D for a formal proof.
3.4 Firm Entry and Exit

**Exit** There are two sources of firm exit in this economy. First, some firms are endogenously liquidated at the debt settlement stage. Denote the fraction of exiting firms by \( F(\psi(\hat{k}', b, m', \hat{a}', z', \psi; s) - 1) \), where \( \psi(\hat{k}', b, m', \hat{a}', z', \psi; s) \) is the default threshold of the demand shock.\(^\text{17}\) Second, a fraction \( 1 - \eta \) of firms are exogenously destroyed after production debt settlement. Firms that receive the exogenous exit signal leave the economy immediately after paying back their debt and pay any remaining profits as dividends to the households. Let \( \mu(dz, d\psi, dk, dx) \) denote the joint distribution of the idiosyncratic technology, demand, capital, and net liquid asset positions across heterogeneous firms at the beginning of period \( t \), and \( \delta^e(\mu(s)) \) denote the total mass of firms exiting during period \( t \), which is given by:

\[
\delta^e(\mu(s)) \equiv \int \left( F(\psi(\hat{k}, b, m, \hat{a}, z, \psi - 1; s - 1)) + \eta \left( 1 - F(\psi(\hat{k}, b, m, \hat{a}, z, \psi - 1; s - 1)) \right) \right) \mu(dz, d\psi, dk, dx).
\]

\(^{33}\)

**Entry** The entry decision in this model amounts to the decision of a firm to go public. The set-up of the potential entrant’s problem is similar to models of firm dynamics with endogenous entry, such as Clementi and Palazzo (2016); Begenau and Salomao (2016); and Clementi, Khan, Palazzo and Thomas (2015). The timing of decisions for potential entrants is illustrated in Figure 4. At the beginning of each period, there is a constant mass \( M > 0 \) of potential entrants. Potential entrants first observe aggregate shocks in the current period \( \{\sigma, \gamma^*\} \). Then each potential firm draws a productivity signal \( q \) that follows a Pareto distribution \( q \sim Q(q) \). More specifically, I posit that \( q \geq 0 \) and that \( Q(q) = (q/q)^\omega, \omega > 1 \). Each potential entrant chooses whether to pay a fixed entry cost \( c_e > 0 \), which ensures that not all firms find it optimal to go public. Consequently it helps to pin down the size distribution of the entering firms.

The entrant only starts operating next period but must decide today with which capital stock

---

\(^{17}\)For an individual firm, the threshold depends on whether the firm employs a NR-contract or an R-contract, whereby \( \psi = \psi_{NR} \) with \( \psi_{NR} \) defined in (11) denoting the threshold such that firms employing NR-contracts and with a productivity \( \psi < \psi_{NR} \) default—either fully or partially—and are liquidated. For a firm employing an R-contract, the default threshold is given by \( \psi = \psi_R \) with \( \psi_R \) defined in (16), such that firms with productivity \( \psi < \psi_R \) defaults.

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it wants to start production tomorrow, conditional on having paid the fixed entry cost \( c_e \). The initial investment can only be financed with equity, (and in the baseline version of the model, entry is the only occasion on which a firm can issue equity). The realization of the idiosyncratic productivity shock and demand shock in the first period of operation depends on the signal \( q \) today and thus follows these processes, respectively:

\[
\log z = \rho_z \log q + \log \epsilon_z; \quad \log \epsilon_z \sim N(-0.5\sigma_z^2, \sigma_z^2),
\]

(34)

and

\[
\log \psi = \rho_\psi \log q + \log \epsilon_\psi; \quad \log \epsilon_\psi \sim N(-0.5\sigma_\psi^2, \sigma_\psi^2),
\]

(35)

and both shocks are independently distributed of each other. Therefore, the value of an entrant can be written as:

\[
V_e(q; s) = \max_{k'_e} \left\{-\gamma_e k'_e + E \left[ \lambda(s, s') \sum_{j=1}^N p_{i,j} V_j^1(z', k'_e, x'_e; s') | s \right] \right\}
\]

(36)

subject to (24), (30), (34), (35) and \( s' = \Gamma(s) \); \( i, j = 1, 2, ..., N \),

where \( \gamma_e \) is the initial cost of issuing equity. Note that \( x'_e = p'(\psi')y' \), as the entrant firm does not hold any financial asset \( (a_f = 0) \), or debt \( (b = 0, m = 0) \). Each potential entrant compares the value of entering \( V_e \) with the cost of entering \( c_e \), after receiving signal \( q \) about its future productivity. Therefore, it will choose to incur the fixed entry cost, and start operating, if and only if:

\[
V_e(q; s) \geq c_e.
\]

Note that \( V \) is weakly increasing in the idiosyncratic level of productivity \( z \), as well as the idiosyncratic level of demand \( \psi \). In other words, a higher signal \( q \) means that the productivity realization \( z \) and the demand realization \( \psi \) are likely to be high. This in turn implies that the conditional distributions of \( z' \) and \( \psi' \) are (independently) decreasing in \( q \). Thus there exists a threshold \( q^* \) such that:

\[
V_e(q^*; s) = c_e.
\]

(37)

If \( q \geq q^* \), the potential entrant is going to enter, and does not enter otherwise. This entry process repeats every period. The entry decision occurs at the end of each period, after financial contracts between existing firms and intermediaries have been settled. Finally, in the next period, the problem of the entrants is identical to the problem of an incumbent firm.

### 3.5 Market Clearing and Aggregation

This section closes the model by specifying conditions required to clear the goods and financial markets. I begin with the problem of the representative household, who solves a standard consumption-savings problem:

\[
W(a_h; s) = \max_{c, a_h'} \left\{ u(c) + \beta E \left[ W(a'_h; s'| s) \right] \right\},
\]

(38)
subject to the budget constraint:

\[ c + q^a a_h + \int c_a \mu_e (ds) \leq \int a_h + [d + F_o k] \mu (ds), \]  

(39)

where \( ds = [dz, d\psi, d\hat{k}, dx] \), where \( s \) summarizes the idiosyncratic state of a firm \( s = [z, \psi, \hat{k}, x] \).

The period-specific utility function \( u(c) \) is assumed to be strictly increasing and strictly concave in consumption \((c)\). To maintain tractability, I assume a simple functional form:

\[ u(c) = \log(c). \]

The household’s intertemporal decisions are determined by the stochastic discount factor,

\[ \lambda(s, s') = \frac{u'(c'(s'))}{u'(c(s))}, \]

where \( u'(\cdot) \) is the marginal utility of consumption.

The budget constraint (39) shows that the household enters the period saving \( a_h \), which is allocated among the financial intermediaries at the end of the previous period together with firms’ savings \( a_f \), and earns a risk-free return \( q^a(s) \). The household is also the owner of firms and the financial intermediaries. It takes the amount of dividends \( d \), the investment in new firms \( c_e \)—which is determined by the condition (37)—as given. Note that the fixed costs of operation are rebated to the household in a lump-sum fashion, hence these costs do not affect the economy-wide resource constraint.

The goods market clearing condition can be expressed as:

\[ c(s) = Y(s) - \int v(s; s) \left[ g(k'(s; s), \hat{k}) + g(\hat{k}; s, k) \right] \mu (ds) - \gamma^b \int b'(s; s) \mu (ds) - \gamma^m \int m'(s; s) \mu (ds) - \int c_e \mu_e (ds) - \int 1_{\psi \leq \bar{\psi}} \times (1 - \chi) y(s, s) \mu (ds), \]  

(40)

where the last term captures the deadweight loss of default; in other words, aggregate consumption plus capital adjustment cost (from both inter- and intra-period optimization), intermediation costs, investment in new firms, and bankruptcy cost equal aggregate output. The deposit market clears at the end of period \( t \) by:

\[ a'_h(s)_{\text{households' savings}} + \int a'_f(s; s)_{\text{firms' savings}} \mu (ds) = a'_b(s) + a'_m(s), \]

(41)

where the left-hand side of (41) is the total deposits collected from household and firms, and the right-hand side is the allocation of deposits to the financial intermediaries, whereby the bank lenders receive \( a'_b \), and the market lenders receive \( a'_m \). The allocation of deposits are determined by the total demands for bank debt and market debt from the firms. The market clearing conditions for bank debt and market debt are, respectively:

\[ \int b'(s; s) \mu (ds) = a'_b(s), \]  

(42)

and

\[ \int m'(s; s) \mu (ds) = a'_m(s), \]  

(43)

where the left-hand side of each condition denotes the total demand for each type of debt from all firms. The recursive equilibrium in this economy can be defined as follows.
Definition 1. (Recursive competitive equilibrium) A recursive competitive equilibrium in this economy is given by:

- policy functions \( C(a_h; s) \) and \( A_h(a_h; s) \), and value function \( W(a_h; s) \) for the representative household;
- policy functions \( B(z, \hat{k}, x, \psi; s) \), \( M(z, \hat{k}, x, \psi; s) \), \( A_f(z, \hat{k}, x, \psi; s) \), \( \hat{A}_f(z, k, a_f, \psi_{-1}; s) \), and \( \hat{K}(z, k, a_f, \psi_{-1}; s) \), and value function \( V^1(z, k, x; s) \) for the incumbent firm;
- prices \( q^m, q^b, \) and \( q^a \);
- an entry scale \( k^e \);
- a measure of incumbent firms \( \mu \);
- a measure of entrants \( \mu_e \);
- a transition mapping for the distribution of firms \( \Gamma \);

such that:

1. the policy functions and value function of the household solve its optimization problem (38) subject to the budget constraint, taking \( q^m, q^b, q^a, \Gamma \) as given;
2. the policy functions and value function of the incumbent firm solve its optimization problem (30), taking \( q^m, q^b, q^a, \Gamma \) as given;
3. the financial intermediaries (bank lenders and market lenders) make expected zero profits for each firm, and determine the optimal asset prices \( q^m, q^b, q^a \) using (13), (18), (14), (19), and (22), taking the household discount factor as given;
4. goods market clearing condition (40) is satisfied;
5. deposits market clearing condition (41) is satisfied;
6. the market clearing conditions for bank debt (42) and market debt (43) are satisfied;
7. the entry scale \( k^e \) and measure of entrants \( \mu_e \) satisfy (36) and (37);
8. the evolution of the distribution of firms follows:

\[
\mu' = \Gamma(\sigma, \gamma^*, \mu, \mu_e)
\]

where \( \mu(z, k, x, \psi) \), \( z \in Z \subset \mathbb{R} \), \( k \in K \subset \mathbb{R} \), \( x \in X \subset \mathbb{R} \), and \( \psi \in \Psi \subset \mathbb{R} \) is the distribution of firms over idiosyncratic technology, capital, net liquid asset position, and idiosyncratic demand shock. \( \Gamma \) is consistent with the policy functions of the firms;
9. Given \( \mu_e \) and \( \Gamma \), the firm measure \( \mu \) is invariant.
4 Quantitative Results

This section describes the numerical results of the model. I first provide details on the calibration of the model, the computation procedure, and the financial policies and firm distribution in the steady state. Then I present the impulse response functions for the economy under two aggregate shocks: (1) financial shocks to the effective supply of bank; (2) time-varying volatility shocks.

4.1 Calibration

The choice of parameters can be divided into three different categories. The first category consists of parameters that are picked according to the literature, such as the decreasing returns to scale parameter. The second group of parameters has a natural data counterpart, such as the volatility and persistence of aggregate shocks. The last group of parameters is calibrated to jointly target moments in the data. The standard approach in the literature (see, for example, Bachmann, Caballero and Engel (2013); Khan and Thomas (2013)) is to match heterogeneous firm models to establishment-level data. As the paper focuses on firm-level financial constraints, the relevant distribution is the firm size distribution.\(^{18}\)

**Standard calibration**  
The time period in the model equals one quarter; accordingly, I set the household’s rate of time preference \(\beta = 0.99\), implying an annualized risk-free rate of 4 percent. The quarterly depreciation rate \(\delta\) is set equal to 0.025. The fixed investment adjustment cost is calibrated to be \(F_{k,0} = 0.01\), a value estimated by Cooper and Haltiwanger (2006). The purchase value of capital \(p_k^+\) is normalized to 1, whilst the resale value of capital \(p_k^-\) is calibrated to match the steady-state level of leverage (discussed below). The decreasing returns to scale parameter \(\alpha\) is set equal to 0.8, which is within the range of estimates of the returns to scale in manufacturing (Lee (2005)). Following Gilchrist, Sim and Zakrajšek (2014), I calibrate the persistence of the idiosyncratic technology process \(\rho\) to be 0.8.

**Calibration to data**  
Financial intermediation costs consist of all non-interest costs that a lender undertakes to operate. As a proxy for the intermediation cost of market debt \(\gamma^m\), I use existing estimates of underwriting fees for corporate bond issuances. Fang (2005) studies a sample of bond issuances in the U.S., and finds an average underwriting fees of 0.95%, while Altınkılıç and Hansen (2000), in a sample including lower-quality issuances, find an average underwriting fee of 1.09%. Given this evidence, I set market debt intermediation costs to \(\gamma^m = 0.01\). Nonetheless, measuring analogously intermediation costs of banks – for example from operating expenses reported in income statements of commercial banks – has two potential drawbacks. First, operating expenses of banks can be associated with a number of non-lending activities. Second, operating expenses may miss some costs associated with credit intermediation by banks, such as equity issuance costs associated with capital and liquidity requirements. Therefore, instead of trying to

\(^{18}\)Table B.2 of the model appendix summarizes the calibration of the model.
construct a direct measure of the wedge between bank and market debt intermediation costs in the steady state ($\bar{\gamma}^*$), I match the average fraction of bank debt among non-financial corporations in the US, as discussed below.

