Time Inconsistency and Financial Covenants*

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Abstract

I investigate how financial covenants influence corporate behavior and firm value by allocating control rights. In a dynamic model with long-term debt, shareholders cannot commit to not expropriate creditors in the future with new debt issuances and risky investments. Creditors intervene upon violations of covenant restrictions and restructure the debt without ex ante commitment. I find that financial covenants significantly increase debt capacity, investment and ex ante firm value by disciplining shareholders. However, I show that lenders' inability to commit to a restructuring plan severely impairs contractual efficiency. My analysis suggests that a further tightening of covenants, relative to the calibrated benchmark, improves their role as a commitment device.

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1 Introduction

Financial covenants are ubiquitous in corporate debt indentures. By allocating certain rights to lenders when borrowers fail to maintain contracted financial ratios, these contingency clauses have long been regarded by theorists, back to [Jensen and Meckling \(1976\)](#) and [Myers \(1977\)](#), as a potential remedy for the agency conflict. Recent empirical advances show that financial covenants are frequently violated and resulting creditor interventions play a critical role in reshaping firm investment and financing going forward.¹ However, four decades later, it remains unclear how these ex post control right reallocations influence ex ante firm behavior and ultimately translate into firm value.

To this end, I develop a tractable model of financial covenants featuring dynamic management of production risk and long-term defaultable debt under limited commitment. I then quantify the ex ante implications and efficiency of these contingency clauses. The key friction that motivates the usage of covenants is shareholders' time inconsistency associated with long-term debt. When able to freely adjust investment and financing without having to repurchase outstanding debt, shareholders end up expropriating legacy lenders. On the financing side, shareholders have the temptation to keep issuing new debt in order to further extract tax shields even if leverage and default risk are already excessive.² On the investment side, they find it beneficial to invest in risky assets when the downside will be primarily borne by legacy lenders. Without covenants, shareholder behavior falls completely out of lenders' control once debt is in place. Lenders thus incorporate the risk of future expropriation into the credit spread, thereby reducing shareholder welfare ex ante.

Financial covenants provide opportunities for lenders to intervene and reduce the risk of being further expropriated. More specifically, when the financial ratio restricted by covenants is breached, usually after negative cash flow shocks, lenders are granted the right to require an additional principal repayment. During the debt restructuring process, shareholders have to comply with whatever acceleration plan lenders find optimal ex post and bear the restructuring costs, before they can retain the control over investment and financing.

There are two channels through which shareholder behavior is disciplined by debt restructuring that covenants introduce. First, the *realization* of a debt restructuring reshapes shareholder behavior going forward. When legacy debt reduces after an acceleration, share-

¹See a review by [Roberts and Sufi \(2009b\)](#).

²See analyses of this “leverage ratchet effect” by [Admati, DeMarzo, Hellwig and Pfleiderer \(2018\)](#).

holders are forced to internalize a larger fraction of the impact of their investment and financing choices. This limits shareholder expropriation in the future and, as a result, increases firm value. Second, shareholder behavior is also affected by the *anticipation* of future contractual violations. When finding debt restructuring privately undesirable, shareholders will slow down debt issuances and risky investment in order to avoid breaching covenants. In that case, precautionary motives alleviate time inconsistency even when violations do not actually realize ex post.

Lenders cannot commit to a restructuring plan. Their ex post incentive to accelerate co-moves negatively with economic fundamentals. First, when default risk is trivial and thus debt trades at a large premium, unreceived interests are safe enough to dissuade lenders from any principal acceleration. Covenant violations in these scenarios are not followed by credit amendment. Secondly, when default risk becomes moderate, long-term lenders would like to discipline future shareholders even at the cost of forgoing some of the default premia. A debt relief takes place and the surplus from a mild debt acceleration is shared between the current equity and debt holders.³ Lastly, for a highly risky firm whose debt trades at a large discount, lenders accelerate outstanding debt aggressively upon violations, which not only generates a huge total surplus but also forces current shareholders to repay some of the debt above the market price.

Quantitative applications of my model deliver the following key results. First of all, currently adopted financial covenants improve the ex ante firm value, before any investment and financing take place, by around 1%. Compared to a net benefit of debt of 4%, such an improvement suggests that the value of discipline imposed on shareholders by existing covenants significantly outweighs the expected incurrence of restructuring costs. In simulations, for the median firm who carries positive amount of debt and risky assets, covenants contribute to 1.5% of the total firm value. There is also a considerable variation across time. About 10% of the time, when fundamentals suddenly deteriorate and thus shareholders have a strong tendency to expropriate legacy lenders, the presence of covenants contributes to more than 2% of firm value. The inclusion of covenants increases leverage ratio by 10% and investment rate by 20%. The average default frequency drops sharply by 30%.