The integration of all exogenous AR(1) processes in the model is approximated by Gaussian quadratures. To calibrate the persistence $\rho_\gamma$ and standard deviation $\sigma_\gamma$ of the financial shock, I follow Bassett, Chosak, Driscoll and Zakrajšek (2012), and utilize data from the Federal Reserve’s Senior Loan Officer Opinion Survey of Bank Lending Practices (SLOOS), which queries participating banks to report whether they have changed their standards during the survey period. Nevertheless, in assessing the supply-side implications of changes in bank lending policies, it is important to bear in mind that the changes in bank lending standards reported in the SLOOS reflect the confluence of demand and supply factors. Recognizing this endogeneity problem, I follow Lown and Morgan (2006) and use VAR-based identification strategies to identify the component of the change in lending standards that is orthogonal to the determinants of loan demand. Specifically, I estimate a VAR(4) specification with quarterly data on four macroeconomic variables—including log real GDP, log GDP deflator, log commodity prices, and the federal funds rate—and the net percent of banks reporting tightening standards. I order the credit variable after the macro variables. Summing the coefficients on lags of the lending standard variables in the lending standard equation itself yields $\rho_\gamma = 0.81$ and $\sigma_\gamma = 0.085$, which are within the range reported in Lown and Morgan (2006). The calibration of the distribution of the idiosyncratic productivity shocks follows Gilchrist, Sim and Zakrajšek (2014). Specifically, I assume four nodes of the idiosyncratic technology shock, such that an increase in volatility generates a greater dispersion in the nodes without changing the conditional expectation of the idiosyncratic technology shock. The steady-state level of uncertainty $\bar{\sigma}_z$ is set to 15 percent (30 percent annualized), which is equal to the sample mean of the uncertainty measure between 2006 and 2014 in the Compustat data (2006:Q1-2015:Q4). Using this proxy, I also estimate an empirical counterpart to equation (2), which yields $\bar{\rho}_\sigma = 0.82$, with the 95-percent confidence interval of [0.69, 0.93]; I set $\rho_\sigma = 0.90$, which is within the estimated range and in line with Bloom (2009). To generate fluctuations in uncertainty in the range between 25 and 50 percent (annualized)—a range consistent with the variability of the uncertainty proxy over the 2006-2014 period—I set the standard deviation of uncertainty shocks $\omega_\sigma$ to 0.05 percent of the steady-state level of uncertainty (1.75 percent annualized). Following Arellano, Bai and Kehoe (2012), I calibrate the persistence of the demand shock to be $\rho_\psi = 0.7$, which is in line with the estimated value by Foster, Haltiwanger and Syverson (2008), whilst the volatility of the demand shock $\sigma_\psi$ is calibrated to target microeconomic data on firms (see below).

**Calibration to target moments** The last seven parameters—including parameters governing firm entry $\{\omega, c_e\}$, the volatility of the idiosyncratic demand shock, $\sigma_\psi$, the resale value of capital $p_k$, the quasi-fixed costs of production $F_o$, the bankruptcy cost parameter $\chi$, and the steady state

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19See Figure B.3 in appendix D.1 for the Net Percentage of Domestic Banks Tightening Standards, and the Net Percentage of Domestic Banks Increasing Spreads of Loan Rates.
level of the positive wedge between the bank and market intermediation costs $\bar{\gamma}^*$—are calibrated to jointly target seven moments. To find these parameters, I first solve the model under a specific set of parameters. Then I simulate data using the policies of the model and compute the target moments. Next, I compare the model implied moments implied by this specific parameter combination. This procedure is repeated until the difference between the data and the model implied target moments has been minimized. The nonlinearities of the model do not allow exact matching of all moments. Nevertheless, Table 1 shows that the model generates target moments similar to those in the data. Six of these moments use Compustat data and are salient features of the microeconomic data on firms: the share of bank debt in the U.S., the mean leverage ratio, the relative sizes of entrants (to the public debt markets) and exiters, and the exit and entry rates. In addition, I also target the mean credit spread on corporate bonds in aggregate data, since the paper focuses on the aggregate impact of firms’ switch to bond financing during the crisis.

The steady state level of the positive wedge between the bank and market intermediation costs $\bar{\gamma}^*$, which captures the relative supply of bank credit, is calibrated to match the bank share of the sample described in Section 2, of U.S. non-financial and non-utility corporations with S&P ratings. The resale value of capital $p_k^-$ is set equal to 0.45, which implies a steady-state level of leverage of 0.39, which is close to the leverage ratio in the Compustat data for this sample of firms over the 2006:Q1-2014:Q4 period. The quasi-fixed operating cost $F_o$ is calibrated to match the exit rate in the data. According to the survey of Business Employment Dynamics, the average yearly survival rate for the establishments that were established between 1994 and 2009 is 0.784, which implies a quarterly survival rate of 0.912. Together with the exogenous exit rate of $1 - \eta = 0.05$, $F_o = 0.12$ implies a total exit rate of 9.6% in the steady state, which implies that 4.6% of firms endogenously exit through financial distress. The entry cost $c_e$ has the largest impact on entry rate, which in equilibrium must equal the exit rate.

The relative sizes of entrants and exiters with respect to survivors are calculated from the

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Parameter</th>
<th>Model</th>
<th>Target</th>
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</thead>
<tbody>
<tr>
<td><strong>Firm-level data</strong></td>
<td></td>
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<tr>
<td>1</td>
<td>Fraction of bank debt (mean)</td>
<td>$\bar{\gamma}^*$ (Wedge in intermediation costs)</td>
<td>35%</td>
</tr>
<tr>
<td>2</td>
<td>Leverage (mean)</td>
<td>$p_k^-$ (Resale value of capital)</td>
<td>39%</td>
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<tr>
<td>3</td>
<td>Exit rate</td>
<td>$F_o$ (Operating cost)</td>
<td>9.6%</td>
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<tr>
<td>4</td>
<td>Entry rate</td>
<td>$c_e$ (Entry cost)</td>
<td>9.6%</td>
</tr>
<tr>
<td>5</td>
<td>Entrants’ relative size</td>
<td>$\omega$ (Pareto exponent)</td>
<td>21%</td>
</tr>
<tr>
<td>6</td>
<td>Exiters’ relative size</td>
<td>$\sigma_\psi$ (Vol. of idiosyncratic demand)</td>
<td>39%</td>
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<tr>
<td><strong>Aggregate data</strong></td>
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<tr>
<td>7</td>
<td>Spread on corporate bonds</td>
<td>$\chi$ (liquidity efficiency)</td>
<td>2.6%</td>
</tr>
</tbody>
</table>
Compustat data, according to the definitions of these metrics in Dunne, Roberts and Samuleson (1988). I follow Clementi and Palazzo (2016) and assume that the distribution of the entry signal $q$ is Pareto, $F(q) = (q/q_0)^{-\omega}$, where $\omega > 1$. Given $\{c_e, \gamma^e\}$, the Pareto exponent $\omega$ is calibrated to target the relative size of entrants with respect to incumbents, whereby the data parallel for entry in the model is the decision of a firm to go public, as in Begenau and Salomao (2016). Furthermore, the relative size of exiters is pinned down by the volatility of the idiosyncratic demand shock $\sigma_{\psi}$. Lastly, given the calibration of the processes for the idiosyncratic uncertainty and the liquidation value of capital assets, I set the degree of frictions in the financial markets—the bankruptcy cost parameter $\chi$—to generate an average credit spread that is close to the mean of the credit spread on Moody’s BAA-rated corporate bonds between 2006 and 2014. Accordingly, I set $\chi = 0.43$, a value close to the micro-level evidence of Bris, Welch and Zhu (2006).

4.2 Computation

The model is solved using the inner-and-outer-loop algorithm of Krusell and Smith (1998), whereby I iterate between an inner loop step and an outer loop step until I isolate forecasting rules consistent with the equilibrium outcomes. Under bounded rationality, the inner-loop problem is solved using value function iteration, which allows for a fully nonlinear global solution under several occasionally binding constraints, including the dividend constraint, partial irreversibility, and nonconvex capital adjustment costs. In the outer loop, I update the aggregate laws of motion using a Monte Carlo simulation, to solve for equilibrium prices that are consistent with market clearing. Once the economy is simulated, I use OLS to update the aggregate laws of motion.

Due to the large dimension of the problem, I assume that agents use only the first moments of log-linearized laws of motions to predict the household’s stochastic discount factor:

$$\begin{bmatrix}
\log \tilde{b}' \\
\log \tilde{m}' \\
\log \tilde{a}_f' \\
\log \tilde{k}' \\
\log c
\end{bmatrix} = \Gamma_0 + \Gamma_1 \begin{bmatrix}
\log \tilde{b} \\
\log \tilde{m} \\
\log \tilde{a}_f \\
\log \tilde{k}
\end{bmatrix} + \Gamma_2 \begin{bmatrix}
\log \sigma_z \\
\log \gamma^* \\
\log \psi
\end{bmatrix}. \quad (44)$$

Specifically, the first moments of the distributions of capital ($\tilde{k}$) and cash holdings ($\tilde{a}_f$) are used to predict the productive capacity of the economy and the liquidity of the firms, whereas the first moments of the distributions of the post-renegotiation values of bank debt ($\tilde{b}$) and market debt ($\tilde{m}$) are for predicting the indebtedness of the corporate sector. As discussed in Section 3, firms would only want to pay dividends if they assign zero probability to being financially constrained in the future, to simplify the computation, I therefore assume that firms never pay out dividends unless they exit.

\footnote{The median estimate of the change in asset values before and after chapter 7 liquidation is 38%, adjusting for the value of collateralized assets that creditors may have seized outside of the formal bankruptcy proceedings.}
The policy functions \( b' = B(z, \psi, \hat{k}, x; \sigma, \gamma^*) \), \( m = M(z, \psi, \hat{k}, x; \sigma, \gamma^*) \), and \( a_f = A_f(z, \psi, \hat{k}, x; \sigma, \gamma^b) \) can be obtained using the result from the value function iteration in the inner loop. Let \( \mu(z_0, \psi_0, k_0, x_0) \) measure the proportion of firms with idiosyncratic technology \( z_0 \), idiosyncratic demand \( \psi_0 \), capital \( k_0 \) and net liquid asset position \( x_0 \). The stationary distribution can be determined by iterating on the following equation:

\[
\mu'(z_0, \psi_0, k_0, x_0) = \int 1_{B(z, \psi, \hat{k}, x)} \int 1_{M(z, \psi, \hat{k}, x)} Q(z, \psi, \hat{k}, x) \left( \mu(dz, d\psi, d\hat{k}, dx) + \mu_e(dz, d\psi, d\hat{k}, dx) \right),
\]

where \( \mu \) is a measure on the space \( Z \times \Psi \times K \times X \), where \( Z \in Z, \Psi \in \Psi, K \in K, X \in X \). \( Z, \Psi, K, \) and \( X \) are the Borel \( \sigma \)-algebras generated by the subsets of \( Z, \Psi, K, \) and \( X \), respectively. I start iterating from a uniform distribution as an initial guess, and \( Q \) denotes the transition matrix implied by the exogenous technology process \( z \), the exogenous demand process \( \psi \), and the policy functions.

4.3 Firm Distribution and Financial Policies in the Steady State

Size distribution

Figure 5 plots the firm size distribution over asset (normalized in the range of 0 and 100). Panel (a) presents the density of logged assets for the full sample of firms described in Section 2. Notably, the sample distribution by assets is negatively skewed, i.e. the mass of the distribution is concentrated on the right; in other words, the investment grade firms account for more than half of the (logged) asset share, even after removing the top 1% of firms. Panel (b) plots the average firm size distribution over the normalized assets for different states of the economy in the model.\(^{21}\) Endogenous entry and exit affect the firm size distribution over time. Firms tend to enter small and more firms enter in non-crisis times during which the distribution gets flatter: the larger firms are larger compared to crisis states during which the size distribution is more concentrated and shifts to the left.

Optimal debt structure

As shown in Panel A of Figure 6, a firm’s asset size is positively related to its share of market debt, defined as the ratio of market debt to total debt. Firms’ debt structures fall under two categories: on the one hand, firms below a certain level of assets (denoted \( a^* \)) choose a “mixed” debt structure, involving a combination of bank debt and market debt (interior solution); larger firms with assets strictly above the threshold choose a “market-only” debt structure (corner solution). Recall from Section 3 that there are two types of debt contracts in the model. As firms grow, they will eventually switch from a mixed debt structure (R-contract) to a market-only debt structure (NR-contract). This optimal debt structure in steady state echoes the results in Crouzet (2015), as well as the evidence provided by Rauh and Sufi (2010), that the degree to which the debt structure of firms is “spread out” across types and priorities is strongly

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\(^{21}\)The book value of assets in the model is defined as \( \pi + p^z \hat{k} \), where \( \pi \) is the gross profit, and \( p^z \hat{k} \) is the book-value of (gross) property, plant, and equipment.
related to firms’ credit ratings: investment-grade firms mostly use senior unsecured debt (bond and program debt), while the speculative grade firms use a combination of secured bank debt, senior unsecured debt, and subordinated bonds.