Further analyses show that the value of covenants is severely impaired by lenders' lack of

³As will become clear later, debt relief is possible in my model because of i) the nonlinearity of default problems and ii) the collective action of lenders in debt restructuring. See related discussions by e.g. [Aguiar, Amador, Hopenhayn and Werning \(forthcoming\)](#).

commitment to enforce a certain restructuring plan. Debt relief, despite being always valuable once realized ex post, makes shareholders ex ante less cautious about debt issuances. Such an undesirable anticipation effect turns out to be quantitatively strong under the existing covenants and significantly exacerbates the expropriative behavior of shareholders in equilibrium. A value improvement is witnessed if lenders could tie their own hands, which is of course time-inconsistent. Interestingly, resource losses incurred during the restructuring constitute a powerful punishment for shareholder misbehavior ex ante. My analyses suggest that such a “burn the boats” strategy ends up being significantly valuable.

I also investigate whether a recalibration of covenants is able to enhance their effectiveness in addressing time inconsistency. First, my analyses lend support to a tightening of covenants. There exists a hump-shaped relation between shareholder welfare and violation threshold. As covenants get tighter, a larger fraction of violations will be expected in low-risk states where discipline generated by realization and anticipation effects both becomes relatively lenient. Under an excessive tightness, the value of discipline fails to justify frequent incurrences of restructuring costs, thus making the access to covenants ex ante undesirable. My model suggests that welfare is maximized at a threshold under which covenants will be breached with a quarterly frequency of 4%, twice more often than what is currently observed.

Second, my experiment shows that granting debt holders all the control in violations is indeed optimal from the ex ante perspective. Lenders’ ex post incentive to accelerate is not perfectly aligned with the firm as a whole because of unreceived coupons. I show that allocating some bargaining power to shareholders partially overcomes this problem and increases the average gain in total firm value created by violations. However, as shareholders receive a larger proportion of the surplus, the anticipation channel is impaired, which turns out to quantitatively dominate the strengthening of the realization channel.

This paper is the first to quantitatively study the contracting of financial covenants. It bridges two large strands of literature in finance and macroeconomics. First, the model highlights an imperfect solution, i.e. covenants, to the well acknowledged time inconsistency problem of long-term debt. Similar to corporate debt models of [Gamba and Triantis \(2014\)](#), [Kuehn and Schmid \(2014\)](#), [Crouzet \(2016\)](#), [Dangl and Zechner \(2016\)](#), [Gomes, Jermann and Schmid \(2016\)](#), [DeMarzo and He \(2017\)](#) and [Admati, DeMarzo, Hellwig and Pfleiderer \(2018\)](#), shareholders in control of the firm cannot commit to debt issuances.⁴ I further enrich

⁴See [Hatchondo and Martinez \(2009\)](#), [Arellano and Ramanarayanan \(2012\)](#), [Chatterjee and Eychengor](#)

the setting by introducing an additional time inconsistency problem on the asset side, similar to what has been considered in [Leland \(1998\)](#) under fixed leverage.

Second, my quantitative findings add to a vast empirical literature on covenants, starting from [Smith and Warner \(1979\)](#).⁵ [Chava and Roberts \(2008\)](#), [Roberts and Sufi \(2009a\)](#), [Nini, Smith and Sufi \(2009, 2012\)](#), [Chava, Fang and Prabhat \(2015\)](#), [Falato and Liang \(2016\)](#), [Chodorow-Reich and Falato \(2017\)](#) and [Ferreira, Ferreira and Mariano \(2018\)](#) focus on studying how firm behavior is altered after financial covenants are violated ex post. [Billett, King and Mauer \(2007\)](#), [Sufi \(2007\)](#), [Drucker and Puri \(2009\)](#), [Demiroglu and James \(2010\)](#), [Hollander and Verriest \(2016\)](#) and [Prilmeier \(2017\)](#) document how covenant usages vary across firms while [Becker and Ivashina \(2016\)](#) and [Berlin, Nini and Yu \(2018\)](#) investigate the recent covenant-light structure in leveraged loans. [Green \(2017\)](#) utilizes a revealed preference approach to structurally estimate the value of bond covenants for speculatively graded firms. [Matvos \(2013\)](#), [Bradley and Roberts \(2015\)](#) and [Feldhutter, Hotchkiss and Karakas \(2016\)](#) focus on estimating the price impact of covenants.