The intuition for this result is as follows. As borrowing from the bank reduces the expected losses associated with financial distress, smaller firms have a stronger incentive to use bank debt, and hence their debt composition is more tilted towards bank loans. If a firm’s default probabilities is sufficiently small, it never restructures bad debt. In that case, the flexibility associated with bank debt is irrelevant to these firms since, in equilibrium they never use that flexibility. But borrowing from banks results in higher intermediation costs. Hence the largest firms always choose the corner solution of a market-only debt structure. Therefore, the trade-off between bank flexibility in times of financial distress and the costs associated with using bank loans in normal times changes with the firm’s size and ultimately its default probability, and hence affect the firm’s choice of debt structure.

**Cash holdings** Panel (b) of Figure 6 plots the (intra-temporal) cash-to-book-asset ratio in the cross-section. First, it indicates that firms employing debt financing simultaneously hold cash balances, and that the stock of internal finance is negatively related to the productivity level of the firm. Intuitively, because of decreasing returns, firms have an ex-ante optimal investment scale. More productive firms are less likely to face financial distress as well as the bankruptcy costs associated with debt financing, and are thus more inclined to avoid the low return of cash holdings and use debt financing to reach the optimal investment scale. This echoes the findings of Riddick and Whited (2009), that firms hold higher precautionary cash balances when external finance is costly.

Figure 6 also shows that when a firm crosses the threshold $a^*$ above which it switches to
Figure 6: Optimal Composition of Debt and Cash Holdings in the Steady State

(a) Debt composition

(b) Cash to book asset

Note: Panel (a) presents the steady-state fraction of market debt \( m \) across the distribution of firms by logged assets, while panel (b) plots the (intra-temporal) cash-to-book-asset ratio across the distribution.

4.4 Macroeconomic Implications of Financial Shocks

This section analyzes the dynamics of the model’s key endogenous variables in response to a negative shock to the effective supply of bank credit (\( \gamma^* \)). The benchmark model economy features a full set of frictions. Subsequently, I add four counterfactual exercises. To assess the magnitude of “precautionary savings” channel in the amplification and propagation of aggregate shocks, I first construct a counterfactual scenario whilst firms can still borrow from both lenders, but cannot optimize the allocation of assets (i.e. holding constant the cash-to-asset ratio). Next, to understand the role of the corporate bond market in a financial crisis, I solve a version of the model that shuts down the effect of debt substitution by holding constant the composition of debt across firms. Third, in order to assess the quantitative role of financial versus real frictions, I solve a version of the model with only financial frictions; in this case, the firms face the same costs of intermediation and deadweight loss in default, as in the benchmark case, except that the firms do not face any capital adjustment frictions, i.e. \( p^- = p^+ = 1 \), \( F_{k,0} = 0 \) and \( F_{k,1} = 0 \). Fourth, I construct a counterfactual scenario with a lower degree of financial market frictions—in other
words, a higher degree of liquidation efficiency $\chi$—in order to examine the extent to which the severity of the recession, as well as the consequent slow recovery, would be alleviated if frictions in the corporate debt market were lower.

In computing the model-implied impulse response functions, I take into account the nonlinearities in the firms’ investment and financial policies that arise naturally in an economy with irreversible investment, fixed capital adjustment costs, and financial distortions. The impulse response functions are constructed as follows: (1) simulate the model twice—first with the idiosyncratic shocks only and then with an aggregate shock layered on top the same set of idiosyncratic shocks; (2) for each simulation, aggregate across each group of firms (defined below) the micro-level impulse responses of endogenous variables of interest; and (3) take the difference between the two sets of aggregate endogenous quantities. To eliminate any sampling bias that may have arisen from drawing idiosyncratic shocks, I repeat these three steps a large number of times and then average the aggregate impulse response functions across replications.

**Firm Heterogeneity in Impulse Responses**  Figure 7 depicts the behavior of the model’s main endogenous variables in response to an adverse shock of about two standard deviations to the bank lending costs in period $t = 5$, which can be interpreted as a negative shock to the effective bank credit supply. Upon impact, the unanticipated increase in bank lending costs reduces the bank debt by 10 percent on average (panel (a)), in line with the Flow of Funds data on aggregate bank lending to non-financial corporations in 2008Q3. In order to maintain comparability with the stylized facts in Section 2, I define two groups of firms, using a fixed threshold on successive cross-sections to characterize their asset sizes, such that in each period, firms with assets more than the threshold amount are the “investment-grade” firms, whereas those with assets less than the threshold amount are the “speculative-grade” one.

Whilst bank lending has declined for all firms (panel (a)), the investment grade firms have substituted towards market debt to a much larger degree (panel (b)). Hence changes in debt composition differ substantially across firms, such that debt substitution is only salient for the larger firms with lower default probabilities. For the speculative grade firms with higher default probabilities, the substitution towards market debt is weak, with bank debt showing the largest decline upon impact. These echo other evidence on the 2007-2009 recession, most notably Adrian, Colla and Shin (2012) and Becker and Ivashina (2014). As a result of the heterogeneity in debt composition after a bank credit supply shock, the speculative grade firms suffer from a much sharper fall in leverage than the investment grade firms (panel (c)).

The striking result is that despite having suffered from a smaller decline in external finance available, the investment grade firms reallocate significantly more of their assets towards cash than productive capital upon the shock, compared to the speculative grade firms, as shown

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22 Over the 2006–2014 period, the total assets of investment grade firms is on average 68% of the total assets of all nonfinancial firms with S&P ratings (excluding the top 1%) by this measure. The number of investment grade firms, according to this definition, fluctuates between 45 and 50 percent of the total number of firms in the model, which is consistent with the fraction (48%) of investment grade firms in the data sample.
Figure 7: Impact of a Financial Shock to Bank Credit Supply
(Baseline with Firm Heterogeneity)

(a) Bank Debt  (b) Market Debt  (c) Total Debt

(d) Intratemporal Capital  (e) Intratemporal Cash  (f) Output

(g) Intertemporal Capital  (h) Intertemporal Cash  (i) Credit spreads

Note: This figure compares the impulse response functions of the investment grade firms (defined below) versus those of the speculative grade firms to a financial shock. A shock reduces the supply of bank loans ($\gamma$) 10 percent upon impact (period 5) on average, a shock of approximately 2 standard deviations; the bank loan supply is then allowed to revert back to its steady-state value following the process in equation (10). The impulse responses are averages of 50,000 simulations, where each simulation is an aggregation of the impulse responses of 10,000 firms. In panels (a)–(h), the blue solid lines depict the impulse response functions of the investment-grade firms, while the red dashed lines depict the impulse responses of the speculative-grade firms; in panel (i), the blue solid line indicates the impulse response function of the spread on corporate bonds, while the red dotted line indicates the spread on bank loans. I define an asset threshold $a_{IS}$ such that, in quarter 0, firms with $a_0 > a_{IS}$ account for a fraction $s_{IS}$ of the total assets in the economy. In the dataset used in Section 2, the ratio of assets averaged across all investment-grade firms to assets averaged across all is 0.68 in 2007. I use this as the cut-off threshold in the model, $s_{IS} = 0.32$. The vertical axis denotes percentage deviation from the steady state and the horizontal axis denotes quarters.
in panels (d) and (g). As a result, firms’ output falls by proportionally more for the investment grade firms (panel (f)). Notably, firms adjust their portfolio of assets before as well as after the realization of idiosyncratic productivity and financial shocks in each period. At the asset reallocation stage (see timing in Figure 3), firms reoptimize their asset portfolio, for a given leverage, by building up precautionary savings in order to avoid costly default (panels (e) & (h)). The investment grade firms increase their cash holdings more than the speculative grade firms, as substitution from a mixed-debt to market-only debt structure entails the loss of ability to restructure debt. Choosing a safer portfolio of assets by holding proportionally more cash, for a given leverage, would optimally offset this. Moreover, due to the presence of convex capital adjustment costs, firms have an incentive to smooth out (dis)investment over time, and hence also adjust the predetermined levels of capital and cash (panels (g) & (h)). Nevertheless, the magnitudes of adjustments are larger at the asset reallocation stage, after the realization of shocks.

In general equilibrium, the increase in precautionary savings depresses the risk-free interest rate, especially in the short run. It is important to note that although the types of credit diverged in quantity, the spreads on both rose sharply, as shown in panel (i)—an empirical feature documented by Adrian, Colla and Shin (2012) and captured in other models of debt substitution such as De Fiore and Uhlig (2015). Market debt has become more costly as now firms of higher default probabilities have switched to an all-market debt structure, as they find the flexibility provided by banks too costly following a bank credit supply shock. With the presence of partial irreversibility of capital, the results here also echo the findings in other models with liquid and illiquid assets (e.g. Guerrieri and Lorenzoni (2015)) that a credit shock can lead, simultaneously, to an increase in demand for the liquid asset and to a reduction in demand for the illiquid asset. This captures a form of “flight to liquidity” on the firm’s side. Nevertheless, as shown below (Figure ??), the presence of real frictions can only explain a small fraction of the increase in cash, whereas the majority of the increase come from the precautionary motive to rebalance asset portfolio towards safe assets.

**Nonlinearity of Impact Across Distribution** Figure 7 demonstrates how the cash holdings and capital investment by the “investment grade” firms appear to be more sensitive to financial shocks than the “speculative grade” ones. In this subsection, I highlight the non-linearity of the effects across firms, by illustrating, in Figure 8, the percentage changes in intratemporal cash (a) and capital (b), in the 10th period after the financial shock, across the distribution of firms by total assets.23 There are three types of firms in the model: (1) firms that were mix-financed before the shock and remain so thereafter (though the proportion of market debt increases); (2) firms that switch from mixed-finance to bond-only after the shock; (3) firms that were already bond-only financed before the shock and remain so thereafter. As Figure 8 shows, the second type of firms experience the largest change in their asset portfolio after the shock, and the intuition is that on the liability side, they have switched from a mixed-debt contract to a riskier bond-only contract

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23As shown in Figure 7, the impulse responses show the largest fall in capital (or rise in cash holdings) approximately 10 periods after the shock.
since the crisis, and lost the ability to restructure debt in the future. This is the “precautionary savings” mechanism associated with the change in debt structure. In the Compustat sample, 17% of firms had switched from mixed-debt to bond-only since the crisis, and in the model, this mechanism affects approximately 15% of the firms in the distribution. For firms that still rely on bank debt (the lower end of the firm distribution), as the probability of default increases after the shock, they also hold more safe assets and decrease investment in risky capital. This is the standard precautionary savings motive, irrespective of any change in debt contract. For the upper end of the distribution, even though these firms did not rely on bank financing before the shock and hence were not directly affected by the crisis, they experience two indirect effect (of the opposite directions) of the shock to bank lending cost. On the one hand, the large increase in safe assets (or deposits) depresses the risk-free rate in general equilibrium, and this reduces the cost of capital and encourages investment; on the other hand, these firms anticipate that, given a sufficiently bad sequence of productivity shocks, they will have to revert to bank borrowing, which is now costlier, they seek to avoid this by increasing their internal finance today. Under the current calibration, the second effect dominates and overall even the upper end of the distribution that were entirely market-financed experience an increase in cash holdings and decrease in investment.

**Mapping Macroeconomic Implications to Data**  
Recall that in estimating the persistence and standard error of $\gamma^b$, I use a VAR(4) specification over the period 1992:Q1–2015:Q4 with the following five quarterly endogenous variables: log real GDP, log GDP deflator, the net percent of banks reporting tightening standards from the Federal Reserve’s Senior Loan Officer Opinion Survey of Bank Lending Practices (SLOOS), the nominal effective federal funds rate, and the 10-year BBB-Treasury credit spread. To identify innovations in bank credit supply, I employ a standard recursive ordering technique, in which innovations in credit supply have an immmedi-
Figure 9: Mapping Impulse Response Functions from the Model to VAR Estimation

(a) Shock to bank lending cost  
(b) Output

(c) Risk-free rate  
(d) Credit Spread on Bond

Note: The panels of the figure compare the impulse response functions to a shock to lending wedge $\gamma^b$ in the model (black solid lines) with the impulse response functions to a shock to lending standard in the VAR estimation (blue dotted lines) for (a) shock to bank lending cost, (b) output, (c) risk-free rate, and (d) corporate bond spread. The size of the shock in both the model and the data is approximately 2 standard deviations, which is chosen to match the 10 percent drop in bank loans in the Flow of Funds data in 2008Q3. Identification scheme in the VAR system corresponds to the following recursive ordering: log real GDP, log GDP deflator, lending standard (shock), federal funds rate, and credit spread; see the text for detail of variables. The shaded bands represent the 95-percent confidence intervals in the VAR estimation based on 1,000 bootstrap replications.
ate impact on credit spreads and short-term interest rates, but they affect economic activity and prices with a lag (Lown and Morgan (2006)). Figure 9 presents the impulse response functions of the VAR variables – lending standard, log real GDP, nominal effective federal funds rate, and 10-year BBB-Treasury credit spread – to the model-implied paths of the corresponding variables: output, relative bank supply, risk-free rate, and bond spread, respectively.

The size of the shock in both the model and the VAR system is approximately two standard deviations, which is chosen to match the 10 percent drop in bank loans in the Flow of Funds data in 2008Q3. Moreover, the persistence of the shock variable is obtained by summing the coefficients on lags of the lending standard variables in the lending standard equation itself in the VAR system, and panel (a) illustrates the matching of the paths of the shock in the model and in the data. Panels (b)–(d) show that qualitatively, the model and the data exhibit similar patterns of responses for the variables in the VAR, although quantitatively, the model predicts a sharper fall (increase) in output and risk-free rate (spread on bonds) upon impact than the VAR estimates.