[Gamba and Triantis \(2014\)](#) investigate the role of financial covenants when shareholders cannot commit to leverage. In their model, violations introduce an exogenous and state-invariant restriction in shareholders' choice. On the contrary, I model lenders' intervention ex post and thus endogenize the violation consequences. A full microfoundation makes it possible to quantify the tradeoff underlying the design of covenants. [Hatchondo, Martinez and Sosa-Padilla \(2016\)](#) experiment with several exogenous non-financial covenants and quantify by how much they can reduce the sovereign dilution problem.

[Hennessy \(2004\)](#), [Moyen \(2007\)](#), [Titman and Tsyplakov \(2007\)](#) and [Gamba, Aranda and Saretto \(2017\)](#) try to quantify agency costs in various settings. More broadly, this work belongs to a thriving literature that studies firm problems with dynamic frameworks. Some examples focusing on positive aspects include [Hayashi \(1982\)](#), [Abel and Eberly \(1994\)](#), [Leland \(1994\)](#), [Goldstein, Ju and Leland \(2001\)](#), [Cooley and Quadrini \(2001\)](#), [Gomes \(2001\)](#), [Hennessy and Whited \(2005\)](#), [Strebulaev \(2007\)](#), [Chen \(2010\)](#), [He and Xiong \(2012\)](#), [Brun-](#)

[\(2012\)](#), [Niepelt \(2014\)](#), [Hatchondo, Martinez and Sosa-Padilla \(2016\)](#), [Bocola and Dovis \(2016\)](#), [Aguiar, Amador, Hopenhayn and Werning \(forthcoming\)](#) and [Dovis \(forthcoming\)](#) for a parallel discussion in the context of sovereign debt.

⁵Theoretical research closely related to financial covenants and more generally control rights includes e.g. [Hart and Moore \(1990\)](#), [Aghion and Bolton \(1992\)](#), [Dewatripont and Tirole \(1994\)](#), [Rajan and Winton \(1995\)](#), [Park \(2000\)](#), [Garleanu and Zwiebel \(2008\)](#). Covenants are also at the center of discussions in accounting literature, with some early examples such as [Beneish and Press \(1993\)](#), [Chen and Wei \(1993\)](#), [Sweeney \(1994\)](#), [Dichev and Skinner \(2002\)](#), etc.

nermeier and Oehmke (2013), He and Milbradt (2014). Some others focusing on contracting issues include Albuquerque and Hopenhayn (2004), DeMarzo and Sannikov (2006), Biais, Mariotti, Plantin and Rochet (2007) and DeMarzo and Fishman (2007).

This paper proceeds as follows. Section 2 presents a stripped-down version of my model for illustrative purposes. I will then integrate it into a richer model in Section 3. Section 4 presents calibration and model assessments. Main counterfactual experiments are carried out in Section 5. Section 6 concludes.

2 Financing with Covenants

In this section, I incorporate covenants into an infinite-horizon long-term debt model with no capital and only independent and identically distributed (i.i.d.) shocks. Such a minimal-style environment allows me to transparently (analytically to a certain extent) illustrate the core mechanisms of the paper – shareholders’ time inconsistency and how the inclusion of covenants reshapes their behavior. It will then be integrated into a richer model which I take to quantitative analyses.

2.1 Shareholders’ Problem

The model is in discrete time and recursively represented, where all agents are risk-neutral. Each period, the firm receives a profit of $Y + z$, where z is i.i.d. with a cumulative density function (c.d.f.) $\Phi(z)$ and support $[-\bar{z}, \bar{z}]$.

I make two tractability assumptions, which are standard in the literature, about rollover arrangements – the firm retires its debt stock b at a rate of λ and new debt is issued *pari passu*⁶. My modeling of financial covenants closely follows the structure adopted by industry with a real-world example presented in Appendix A. More specifically, it requires maintaining a minimum market capitalization to debt ratio of $\kappa > 0$. When it is violated, lenders have the right to require shareholders to repay an additional $\alpha(b, z) \in [0, 1 - \lambda]$ fraction of the principal at par, which will be endogenously determined *ex post* by lenders in a restructuring. As a result, the effective debt maturity becomes state-contingent if $\alpha(\cdot) > 0$ for some (b, z) .