**Precautionary Savings Channel** The model captures two channels in the firms’ response to a contraction in the supply of bank credit. The first is the traditional “financial constraint” channel, by which constrained firms are forced to deleverage and reduce investment. The second is the “precautionary savings” channel, by which unconstrained firms that substitute bank loans with bond issues reallocate their assets from productive capital to savings as a buffer against future shocks. To isolate the component of the shock’s aggregate effect that is attributable to this new channel, I compare the aggregate impulse responses—without differentiating between different types of firms—in the baseline model with the aggregate impulse responses of an alternative economy in which firms are allowed to switch debt instruments, but holding constant the fraction of assets that is saved. The corresponding impulse responses are reported in Figure 10.

There are two key differences from the baseline model. First, without the ability to optimize asset allocation, the fall in total debt is larger than in the baseline. This is in line with the results in Crouzet (2015), that switching to a market-only debt structure while keeping total leverage constant would expose firms to a larger risk of financial distress; they offset this by reducing total borrowing further, in addition to the reduction in leverage associated with the tradition “financial constraint” channel. Intuitively, holding safe assets increases the survival probability of the firm, and thus partially offsets the higher default risks associated with the change in debt structure, so the need for deleverage is lower. Moreover, without the ability to hold more safe assets, for a given leverage, the substitution towards market debt is of a smaller magnitude, as firms would value the flexibility of the bank debt more. Second, even though total debt falls by more, capital and output actually fall by less in the counterfactual example than in the baseline, as all the external finance raised is allocated towards capital. Therefore, this counterfactual exercise shows that the precautionary savings channel is quantitatively important for generating the amplification and persistence of an aggregate financial shock, as debt substitution induces firms to hoard large amounts of cash for precautionary purpose, and not use them to finance investment. Quantitatively, this channel can account for about 40% of the total decline in aggregate output in
Figure 10: Impact of a Financial Shock to Bank Credit Supply
(Counterfactual: Precautionary Savings Channel)

(a) Bank Debt
(b) Market Debt
(c) Total Debt
(d) Intratemporal Capital
(e) Intratemporal Cash
(f) Output
(g) Intertemporal Capital
(h) Intertemporal Cash
(i) Credit spreads

Note: This figure compares the (aggregate) impulse response functions in the baseline model with those in the counterfactual exercise, whereby firms are allowed to switch debt instruments whilst holding constant the fraction of assets that is saved at the steady state level. A shock reduces the supply of bank loans ($\gamma^*$) 10 percent upon impact (period 5) on average, a shock of approximately 2 standard deviations; the bank loan supply is then allowed to revert back to its steady-state value following the process in equation (10). The impulse responses are averages of 50,000 simulations, where each simulation is an aggregation of the impulse responses of 10,000 firms. The black solid lines depict the impulse response functions in the baseline, while the green dashed lines depict the impulse responses in the counterfactual exercise. The vertical axis denotes percentage deviation from the steady state and the horizontal axis denotes quarters.
the first two years of the crisis, and more than one-half of the decline in the following five years.

Furthermore, this counterfactual exercise also shows that introducing endogenous asset allocation generates a more persistent response in output than a model without cash holdings. Persistence arises because both the debt composition and portfolio allocation decisions are endogenously determined. Once the firm increases the proportion of cash holdings in response to switching to a market-only debt structure, this partially offsets the change in default risk associated with the change in debt structure. Consequently, the flexibility of bank debt would appeal less to the firm compared to the scenario where the only asset is productive capital, and this in turn slows down the adjustment of bank borrowing, and triggers another high cash-to-asset ratio in the following period. Therefore, turning off cash holdings by firms produces the counterfactual results of a larger decline in total leverage during the crisis but a relatively faster recovery in output thereafter.

The Role of the Bond Market

The next counterfactual exercise investigates the role of the bond market as a “spare tyre” to the traditional, bank-based intermediation at times when the latter is impaired. To do so, I compute the aggregate impulse responses of an alternative economy in which the debt composition is held constant at the steady state level; in other words, firms are not allowed to switch to bonds after the bank credit supply shock (but are allowed to save). As shown in Figure 11, there is no substitution towards market debt following a bank credit supply shock; instead, bank debt and market debt fall by the same proportion.

There are three key observations from this counterfactual exercise. First, without the ability to substitute into bonds, total debt falls by more. Intuitively, both types of firms are affected by the “financial constraint” channel. Second, cash holdings increase in the economy, but by significantly less, indicating that a significant proportion of the increase in cash in the baseline is associated with changes in debt structure. Third, the negative impact of the large fall in total external finance available outweighs the counteractive force associated with a milder reallocation of funds into cash. As a result, capital and output fall by significantly more in the counterfactual example. Therefore, the model lends support to the conventional wisdom that firms’ ability to substitute among alternative debt instruments is important to shield the economy from adverse real effects of a financial crisis. Nevertheless, as indicated in the Figure 10, containing firms’ cash holdings incentive to offset the “precautionary savings” channel associated with the change in debt structure can improve the efficacy of the corporate bond markets in times of financial crisis.

4.5 Macroeconomic Implications of Uncertainty Shocks

A common explanation for the slow recovery from the 2007-2009 recession is that uncertainty over business conditions limits investment—either through the “real options” effects on the demand for capital, or via changes in credit spreads—and induces firms to hoard cash and cut debt

24The persistence effect of introducing cash holdings is more prominent in a model where firms are not allowed to save at all, and the results are available upon request.
Figure 11: Impact of a Financial Shock to Bank Credit Supply
(Counterfactual: The Role of Bond Market)

Note: This figure compares the (aggregate) impulse response functions in the baseline model with those in the counterfactual exercise, holding the debt composition constant at the steady state level. A shock reduces the supply of bank loans ($\gamma^\ast$ 10 percent upon impact (period 5) on average, a shock of approximately 2 standard deviations; the bank loan supply is then allowed to revert back to its steady-state value following the process in equation (10). The impulse responses are averages of 50,000 simulations, where each simulation is an aggregation of the impulse responses of 10,000 firms. The black solid lines depict the impulse response functions in the baseline, while the green dashed lines depict the impulse responses in the counterfactual exercise. The vertical axis denotes percentage deviation from the steady state and the horizontal axis denotes quarters.
Figure 12: Impact of an Uncertainty Shock

(a) Bank Debt

(b) Market Debt

(c) Total Debt

(d) Capital ($k$)

(e) Cash ($a_f$)

(f) Output

(g) Capital ($k'$)

(h) Cash ($a'_f$)

(i) Credit spreads

Note: This figure compares the impulse response functions of the investment grade firms (defined below) versus those of the speculative grade firms to an uncertainty shock. A shock increases the volatility of the idiosyncratic technology shock ($\sigma_z$) 3 percentage points (annualized) upon impact (period 5), a shock about 2.5 standard deviations; the volatility is then allowed to revert back to its steady-state value following the process in equation (2). The impulse responses are averages of 50,000 simulations, where each simulation is an aggregation of the impulse responses of 10,000 firms. In panels (a)–(g), the blue dashed lines depict the impulse response functions of the investment-grade firms, while the red dashed lines depict the impulse responses of the speculative-grade firms; in panel (i), the blue dotted line indicates the impulse response function of the spread on corporate bonds, while the red dotted line indicates the spread on bank loans. I define an asset threshold $a_{IS}$ such that, in quarter 0, firms with $a_0 > a_{IS}$ account for a fraction $s_{IS}$ of the total assets in the economy. In the dataset used in Section 2, the ratio of assets averaged across all investment-grade firms to assets averaged across all, in 2007, I use this as the cut-off threshold in the model, $s_{IS} = 0.32$. The vertical axis denotes percentage deviation from the steady state and the horizontal axis denotes quarters.
to hedge against future shocks. To formalize the mechanism, much of the literature has focused on total debt issued by firms. This paper, however, highlights the importance of looking at debt composition, and doing so reveals that it is not clear that fluctuations in idiosyncratic uncertainty are the sources of the 2007-2009 recession or the slow recovery thereafter.

The macroeconomic implications of uncertainty shocks are presented in Figure 12. Even though the aggregate impulse responses are in line with the conventional wisdom—that an increase in volatility is associated with less debt, higher cash holdings, lower investment and output—a closer look at the impulse responses by subsets of firms reveals that a recession driven by an increase in idiosyncratic volatility generates two striking results in the model that are odds with the data. First, instead of retiring bank loans whilst increasing bonds—as shown in the data—all firms increase the fraction of their bank debt following an increase in idiosyncratic volatility. Moreover, both types of firms significantly retire their market debt, which is at odds with the evidence in Section 2. This reflects the greater demand for debt flexibility from both small and large firms when firm-level uncertainty is high.

Second, as an increase in idiosyncratic volatility is an aggregate shock that affects all firms, it induces the smaller firms with higher default probabilities to increase their cash holdings much more than the larger firms. At the aggregate level, this is in line with the conventional wisdom: higher uncertainty raises the importance of waiting and staging flexibility when making investment decisions; consequently, the proportion of firms in the inactive region rises, and all firms find it optimal to hold proportionally more cash. However, again, a closer look at the heterogeneity in firm dynamics reveals that the results are the opposite of the empirical evidence: the stylized facts in Figures 1 and 2 show that the investment grade firms increased their cash holdings much more, and had a slower recovery than the speculative grade firms. The impulse responses to an uncertainty shock, however, suggest that the investment by larger firms would recover faster. Therefore, the model generates firm dynamics that are consistent with the data following a credit supply shock, but not an increase in the volatility of the idiosyncratic technology process.

5 Conclusion

This paper has explored the role of firms’ balance sheet adjustment on the propagation of aggregate shocks. Using a micro-level dataset on the public U.S. firms’ debt compositions between 2006 and 2015, I find that the substitution of corporate bonds for bank loans since the Great Recession has been associated with a substantial reallocation of firms’ assets from capital to cash holdings. As a result, remarkably, firms that had tapped the bond market in large quantities since the 2007-2009 recession have experienced a more severe recession and a slower recovery.

I evaluate the economic mechanisms that mediate the above relationship using a quantitative general equilibrium model of firm dynamics, where firms choose both the scale and composition of debt, and simultaneously hold cash balances. In choosing between bank and bond financ-
ing, firms trade-off the greater flexibility of banks in case of financial distress against the lower marginal costs of bond issuances. Moreover, for a given debt structure, firms face a trade-off between investing more and getting higher profits in the future—conditional on receiving a favorable demand shock and not defaulting—and holding more cash which implies a lower variance of return and thus a higher chance of survival. As a result, the model endogenously generates a distribution of firms across levels of productivity in the steady state, and predicts a tight link between the likelihood of financial distress, the level of cash balances, and the composition of debt, consistent with the evidence that firms hold higher precautionary cash balances when external finance is costly, and that firms tend to increase their reliance on bank loans as credit quality declines. Furthermore, substituting market debt for bank debt exposes firms to a larger default risk, thus incentivizing them to reallocate assets from capital to cash holdings.

The novelty of the model framework is twofold. First, to the best of my knowledge, this is the first model to generate simultaneous borrowing and saving by risk-neutral firms with limited liability. The latter is difficult to generate in this class of models and requires more than simply adding an additional state variable to an otherwise standard firm investment model, as the risk-free return on saving is weakly dominated by the cost of risky debt, which prices in the probability of firm defaults. Therefore, firms that maximize expected payoffs have little incentive to borrow for the purpose of savings, if liability and asset decisions were simultaneously determined. To overcome this, I introduce a sequence of shocks to separate the decisions on the liability side from those on the asset side, such that firms choose the optimal asset allocation for a given liability structure in the middle of a period; by backward induction, firms rationally expect the optimal asset allocation later on and choose the optimal liability structure at the beginning of a period. Second, the model provides the first unified framework to study the macroeconomic implications of balance sheet adjustments on both the asset- and liability-side. Due to the imperfect substitutability between cash-financing and debt financing, and the trade-offs among different debt instruments, firms’ balance sheet policies in response to aggregate shocks can have important implications on future macroeconomic dynamics.

The model provides a useful framework to study the transmission of aggregate shocks and the macroeconomic implications of debt heterogeneity. In studying the transmission of a financial shock that alters the effective supply of bank credit, I use the model to quantitatively evaluate the “precautionary savings” channel associated with the change in debt composition, vis-à-vis the more traditional “financial constraint” channel that manifests itself in the decline of total quantity of debt. The counterfactual scenario in which I isolate the “precautionary savings” channel by not allowing firms to reallocate assets suggests that the channel of balance sheet restructuring can account for 40% of the decline in aggregate investment in the first two years of the crisis, and more than one-half of the decline in the following five years.

Furthermore, I also examine through the lens of the model whether financial frictions manifest themselves through shocks to the demand for credit or to its supply in the Great Recession. A recession driven by an increase in idiosyncratic volatility generates results in the model that are odds with the data. Therefore, the model generates firm dynamics that are consistent with the
data following a credit supply shock, but not an increase in the volatility of the idiosyncratic technology process, suggesting that financial frictions have manifested themselves mainly through shocks to the supply of credit rather than the demand for credit during the Great Recession. Therefore, introducing corporate balance sheet adjustment to an otherwise standard business cycle model may serve as one way to disentangle shocks to credit demand from shocks to credit supply.

Finally, although I abstract from policy analysis here, the model provides a useful framework for studying the transmission of monetary policy with heterogeneous firms. Due to firms’ different balance sheet structures, some firms may respond more strongly to monetary policy shocks than others, and this in turn can have consequences for the distribution of net worth across firms, and the design of monetary policy. I leave the precise analysis of how firms’ balance sheet dynamics affect the transmission of monetary policy as a potential avenue for future research.
References


Appendices

A  Data Appendix

In this appendix, I describe the data used in the empirical analysis and provide additional regression results. Subsection A.1 provides the details of the sources and construction of the data series. Subsection A.2 shows additional univariate evidence, including medians of the series shown in Figures 1 and 2 of the main text, and the fractions of firms by industry. Subsection A.3 shows that unlike the diverging patterns of debt financing and cash holdings shown in Section 2 of the main text, the investment- and speculative-grade firms had more similar patterns of equity financing and dividend payment since the crisis. Finally, in subsection A.4, I corroborate the descriptive stylized facts presented in the main text using panel data analysis.

A.1  Description of Variables

Aggregate balance sheet data for the U.S. is from Table L.102 of the Flow of Funds, the balance sheet of the nonfinancial corporate sector. Data on aggregate investment is from Table F.103 of the Flow of Funds, and credit spread data is from the Bureau of Economic Analysis, and Federal Reserve Bank of St. Louis. Firm characteristics are from Compustat (numbers in parentheses refer to the corresponding Compustat data item). Debt structure variables are from Capital IQ, which decomposes total debt into seven mutually exclusive debt types: commercial paper, drawn credit lines, term loans, senior bonds and notes, subordinated bonds and notes, capital leases, and other debt.

In the Compustat dataset, for firms with a fiscal year ending in the beginning of the year, i.e. in the months January through May, I shift the observation to align it better with the observation for the macroeconomic variables. A year $t$ observation for a firm with a fiscal year ending in May corresponds to the period from June of year $t-1$ to May of year $t$. This observation enters the sample in year $t-1$. The same change in date is used for firms with a fiscal year ending in the months January through April. Details of the data series are listed below.

Aggregate data

- *Corporate bonds outstanding* is the sum of corporate bonds (line 23, Table L.102) and commercial paper (line 21, Table L.102)
- *Bank loans outstanding* is the sum of depository institution loans (line 27) and other loans and advances (line 18, Table L.102)
- *Debt outstanding* is total credit market instruments outstanding (line 23, Table L.102)
- *Investment* is capital expenditures of private nonfinancial corporations (line 11, Table F.103)
- *Credit Spread* is Moody’s Seasoned Baa Corporate Bond Yield Relative to Yield on 10-Year Treasury Constant Maturity
Compustat data on firm characteristics

- **Cash-to-asset** is the ratio of cash and marketable securities (#1) to book assets (#6)
- **Cash-to-debt** is the ratio of cash and marketable securities (#1) to the sum of long-term debt (#9) and debt in current liabilities (#34)
- **Firm size** is the natural logarithm of book assets (#6) in 2009 dollars (using GDP deflator)
- **Leverage** is the ratio of long-term debt (#9) plus debt in current liabilities (#34) to book assets (#6)
- **Net leverage** is the ratio of long-term debt (#9) plus debt in current liabilities (#34) minus cash and marketable securities (#1) to book assets (#6)
- **Total investment-to-asset** is the ratio of the sum of capital expenditures (#128) and acquisitions (#129) less the sale of property (#107), to book assets (#6)
- **Capital expenditures-to-asset** is the ratio of the sum of capital expenditures (#128) and R&D expenditures (#46) to book assets (#6)
- **Cash flow** is earnings after interest, dividends, and taxes before depreciation divided by book assets ((#13-#15-#16-#21)/#6)
- **Market-to-book** is the ratio of the book value of assets (#6) minus the book value of equity (#60) plus the market value of equity (#199 × #25) to the book value of assets (#6)
- **Net working capital** is the ratio of net working capital (#179) minus cash and marketable securities (#1) to book assets (#6)
- **Tangibility** is the ratio of net property, plant and equipment (#8) to book assets (#6)
- **R&D** is the ratio of R&D expenditures (#46) to book assets (#6)
- **Dividend** is a dummy variable equal to one in years in which a firm pays a common dividend (#21)
- **Acquisitions** is the ratio of acquisitions (#129) to book assets (#6)
- **Rating** is the yearly average of the monthly S&P long-term issuer rating (splticrm), where we assign an integer number ranging from 1 (SD or D) to 22 (AAA) to each monthly rating and take the yearly average
- **WW-Index** is based on Whited and Wu (2006) and is computed as follows: $-0.091 \times \text{Cash flow} - 0.062 \times \text{Dividend} + 0.021 \times \text{Leverage} - 0.044 \times \text{Size} + 0.102 \times \text{Industry Growth} - 0.035 \times \text{Growth}$, where **Industry Growth** is the 4-SIC industry sales growth, **Growth** is own-firm’s real sales growth, and the other variables are as defined above
• **Asset Liquidation Value** is based on Berger, Ofek and Swary (1996), and is computed as follows: $0.715 \times \text{Receivables (2)} + 0.547 \times \text{Inventory (3)} + 0.535 \times \text{Capital (8)}$

• **Industry Frequency of Investment Inaction** is defined at the firm level based on Cooper and Haltiwanger (2006) as the number of firm-year observations with $|\text{Total investment/book assets}| < 0.01$, over the total number of observations in the 4-SIC industry.

• **Investment Spikes in the Industry** is defined as the number of firm-year observations with $|\text{Total investment/book assets}| \geq 0.2$.

• **Time-Series Skewness (Kurtosis) of Industry Investment** is based on Caballero (1999) and calculated as the skewness (kurtosis) of average annual capital expenditures to book asset ratios in each 4-SIC industry.

**Capital IQ data on debt structure**

• **Market debt** is the sum of commercial paper, senior bonds and notes, and subordinated bonds and notes. In panel (a) of Figure ??, the series “market debt” is given by: $\frac{m_{t_0}}{b_{t_0} + m_{t_0}} (\frac{m_{t_0}}{m_{t_0}} - 1)$, where $t_0$ corresponds to the level in 2008Q3.

• **Bank debt** is the sum of drawn credit lines and term loans. In panel (a) of Figure ??, the series “bank debt” is given by: $\frac{b_{t_0}}{b_{t_0} + m_{t_0}} (\frac{b_{t_0}}{b_{t_0}} - 1)$, where $t_0$ corresponds to the level in 2008Q3.

• **Market fraction** is the ratio of market debt to the sum of market debt and bank debt.

• **Market only** is a dummy variable equal to one if the current year’s fraction of market debt is 100 percent and the previous year’s fraction is less than 100 percent.
### A.2 Additional Univariate Evidence

**Table A.1: Stylized Facts on Financial Policies and Firm Dynamics**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Investment (1) Speculative (2) p-value (3)</td>
<td>Investment (4) Speculative (5) p-value (6)</td>
<td>Investment (7) p-value Speculative (8) p-value (9)</td>
</tr>
<tr>
<td><strong>Assets</strong></td>
<td>7.55 (1) 1.83 (2) 0.00 (3)</td>
<td>9.06 (4) 1.94 (5) 0.00 (6)</td>
<td>1.51 (7) 0.00 (8) 0.11 (9) 0.00 (10)</td>
</tr>
<tr>
<td><strong>Market fraction</strong></td>
<td>0.80 (1) 0.61 (2) 0.00 (3)</td>
<td>0.94 (4) 0.64 (5) 0.00 (6)</td>
<td>0.14 (7) 0.00 (8) 0.03 (9) 0.00 (10)</td>
</tr>
<tr>
<td><strong>Market only</strong></td>
<td>0.10 (1) 0.07 (2) 0.00 (3)</td>
<td>0.27 (4) 0.12 (5) 0.00 (6)</td>
<td>0.17 (7) 0.00 (8) 0.05 (9) 0.00 (10)</td>
</tr>
<tr>
<td><strong>Cash to asset</strong></td>
<td>0.08 (1) 0.09 (2) 0.06 (3)</td>
<td>0.11 (4) 0.08 (5) 0.00 (6)</td>
<td>0.03 (7) 0.00 (8) -0.01 (9) 0.05 (10)</td>
</tr>
<tr>
<td><strong>Leverage</strong></td>
<td>0.30 (1) 0.43 (2) 0.00 (3)</td>
<td>0.39 (4) 0.39 (5) 0.00 (6)</td>
<td>0.09 (7) 0.00 (8) -0.04 (9) 0.00 (10)</td>
</tr>
<tr>
<td><strong>Capex to asset</strong></td>
<td>0.05 (1) 0.04 (2) 0.06 (3)</td>
<td>0.03 (4) 0.05 (5) 0.00 (6)</td>
<td>-0.02 (7) 0.00 (8) 0.01 (9) 0.06 (10)</td>
</tr>
<tr>
<td><strong>Sales to asset</strong></td>
<td>0.24 (1) 0.18 (2) 0.00 (3)</td>
<td>0.20 (4) 0.21 (5) 0.00 (6)</td>
<td>-0.04 (7) 0.00 (8) 0.03 (9) 0.00 (10)</td>
</tr>
<tr>
<td><strong># Observations</strong></td>
<td>1,318 (1) 1,452 (2) 9,253 (3)</td>
<td>9,253 (4) 9,379 (5) 9,253 (6)</td>
<td></td>
</tr>
</tbody>
</table>

Note: This table presents the medians aggregated across all quarters before the crisis (columns (1)–(3)) and after the crisis (columns (4)–(6)), among all investment grade firms (columns (1) and (4)) and speculative grade firms (columns (2) and (5)). The p-values for the differences in medians between the two groups of firms are reported in columns (3) and (6) for the pre- and post-crisis subsamples, respectively. The differences in medians between the pre- and post-crisis subsamples are reported in column (7) for the investment grade firms, and column (8) for the speculative grade firms, and the corresponding p-values are reported in columns (9) and (10), respectively. The sample includes all Compustat firm-year observations from 2006Q1 to 2015Q4 with positive values for the book value of total assets, and data available on debt structure from Capital IQ, for firms with Standard & Poor’s ratings incorporated in the United States. Financial firms (SIC code 6000-6999), utilities (SIC 4900-4949) and the largest 25 cash holders are excluded from the sample, yielding a panel of 21,402 firm-quarter observations for 938 unique firms. Assets are in billions of 2009 dollars. Market fraction is the percentage of market debt to the sum of bank and market debt. Market only is the fraction of firms with only have market debt in a particular quarter. Cash to asset, Leverage, Capex to asset, and Sales to asset are expressed as percentages of book assets. Detailed variable definitions are provided in Appendix A.1.

**Table A.2: Fraction of Firms by Industry in Each Subsample**

<table>
<thead>
<tr>
<th>Industry (%)</th>
<th>SIC</th>
<th>Investment grade</th>
<th>Speculative grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture &amp; Forestry</td>
<td>0100-0999</td>
<td>0.37 (1)</td>
<td>0.21 (2)</td>
</tr>
<tr>
<td>Mining</td>
<td>1000-1499</td>
<td>10.25 (3)</td>
<td>14.62 (4)</td>
</tr>
<tr>
<td>Construction</td>
<td>1500-1799</td>
<td>0.93 (5)</td>
<td>3.80 (6)</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>2000-3999</td>
<td>50.85 (7)</td>
<td>40.25 (8)</td>
</tr>
<tr>
<td>Transportation &amp; Electricity</td>
<td>4000-4999</td>
<td>9.76 (9)</td>
<td>12.41 (10)</td>
</tr>
<tr>
<td>Wholesale Trade</td>
<td>5000-5199</td>
<td>4.11 (11)</td>
<td>3.87 (12)</td>
</tr>
<tr>
<td>Retail Trade</td>
<td>5200-5999</td>
<td>9.98 (13)</td>
<td>8.17 (14)</td>
</tr>
<tr>
<td>Services</td>
<td>7000-8999</td>
<td>13.22 (15)</td>
<td>16.59 (16)</td>
</tr>
</tbody>
</table>

Note: This table presents the medians aggregated across all quarters before the crisis (columns (1)–(3)) and after the crisis (columns (4)–(6)), among all investment grade firms (columns (1) and (4)) and speculative grade firms (columns (2) and (5)). The p-values for the differences in medians between the two groups of firms are reported in columns (3) and (6) for the pre- and post-crisis subsamples, respectively. The differences in medians between the pre- and post-crisis subsamples are reported in column (7) for the investment grade firms, and column (8) for the speculative grade firms, and the corresponding p-values are reported in columns (9) and (10), respectively. The sample includes all Compustat firm-year observations from 2006Q1 to 2015Q4 with positive values for the book value of total assets, and data available on debt structure from Capital IQ, for firms with Standard & Poor’s ratings incorporated in the United States. Financial firms (SIC code 6000-6999), utilities (SIC 4900-4949) and the largest 25 cash holders are excluded from the sample, yielding a panel of 21,402 firm-quarter observations for 938 unique firms. A.1.
Figure A.1: Firm Heterogeneity in Debt Composition and Firms’ Balance Sheet Policies (Sample split by quartiles of firms’ total assets)

Note: The sample includes all Compustat firm-quarter observations from 2006Q1 to 2015Q4 with positive values for the book value of total assets, and data available on debt structure from Capital IQ, for firms with Standard & Poor’s ratings incorporated in the United States. Financial firms (SIC code 6000-6999), utilities (SIC 4900-4949) and the largest 25 cash holders are excluded from the sample, yielding a panel of 21,402 firm-quarter observations for 938 unique firms. To remove seasonality in financing activities, all panels report the raw series (dashed lines) and its smoothed version (solid lines) as a moving average straddling the current term with two lagged and two forward terms. Variable definitions are provided in Appendix A.1.
A.3 Equity Financing: Data

Equity financing is measured by the total net amount of equity raised (panel (a)). Following Fama and French (2005), I measure the total net amount of equity raised with the change in the book value of equity.\textsuperscript{25} Dividend payment in panel (b) is measured by the number of shares outstanding times the dollar value per share, averaged across firms in a particular category in a particular period. As shown below, the two groups of firms had similar patterns of equity financing and dividend payment since the crisis, unlike the divergence in bond financing and cash holdings captured in Figure 1.

**Figure A.2:** Equity Financing by Credit Ratings

<table>
<thead>
<tr>
<th>(a) Net equity raised</th>
<th>(b) Dividend payment</th>
</tr>
</thead>
</table>

**Note:** The sample includes all Compustat firm-quarter observations from 2006Q1 to 2015Q4 with positive values for the book value of total assets, and data available on debt structure from Capital IQ, for firms with Standard & Poor’s ratings incorporated in the United States. Financial firms (SIC code 6000-6999), utilities (SIC 4900-4949) and the largest 25 cash holders are excluded from the sample, yielding a panel of 21,402 firm-quarter observations for 938 unique firms. To remove seasonality in financing activities, all panels report the raw series (dashed lines) and its smoothed version (solid lines) as a moving average straddling the current term with two lagged and two forward terms. Variable definitions are provided in Appendix A.1.

\textsuperscript{25}For the purpose of studying firm financing over the business cycle, it is important to use the book value, as we are interested in measuring how much funds firms raise, not in changes in the valuation of existing equity. Moreover, this is an all encompassing measure of equity, that does not only capture the sale of stock and repurchases, but also equity raised through options and warrants being exercised.
A.4 Panel Evidence

This subsection provides evidence from the following panel regressions to corroborate the descriptive statistics:

\[
\text{Cash}_{i,t} = \beta_1 \text{DebtStructure}_{i,t-1} + \theta \text{Controls}_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t},
\]

where the independent variable of interest, DebtStructure\(_{i,t-1}\), is a proxy for firm \(i\)'s debt structure in year \(t-1\), and Controls\(_{i,t-1}\) is a vector of firm-level controls including firm size, cash flow, leverage, market-to-book ratio, capital expenditures, net working capital, R&D, acquisition expenditures, asset tangibility, and a dummy for whether the firm pays dividend in any given year (e.g., Bates, Kahle and Stulz (2009)). Equation (A.1) also includes a firm fixed effect \(\eta_i\) and a time fixed effect \(\lambda_t\). For robustness, I consider two measures of debt structure. The first measure, MarketFraction\(_{i,t-1}\), is the ratio of market debt to the sum of market debt and bank debt for firm \(i\) in year \(t\). In addition, I also construct an indicator variable MarketOnly\(_{i,t-1}\) for each firm-year \((i,t)\) in the sample, which is equal to one if MarketFraction\(_{i,t-1}\) is one and zero otherwise. Statistical significance is evaluated using robust clustered standard errors adjusted for non-independence of observations within firms.

| Table A.3: Panel Evidence on Corporate Debt Composition and Firm Financing |
|---|---|---|---|---|---|
|  | Full sample |  | Investment grades |  | Speculative grades |  |
|  | (1) | (2) | (1) | (2) | (1) | (2) |
| DebtStructure\(_{i,t-1}\) | 0.027*** | 0.019** | 0.023*** | 0.015** | 0.034** | 0.024** |
|  | (0.002) | (0.010) | (0.003) | (0.023) | (0.012) | (0.028) |
| # Observations | 4,683 | 4,683 | 2,178 | 2,178 | 2,505 | 2,505 |
| # Clusters (firms) | 867 | 867 | 327 | 327 | 540 | 540 |
| Year fixed effects | Yes | Yes | Yes | Yes | Yes | Yes |
| Firm controls | Yes | Yes | Yes | Yes | Yes | Yes |
| Within R\(^2\) | 0.775 | 0.756 | 0.820 | 0.796 | 0.746 | 0.718 |

Note: The sample includes all Compustat firm-year observations from 2006 to 2015 with positive values for the book value of total assets, and data available on debt structure from Capital IQ, for firms with Standard & Poor’s ratings incorporated in the United States. Financial firms (SIC code 6000-6999), utilities (SIC 4900-4949) and the largest 25 cash holders are excluded from the sample. Columns (1) report the estimates from panel regressions of cash holdings to book assets on MarketFraction\(_{i,t-1}\), and columns (2) report estimates from similar regressions but replaces MarketFraction\(_{i,t-1}\) by the indicator variable MarketOnly\(_{i,t-1}\). Year dummies as well as firm-level controls for standard determinants of cash holdings are included in all regressions. P-values are in parentheses and are clustered at the firm level. Detailed variable definitions are in Appendix A.1. ***, **, and * denote statistical significance at the 1%, 5% and 10% levels, respectively.

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26In line with the empirical literature on the determinants of corporate cash holdings, I use the annual counterpart to the quarterly dataset described in the previous section in these regressions, and one period lagged explanatory variables are used to reduce endogeneity concerns associated with using contemporaneous explanatory variables.
B Model Appendix

In this appendix, I provide details regarding the key elements of the model in Sections 3 and 4. Subsection B.1 provides evidence for the key assumption in the model, on the timing of portfolio adjustment. Subsection C lays out the debt settlement outcomes in the presence of two types of lenders. Subsection D provides the theoretical results on dividends. Subsection D.1 explains the calibration of the financial shock, while D.2 summarizes the calibration details. Subsections D.3 and D.4 illustrate two sets of robustness results for the mechanism of the theoretical model: one uses the working capital constraint set up (Jermann and Quadrini (2012a)), and the other introduces equity financing.

B.1 Evidence for Portfolio Adjustment Assumption

To test the validity of assumption 1, I compute, for the median firm of each quartile of firms by assets, the coefficient of variation (standard deviation divided by the mean) for: (i) cash as a proportion of total assets (column (1)), (ii) market debt as a proportion of total assets (columns (2)–(3)), (iii) bank debt as a proportion of total assets (columns (4)–(5)), (iv) total debt as a proportion of total assets (columns (6)–(7)), using the annual Compustat dataset between 2006 and 2015 as described in Section 2. The results are reported in Table B.1, which shows that the coefficient of variation for cash is consistently higher than the correlation of variation for debt, across all definitions of debt and all quartiles of firms, and the differences are significant at the 1% level (with the p-values reported in columns (3), (5), and (7), respectively). As shown in Table B.1, market debt has the lowest coefficient of variation.

<table>
<thead>
<tr>
<th></th>
<th>Cash</th>
<th>Market debt</th>
<th>Bank debt</th>
<th>Total debt</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>stdev</td>
<td>mean</td>
<td>stdev</td>
<td>mean</td>
</tr>
<tr>
<td>Q1</td>
<td>0.57</td>
<td>0.21</td>
<td>(0.00)</td>
<td>0.43</td>
</tr>
<tr>
<td>Q2</td>
<td>0.53</td>
<td>0.24</td>
<td>(0.00)</td>
<td>0.41</td>
</tr>
<tr>
<td>Q3</td>
<td>0.54</td>
<td>0.26</td>
<td>(0.00)</td>
<td>0.39</td>
</tr>
<tr>
<td>Q4</td>
<td>0.58</td>
<td>0.31</td>
<td>(0.00)</td>
<td>0.42</td>
</tr>
<tr>
<td>Full sample</td>
<td>0.56</td>
<td>0.28</td>
<td>(0.00)</td>
<td>0.40</td>
</tr>
</tbody>
</table>

Note: The data sample includes all Compustat firm-year observations from 2006 to 2015 with positive values for the book value of total assets, and data available on debt structure from Capital IQ, for firms with Standard & Poor’s ratings incorporated in the United States. Financial firms (SIC code 6000-6999), utilities (SIC 4900-4949) and the top 1% of companies (the largest 25 cash holders) are excluded from the sample. Columns (1), (2), (4) and (6) report the coefficient of variation (standard deviation divided by the mean) for the variable of interest as a proportion of total assets. Columns (3), (5) and (7) report the p-values of the differences between the coefficients of variation of the corresponding type of debt and cash. Results also hold using the quarterly sample counterpart.
C  Debt Settlement Outcomes

Lemma 1. There are two types of debt settlement outcomes, depending on the relative amount of the market debt vis-à-vis bank debt in the firm’s portfolio:

- (R-contract) If $\frac{b'}{\chi} \geq \frac{m'}{1-\chi}$, the firm chooses to repay its creditors in full if and only if $\pi' \geq \frac{b'}{\chi}$. It successfully restructures its debt if and only if $\frac{b'}{\chi} < \pi' < \frac{b'}{\chi}$, and it is liquidated when $\pi' < \frac{b'}{1-\chi}$.

The restructured amount of bank debt is $b'_R = \chi \pi'$.

- (NR-contract) If $\frac{b'}{\chi} < \frac{m'}{1-\chi}$, the firm repays its creditors in full if and only if $\pi' \geq b' + m'$, and it is liquidated otherwise.

where $\pi' = p'(\psi')y + p'((1-\delta)\hat{k} + \hat{d}_f)$. In all debt settlement outcomes resulting in liquidations, the firm’s payoff is $V' = 0$.

Proof. The proof proceeds by comparing the value of the firm at the debt settlement stage, under repayment $V''_P$, restructuring $V''_R$, and liquidation $V''_L$. Recall that:

$$V''_P(k', x'; s') = (1 - \eta)(\pi' - b' - m') + \eta V'(k', x'; s')$$

$$\geq (1 - \eta)(\pi' - b' - m'),$$

and

$$V''_R(k', x'_R; s') = (1 - \eta)(\pi' - b'_R - m') + \eta V'(k', x'; s')$$

$$\geq (1 - \eta)(\pi' - b'_R - m').$$

By assumption, the firm is forced into liquidation by the lenders if its current period resources $\pi'$ cannot repay both types of debt, even with restructuring. As the firm is the last residual claimant, the payoff to the firm in liquidation is given by:

$$V''_L(k', x'_R; s') = \max (0, (1 - \eta)(\chi \pi' - b' - m')),$$

where $V'(k', x'; s')$ is the continuation value of the firm that does not default in period $t + 1$ (i.e. after debt settlement), and $b'_R = \min(b', \chi \pi')$. There are two types of contracts:

1. R-contract: $\frac{m'}{1-\chi} \leq \frac{b'}{\chi}$
   - when $\pi' \geq \frac{b' + m'}{\chi}$;
     $$V''_L = (1 - \eta)(\chi \pi' - b' - m') < (1 - \eta)(\pi' - b' - m') \leq V''_P,$$
     and $V''_P = V''_R$, as $b'_R = \min(b', \chi \pi') = b'$;  \(\implies\) Repayment;
   - when $\frac{b' + m'}{\chi} > \pi' \geq \frac{b'}{\chi}$;
     $\pi' \geq \frac{b'}{\chi} \geq b' + m'$, so $V''_P \geq (1 - \eta)(\pi' - b' - m') > 0 = V''_L$, and $V''_R = V''_P$ as $b'_R = b'$;  \(\implies\) Repayment;
   - when $\frac{b'}{\chi} > \pi' \geq b' + m'$;
     $$V''_R \geq (1 - \eta)((1 - \chi)\pi' - m'),$$
     as $b'_R = \min(b', \chi \pi') = \chi \pi'$, so $V''_R > (1 - \eta)(\pi' - b' - m') \geq 0$ and thus $V''_R \geq V''_P; V''_L = 0 < V''_R; \implies\) Restructuring;
• when \( b' + m' \geq \pi' \geq \frac{m'}{1-\chi} \):
  \[ V_L^{\theta'} = 0 \leq (1-\eta)((1-\chi)\pi' - m') \leq V_R^{\theta'}, \]
  as \( b' = \min(\chi\pi', b') = \chi\pi' \), and \( V_P^{\theta'} < V_R^{\theta'} \);
  \( \implies \) Restructuring;

• when \( \frac{m'}{1-\chi} > \pi' \):
  \[ V_L^{\theta'} = 0 \text{ and again } V_P^{\theta'} < V_R^{\theta'}; \] moreover, \( b_R' = \chi\pi' \), but \( (1-\eta)((1-\chi)\pi' - m') < 0 \), so even with restructuring, firm cannot repay both types of debt with the current period’s resources; \( \implies \) Liquidation.

2. NR-contract: \( \frac{m'}{1-\chi} > \frac{b'}{\chi} \):

• when \( \pi' \geq \frac{b' + m'}{\chi} \):
  \[ V_L^{\theta'} = (1-\eta)(\chi\pi' - b' - m') < (1-\eta)(\pi' - b' - m') \leq V_P^{\theta'}; \]
  \( b_R' = \min(b', \chi\pi') = b' \) so \( V_L^{\theta'} = V_P^{\theta'} \); \( \implies \) Repayment.

• when \( \frac{b' + m'}{\chi} > \pi' \geq b' + m' \):
  again \( b_R' = \min(b', \chi\pi') = b' \) so \( V_L^{\theta'} = V_P^{\theta'} > 0; V_L^{\theta'} = 0 \implies \) Repayment.

• when \( b' + m' > \pi' \):
  \[ V_L^{\theta'} = 0; b_R' = \min(b', \chi\pi') = b' \] so \( V_P^{\theta'} = V_R^{\theta'} \), but \( (1-\eta)(\pi' - b' - m') < 0 \) i.e. there is no gain from restructuring, and the firm cannot repay both types of debt with the current period’s resources; \( \implies \) Liquidation.

Therefore, in an R-contract, the firm repays when \( \pi' \geq \frac{b'}{\chi} \), restructures when \( \frac{b'}{\chi} \geq \pi > \frac{m'}{1-\chi} \) with the renegotiated amount of bank debt equal to \( b_R' = \chi\pi' \), and liquidated otherwise; in a NR-contract, the firm repays when \( \pi' \geq b' + m' \), and liquidates otherwise. In default, \( V_L^{\theta} = 0 \), regardless the contract chosen, because otherwise \( \pi' - b' - m' \geq \chi\pi' - b' - m' > 0 \), in which case the firm would be better off by paying its creditors.

---

**D Dividend Policy**

**Lemma 2.** It is optimal that continuing firms do not pay dividends to households, unless they assign a zero probability to a binding dividend constraint in the future.

**Proof.** For the sake of argument, assume that the optimal borrowing conditions can be obtained directly by differentiating the Bellmann equation (30) with respect to \( b' \) and \( m' \), thus ignoring any nondifferentiabilities. Let \( \mu \) denote the multiplier to the non-negativity dividend constraint.

First, suppose that the firm takes on positive amount of debt \((b' > 0 \text{ or } m' > 0)\). If it chooses a
Recall that the prices of bank debt $b$, and the first order condition by differentiating (31) with respect to NR-contract $m$ is:

$$
(1 + \mu) \left[ q^b + \frac{\partial q^b}{\partial b'} b' + \frac{\partial q^m}{\partial b'} m' \right] + E \left[ \lambda(s, s') \left( \int_{\psi_{NR}}^{+\infty} \frac{\partial V^\psi}{\partial b'} dF(\psi') - V^\psi(\psi_{NR}) f(\psi_{NR}) \frac{\partial \psi_{NR}}{\partial b'} \right) \right] = 0.
$$

(B.1)

It has to be true that $V^\psi(\psi_{NR}) = 0$, as lemma 1 established that the firm value default is zero, and $\psi_{NR}$ is the threshold value of the demand (profit) shock $\psi'$ below which the firm starts to default. Moreover, the associated Benveniste-Scheinkman condition is given by:

$$
\frac{\partial V^\psi}{\partial b'} = -(1 + \eta b'),
$$

so the first order condition (B.1) becomes:

$$
(1 + \mu) \left[ q^b + \frac{\partial q^b}{\partial b'} b' + \frac{\partial q^m}{\partial b'} m' \right] = E \left[ \lambda(s, s') \left( 1 + \eta b' \right) dF(\psi') \right] - \gamma^b,
$$

(B.2)

Recall that the prices of bank debt $q^b$ and market debt $q^m$ in a NR-contract are:

$$
q^b(k', b', a', z, \psi; s) = E \left[ \lambda(s, s') \left( 1 - F(\psi_{NR}) \right) + \int_{-\infty}^{\psi_{NR}} \frac{\chi \pi'}{b'} dF(\psi') \right] - \gamma^b,
$$

(B.3)

and

$$
q^m(k', b', m', a', z, \psi; s) = E \left[ \lambda(s, s') \left( 1 - F(\psi_{NR}) \right) + \int_{-\infty}^{\psi_{NR}} \frac{\chi \pi' - b'}{m'} dF(\psi') \right] - \gamma^m,
$$

(B.4)

and $\psi_{NR}$ and $\psi_{NR}'$ are the full default and partial default thresholds, respectively. Note that $\psi_{NR}' \leq \psi_{NR}$, as the firm defaults upon market debt first (if $\psi_{NR}' \leq \psi' \leq \psi_{NR}$), given the assumption that bank debt is more senior than market debt.

$\frac{\partial q^b}{\partial b'}$ and $\frac{\partial q^m}{\partial b'}$ can be determined using the Leibniz rule:

$$
\frac{\partial q^b}{\partial b'} = E \left[ \lambda(s, s') \left( - \int_{-\infty}^{\psi_{NR}} \frac{\chi \pi'}{b'} dF(\psi') - \left( 1 - \frac{\chi \pi'}{b'} \right) b' f(\psi_{NR}) \frac{\partial \psi_{NR}}{\partial b'} \right) \right],
$$

(B.5)

and

$$
\frac{\partial q^m}{\partial b'} = E \left[ \lambda(s, s') \left( - \left( F(\psi_{NR}) - F(\psi_{NR}') \right) \left( m' - (\chi \pi' - b') \right) f(\psi_{NR}) \frac{\partial \psi_{NR}}{\partial b'} - (\chi \pi' - b') f(\psi_{NR}) \frac{\partial \psi_{NR}}{\partial b'} \right) \right].
$$

(B.6)

Substitute equations (B.3), (B.5), and (B.6) in the first order condition (B.2), and simplify:

$$
\mu \left( q^b + \frac{\partial q^b}{\partial b'} b' + \frac{\partial q^m}{\partial b'} m' \right) = E \left[ \lambda(s, s') \left( \eta b' \left( 1 - F(\psi_{NR}) \right) + \left( 1 - \frac{\chi \pi' - b'}{m'} \right) m' f(\psi_{NR}) \frac{\partial \psi_{NR}}{\partial b'} \right) \right] + \gamma^b.
$$

(B.7)

\(^{27}\)If firm only takes on positive market debt $m' > 0$, the proof goes through by differentiating (31) with respect to $m'$. The results of are available upon request.
Thus, condition (B.7) implies that unless the firm puts probability 1 on \( \mu' = 0 \), \( \mu \) can never be 0 for the first order condition to hold; in other words, if the firm chooses an NR-contract, the dividend constraint has to bind at all times.

The proof for the case of R-contract goes through by the same logic. The first order condition by differentiating (32) with respect to \( b' \) is:

\[
(1 + \mu) \left[ q^b + \frac{\partial q^b}{\partial b'} b' \right] + E \left[ \lambda(s, s') \left( \int_{\psi_R}^{+\infty} \frac{\partial V^0(k', x'; s')}{\partial b'} dF(\psi') - V^0(\psi') f(\psi') \frac{\partial V'_R}{\partial b'} \right) \right] = 0. 
\]  
(B.8)

As \( \psi_R \) is the default threshold in the R-contract, so \( V^0(\psi_R) = 0 \), and the first order condition (B.8) can be simplified to:

\[
(1 + \mu) \left[ q^b + \frac{\partial q^b}{\partial b'} b' \right] + E \left[ \lambda(s, s') \left( \int_{\psi_R}^{+\infty} \frac{\partial V^0(k', x'; s')}{\partial b'} dF(\psi') + \int_{\psi_R}^0 \frac{\partial V^0(k', x'; s')}{\partial b'} dF(\psi') \right) \right] = 0.
\]

The associated Benveniste-Scheinkman conditions are:

\[
\frac{\partial V^0(k', x'; s')}{\partial b'} = -(1 + \eta \mu'), \quad \frac{\partial V^0(k', x'; s')}{\partial b} = 0,
\]

and the latter arises because \( x'_R = p'(\psi)g' - F_ch - b'R - m' + \tilde{a}, \) and \( b'_R = \chi \pi' \) as shown in lemma 1. Therefore, the first order condition can be further simplified to:

\[
(1 + \mu) \left[ q^b + \frac{\partial q^b}{\partial b'} b' \right] = E \left[ \lambda(s, s') \left( \int_{\psi_R}^{+\infty} \left( (1 + \eta \mu') dF(\psi') \right) \right) \right]
\]

(B.9)

The price of bank debt \( q^b \) in an R-contract is:

\[
q^b(k', b', a'; s) = E \left[ \lambda(s, s') \left( 1 - F(\psi_R) \right) + \lambda(s, s') \eta \mu' \left( 1 - F(\psi_R) \right) \right] - \gamma^b,
\]  
(B.10)

and \( \psi_R \) is the restructuring threshold, and the bank lender’s payoff is the same under restructur- ing and default, due to the combination of bank seniority in default and the bargaining outcome in renegotiation. Again, \( \frac{\partial q^b}{\partial b'} \) can be determined using the Leibniz rule:

\[
E \left[ \lambda(s, s') \left( - \int_{-\infty}^{\psi_R} \frac{\chi \pi'}{b'} dF(\psi') - \left( 1 - \frac{\chi \pi'}{b'} \right) b' f(\psi_R) \frac{\partial \psi_R}{\partial b'} \right) \right].
\]  
(B.11)

Substitute equations (B.10) and (B.11) in the first order condition (B.9), and simplify:

\[
\mu \left( q^b + \frac{\partial q^b}{\partial b'} b' \right) = E \left[ \lambda(s, s') \left( \eta \mu' \left( 1 - F(\psi_R) \right) + \left( 1 - \frac{\chi \pi'}{b'} \right) b' f(\psi_R) \frac{\partial \psi_R}{\partial b'} \right) \right] + \gamma^b.
\]  
(B.12)
Again, condition (B.12) implies that unless the firm puts probability 1 on \( \mu' = 0 \), \( \mu \) can never be 0 for the first order condition to hold. Thus regardless the contract chosen, for a firm with positive debt, the dividend constraint has to bind at all times, i.e. firms are always better off paying back their debt than issuing dividends.

Next, consider the case where the firm only saves \((a_f' > 0)\) at price \( q^a = E[\lambda(s, s')] \) and does not borrow \((b' = m' = 0)\). The Bellman equation becomes:

\[
V^1(\hat{k}, x; s) = \max_{v, k', a_f'} \left\{ d + E \left[ \lambda(s, s') \hat{V}^0(k', a_f', \psi', \alpha'; s') | s \right] \right\} \\
= \max_{v, k', a_f'} \left\{ d + E \left[ \lambda(s, s') \max_{k', a_f'} \hat{V}^0(k', x'; s') | s \right] \right\}
\]

subject to (1), (3), (21), (22), (23), and \( s' = \Gamma(s); \quad i, j = 1, 2, ..., N \).

The first order condition with respect to \( a_f' \) yields:

\[
(1 + \mu)q^a = E[\lambda(s, s')(1 + \eta\mu')]
\]

\[
\mu E[\lambda(s, s')] = E[\lambda(s, s')(\eta\mu')]
\]

Therefore, \( \mu = 0 \) if and only if the firm assigns zero probability to the event that the dividend constraint will be binding in the next period. Therefore, firms with no borrowing are at most indifferent between paying dividends and savings, as they share the same discount factor as households.
D.1 Indicators of Changes In the Supply of Bank Intermediated Credit

The data used to calibrate the persistence and standard deviation of the bank credit supply shock comes from the Federal Reserve’s Senior Loan Officer Opinion Survey of Bank Lending Practices (SLOOS). This survey has queried banks about changes in their lending standards for the major categories of loans to households and businesses beginning with the April 1990 survey and about changes in demand for most of those types of loans starting with the October 1991 survey. The SLOOS is usually conducted four times per year by the Federal Reserve Board, and up to 80 U.S. commercial banks participate in each survey. Participating banks are asked to report whether they have changed their standards during the survey period in several categories of core loans.

**Figure B.3: Proxy for Bank Credit Supply**

(a) Net Percentage of Domestic Banks Tightening Standards

(b) Net Percentage of Domestic Banks Increasing Spreads of Loan Rates

*Note:* Quarterly data is obtained from the Senior Loan Officer Opinion Survey on Bank Lending Practices release of the Board of Governors of the Federal Reserve System. Panel (a) plots the net percentage of domestic banks tightening standards for commercial and industrial loans to large and middle-market firms (dashed blue line) and small firms (solid red line), where “net percentage” refers to the fraction of banks that reported having tightened (“tightened considerably” or “tightened somewhat”) minus the fraction of banks that reported having eased (“eased considerably” or “eased somewhat”). Panel (b) plots the net percentage of domestic banks increasing spreads of loan rates over banks’ cost of funds for large and middle-market firms (dashed blue line) and small firms (solid red line), where “net percentage” refers to the fraction of banks that reported having increased the spreads minus the fraction of banks that reported having reduced the spreads.

Nevertheless, in assessing the supply-side implications of changes in bank lending policies, it is important to bear in mind that the changes in bank lending standards reported in the SLOOS reflect the confluence of demand and supply factors. Recognizing this endogeneity problem, I use VAR-based identification strategies to identify the component of the change in lending standards that is orthogonal to the determinants of loan demand (see, for example, Lown and Morgan (2006)). Specifically, I estimate for the relative supply of bank credit $\gamma^b$ using a VAR(4) specification with four quarterly macroeconomic variables—including log real GDP, log GDP deflator, log commodity prices, and the federal funds rate—and the net percent of banks reporting tightening standards. I order the credit variable after the macro variables. Summing the coefficients on lags of the lending standard variables yields $\hat{\rho}_\gamma = 0.81$ and $\hat{\sigma}_\gamma = 0.085$, which are the values used in the calibration. For robustness, I also estimate an AR(1) estimation using the change in the loan spreads (panel (b)), which gives $\hat{\rho}_\gamma = 0.89$ and $\hat{\sigma}_\gamma = 0.072$. 
### D.2 Calibration Summary

<table>
<thead>
<tr>
<th>Model Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Production and capital accumulation</strong></td>
<td></td>
</tr>
<tr>
<td>Value-added share of capital ($\alpha$)</td>
<td>0.30</td>
</tr>
<tr>
<td>Decreasing returns to scale ($\alpha$)</td>
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</tr>
<tr>
<td>Depreciation rate ($\delta$)</td>
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</tr>
<tr>
<td>Quasi-fixed costs of production ($F_0$)</td>
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<tr>
<td>Quasi-fixed costs of investment ($F_{k,0}$)</td>
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<tr>
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<tr>
<td>Resale price of capital ($p^-$)</td>
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<tr>
<td><strong>Firm entry and exogenous exit</strong></td>
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</tr>
<tr>
<td>Survival probability ($\eta$)</td>
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</tr>
<tr>
<td>Pareto distribution ($\omega$)</td>
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<tr>
<td>Initial equity issuance cost ($\gamma^e$)</td>
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<tr>
<td>Entry cost ($c_e$)</td>
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</tr>
<tr>
<td>Mass of potential entrants ($M$)</td>
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</tr>
<tr>
<td><strong>Financial markets</strong></td>
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</tr>
<tr>
<td>Market debt intermediation cost ($\gamma^m$)</td>
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</tr>
<tr>
<td>Efficiency of liquidation ($\chi$)</td>
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</tr>
<tr>
<td><strong>Representative household</strong></td>
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</tr>
<tr>
<td>Discount factor ($\beta$)</td>
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</tr>
<tr>
<td><strong>Exogenous shocks</strong></td>
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</tr>
<tr>
<td>Persistence of the shock to the wedge in intermediation costs ($\rho_\gamma$)</td>
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</tr>
<tr>
<td>Volatility of the innovations of the wedge in intermediation costs ($\sigma_\gamma$)</td>
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</tr>
<tr>
<td>Steady state level of the wedge in intermediation costs ($\bar{\gamma}$)</td>
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</tr>
<tr>
<td>Persistence of the idiosyncratic technology shock process ($\rho_\epsilon$)</td>
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<tr>
<td>Steady-state level of idiosyncratic uncertainty ($\bar{\sigma}_\epsilon$)</td>
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<tr>
<td>Persistence of the idiosyncratic uncertainty ($\rho_\sigma$)</td>
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<td>Volatility of innovations of the idiosyncratic uncertainty process ($\omega_\sigma$)</td>
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<td>Persistence of the idiosyncratic demand shock process ($\rho_\psi$)</td>
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<tr>
<td>Volatility of the idiosyncratic demand shock process ($\sigma_\psi$)</td>
<td>0.23</td>
</tr>
</tbody>
</table>
financial & productivity shocks \((\sigma, \gamma^*, z)\) \hspace{1cm} Debt Settlement \hspace{1cm} choose \((k', a'_f)\)

\begin{align*}
\text{Production} & \quad \text{Debt Settlement} & \quad \text{choose} \quad (k', a'_f) \\
\text{t} & \quad \text{t+1} \\
(k, a_f) & \quad \text{(exogenous exit} \; 1 - \eta) \\
\end{align*}

\begin{align*}
\text{demand} & \quad \text{shock} \; \psi \\
\text{choose} & \quad h \\
\text{and} & \quad \text{can borrow} \\
(b, m) & \quad \text{to pay} \\
\text{w, } F_o k & \\
\text{(working capital constraint)} \\
\end{align*}

\textbf{Figure B.4:} Overview and Timing of Intermediate Goods Firms’ Problem

\section*{D.3 Robustness: Working Capital Constraint Set-up}

In this section, I present an alternative set-up of the theoretical framework that motivates firms to borrow and simultaneously hold cash balances. The model builds on Jermann and Quadrini (2012), and the key assumption is that firms have to finance their working capital before they receive revenues from sales, and can do so through accumulated cash holdings, or by issuing two types of debt: bank loans and market debt. Due to the separation between the time of financing and the receipt of revenues, any idiosyncratic shocks, such as demand shocks, that occur between these times can cause costly default of firms.

\textbf{Timing}  \ Figure B.4 summarizes the timing of each intermediate goods firm’s problem. At the beginning of each period, all shocks pertaining to the production and borrowing decisions—including the level of idiosyncratic uncertainty \((\sigma)\), the relative supply of bank credit \((\gamma^*)\), and the level of idiosyncratic technology \((z)\)—are realized. The volatility level \(\sigma\) determines the distribution of \(z'(\sigma)\) in the next period (Bloom (2009); Gilchrist, Sim and Zakrajšek (2014)). Thus, from the agents’ perspective, an increase in \(\sigma\) represents “news” regarding the distribution of profits tomorrow. Consistent with the typical timing convention, capital \((k)\) is predetermined, whereas the input of labor \((h)\) is chosen after the realization of shocks \((z, \sigma, \gamma^*)\), from:

\[
\tilde{\pi}(z, w, k) = \max_{h \geq 0} \left\{ z(1 - \alpha) \chi \left( k^{\alpha} h^{1 - \alpha} \chi - F_o k - wh \right) \right\} 
= z \Phi(w) k^\phi - F_o k - wh,
\]

where: \(\phi = \frac{\alpha \chi}{1 - (1 - \alpha) \chi}\) and \(\Phi(w) = [1 - (1 - \alpha) \chi] \left( \frac{(1 - \alpha) \chi}{w} \right)^{(1 - \alpha) \chi}\)

subject to:

\[
a_f^{\text{internal funds}} + q^b b + q^m m \geq wh + F_o k. \quad \text{(B.15)}
\]

Hence, equation (B.15) is a working capital constraint that motivates firms to borrow, if accumulated savings cannot cover the payments fully (Jermann and Quadrini (2012b)). Specifically, firms have to pay the wage bill and operating costs that are proportional to the predetermined capital stock \(F_o k\) before their revenues are realized, using a combination of the predetermined
Figure B.5: Impact of a Financial Shock to Bank Credit Supply (Baseline: Working Capital Constraint Version)

Note: A shock reduces the supply of bank loans ($\gamma^b$) 10 percent upon impact (period 5) on average, a shock of approximately 2 standard deviations; the bank loan supply is then allowed to revert back to its steady-state value following the process in equation (10). The impulse responses are averages of 50,000 simulations, where each simulation is an aggregation of the impulse responses of 10,000 firms. In panels (a)–(g), the blue dashed lines depict the impulse response functions of the investment-grade firms, while the red dashed lines depict the impulse responses of the speculative-grade firms; in panel (i), the blue dotted line indicates the impulse response function of the spread on corporate bonds, while the red dotted line indicates the spread on bank loans.
accumulated cash \((a_f)\), and external funds consisting of a combination of (intra-temporal) bank debt \((b)\) and market debt \((m)\), at prices \((q^b)\) and \((q^m)\), respectively.

After the payments of wage bill and operating costs, the firm produces output using the technology described in (1). After production, idiosyncratic demand shocks \((\psi)\) are realized. At the debt settlement stage, the firm can either repay both types of debt, restructure bank debt, or default, in which case it exits endogenously. As the terms of debt contracts \((q^b, q^n)\) cannot be indexed by \(\psi\), they demand a premium because of the agency costs associated with default. Finally, firms choose the amount of capital \(k'\) and cash holdings \(s'\) that they want to take into next period. Importantly, as cash holdings decisions are made before the realization of shocks, this gives rise to precautionary incentive to accumulate cash. Even though this incentive would be softened by the possibility to issue intra-temporal debt after the realization of production and financial shocks, the presence of financial and real frictions act in the opposite direction, amplifying firms’ incentive to save pre-emptively in order to reduce their reliance on external finance. The impulse response functions to a financial shock are presented in Figure B.5.

D.4 Robustness: Equity Financing

In Section 3, I assume that firms can only issue equity upon entry and not in subsequent periods. In order to assess the role of equity financing and the extent to which the ability to issue equity would dampen the precautionary motives by firms, here I introduce equity financing in the model.

Unless there is friction in equity financing, firms would not issue debt. To introduce frictions in the stock market, I assume that equity issuance is costly, in the sense that the value of existing shares is reduced by more than the amount of newly issued shares. I capture this distortion by assuming a constant marginal cost of equity issuance, commonly referred to as equity dilution costs in the corporate finance literature (see, for instance, Cooley and Quadrini (2001), Gomes (2001), Hennessy and Whited (2007)). Formally, the loss in the value of existing shares associated with the amount \(e\) of newly issued equity is given by:

\[
\bar{\xi}(e) = e + \xi \max\{e, 0\}
\]

where \(\xi\) measures the degree of frictions in the stock market, and \(e\) denotes the value of newly issued shares when positive and the value of share repurchases when negative. Share repurchases \((e < 0)\), in contrast, are assumed equivalent to dividend payments and thus do not involve any dilution costs. Consistent with the prevalence of corporate dividend-smoothing policies, I posit that the firms face a minimum dividend constraint:

\[
d \geq d \geq 0.
\]

(B.16)

Thus, when the firm’s internal funds and the proceeds raised from bond financing fall short of its financing needs, the firm must raise outside equity to satisfy the dividend constraint.\(^{28}\)

\(^{28}\)As documented by Fama and French (2005), in an average year below 1973 and 2002, almost 60 percent of dividend
The firm’s dividend can now be written as:

\[ d = p(\psi) g(z_i(\sigma_{-1})) - F_\delta \hat{k} - b_R - m + \hat{a}_f - v g(k', \hat{k}) + q^b b' + q^m m' - q^a a'_f + e \]

\[ = x - v g(k', \hat{k}) + q^b b' + q^m m' - q^a a'_f + e. \]

The firm’s recursive problem becomes:

\[ V_1^1(\hat{k}, x; s) = \max_{x, k', \beta', \alpha', q} \left\{ d_{NR} + \phi(d - \bar{d}) - \bar{\xi}(e) + E \left[ \lambda(s, s') \sum_{j=1}^{N} p_{ij} \hat{V}_0(k', a'_f, \psi', z'; s') \right] | s \right\} \]

As the model is in general equilibrium, the addition of equity financing also affects the household’s problem, who solves:

\[ W(a_h; s) = \max_{a_h, s'} \left\{ u(c) + \beta E \left[ W(a_h'; s'| s) \right] \right\}, \]

subject to a new budget constraint:

\[ c + q^a a'_h + \int c_s \mu_s(ds) + \int p_s s' \mu(ds) \leq \int \left[ a_h + F_0 \hat{k} + (d + \bar{p}_s) s \right] \mu(ds), \]

where \( p_s \) is the ex-dividend value of equity, \( \bar{p}_s \) is the current market value of equity, and \( 0 \leq s \leq 1 \) is the fraction of outstanding shares owned by the household. Recall that \( ds = [dz, d\psi, d\hat{k}, dx] \), where \( s \) summarizes the idiosyncratic state of a firm \( s = [z, \psi, \hat{k}, x] \). The equity valuation terms are linked by the accounting identity:

\[ \bar{\xi}(e) = p_s - \bar{\xi}(e), \]

where \( \bar{\xi}(e) \geq e \) represents the cost of issuing new shares. The dynamic efficiency conditions associated with the household’s problem pin down equity prices, according to:

\[ p_s(z, \psi, \hat{k}, x; s) = E \left[ m(s, s') \left( d' - \bar{\xi}(e') + p'_s(z', \psi', \hat{k}', x'; s') \right) \right] | s \].

The estimates of the cost of seasoned equity issuance vary substantially in the literature, from a low of 0.08 in Gomes (2001) to a high of 0.30 in Cooley and Quadrini (2001). In line with Gilchrist et al. (2014), I make a conservative choice by letting \( \xi = 0.12 \). Given this value, I choose the lower bound of dividends \( d \) such that 15 percent of firms, on average, issue new shares in each quarter, a proportion that is roughly in line with that implied by the Compustat data. The impulse response functions to a financial shock are presented in Figure B.6. As long as equity financing is costly, the trade-off between debt and equity financing exists, and hence the precautionary savings channel cannot be mitigated. Under the conservative choice of equity financing cost, the size of this channel is reduced by approximately one-quarter to one-third.

paying firms also issued new shares in the same year, on net, evidence that seems difficult to reconcile with the assumption of frictionless financial markets. However, another possibility is that the lower bound on dividends \( d \) is strictly positive for a fraction of firms that engage in dividend smoothing behavior for reasons unrelated to financial market frictions, which is the interpretation adopted here.
Figure B.6: Impact of a Financial Shock to Bank Credit Supply
(Extended Model: With Equity Financing)

(a) Market Debt  (b) Bank Debt  (c) Total Debt  
(d) Capital  (e) Cash  (f) Investment  
(g) Output  (h) Equity issuance  (i) Credit spreads

Note: This figure compares the (aggregate) impulse response functions in the baseline model with those in the counterfactual exercise, whereby firms are allowed to issue equity, subject to an equity issuance cost. A shock reduces the supply of bank loans (γ∗) 10 percent upon impact (period 5) on average, a shock of approximately 2 standard deviations; the bank loan supply is then allowed to revert back to its steady-state value following the process in equation (10). The impulse responses are averages of 50,000 simulations, where each simulation is an aggregation of the impulse responses of 10,000 firms. The black solid lines depict the impulse response functions in the baseline, while the green dashed lines depict the impulse responses in the counterfactual exercise. The vertical axis denotes percentage deviation from the steady state and the horizontal axis denotes quarters.