⁶With bankruptcy costs, imposing a strict seniority rule is insufficient to shelter lenders away from the commitment problem. A related discussion is provided by Bizer and DeMarzo (1992).

Figure 1: Timeline

Conditional on repaying αb additionally, shareholder value is given by:

$$V^e(b, z; \alpha) = (1 - \tau)(Y + z) - [(1 - \tau)c + \lambda + \alpha]b + J\left((1 - \lambda - \alpha)b\right),$$

where τ and c are respectively tax and coupon rates. $J(\cdot)$ represents the continuation value of equity.

Financial covenants are violated if:

$$\frac{V^e(b, z; 0)}{b} \leq \kappa,$$

where κ is the violation threshold. Protected by limited liability, shareholders choose not to repay if their value becomes weakly negative after going through the debt restructuring:

$$V^e(b, z; \alpha) \leq 0.$$

The above two inequalities can be represented by two state-dependent cutoff values of the z shock – $z_v : V^e(b, z_v; 0) = \kappa b$ and $z_d : V^e(b, z_d; \alpha) = 0$ – that stand respectively for covenant violation and default. It will be true both in the data and in my numerical analyses that covenant violations take place more frequently than defaults, i.e. $z_v \geq z_d$. The continuation

value can thus be expressed as:

$$J(\tilde{b}) = \max_{b'} (b' - \tilde{b})Q(b') + \beta \mathbb{E} \left[\int_{z'_v}^{\bar{z}} V^e(b', z'; 0) d\Phi(z') + \int_{z'_d}^{z'_v} V^e(b', z'; \alpha') d\Phi(z') \right], \quad (1)$$

where $\tilde{b}$ stands for the legacy debt and $Q(\cdot)$ the debt pricing schedule. The first term represents the gain from selling new debt while the latter two denote the discounted shareholder value next period. It reveals the fact that shareholders only internalize the price impact of their issuances on new debt, i.e. $b' - \tilde{b}$, rather than the entire stock b' .

2.2 Lenders' Problem

Again, conditional on α and no default, debt value is given by:

$$V^b(b; \alpha) = (c + \lambda + \alpha)b + Q\left(b'((1 - \lambda - \alpha)b)\right)(1 - \lambda - \alpha)b,$$

where $b'(\cdot)$ is equity holders' debt policy – the solution to equation (1).

Nothing is recovered upon defaults, which will be relaxed in the full model. There is perfect competition at the moment of lending. As a result, lenders' zero-profit condition pins down the debt pricing schedule $Q(\cdot)$:

$$Q(b')b' = \beta \mathbb{E} \left[\int_{z'_v}^{\bar{z}} V^b(b'; 0) d\Phi(z') + \int_{z'_d}^{z'_v} V^b(b'; \alpha') d\Phi(z') \right]. \quad (2)$$

2.3 Debt Restructuring

After covenants have been violated, lenders are granted all the control right and decide on how much principal to accelerate so that their collective value given repayment can be maximized. More specifically, the payment acceleration policy $\alpha(b, z)$ is determined by:

$$\alpha(b, z) = \operatorname{argmax}_{\tilde{\alpha} \in [0, 1 - \lambda]} V^b(b; \tilde{\alpha}), \quad \forall z. \quad (3)$$

Conditional on repayment, equity holders retain full control over the firm and are able to reissue debt in the market. Equity policy and debt pricing functions, embedded in $V^b(\cdot)$, are taken as given in equation (3). As will be illustrated later, the stickiness of debt caused

by time inconsistency prevents such a one-time acceleration from being completely unwound by re-issuance. As a result, debt dynamics of the firm going forward are altered.

Acceleration makes a default more likely if shareholders find an additional principal retirement costly. Given the existence of liquidation costs, one might therefore think that lenders can do even better by imposing a more lenient acceleration for some (b, z) if a default could have been avoided. However, as long as equity holders decide to default when continuation value equals zero, which is what I have assumed, those scenarios cannot be an equilibrium because lenders always have a strict incentive to extract a little bit more. In other words, adopting an alternative restructuring problem where $\alpha(b, z) = \operatorname{argmax}_{\tilde{\alpha}} \mathbb{1}_{V^e(b, z; \tilde{\alpha}) > 0} V^b(b; \tilde{\alpha})$ yields identical results.⁷

Equation (3) also reflects an important distinction between debt adjustment through restructurings and that via market operations. When shareholders buyback their debt in the market, each lender is able to hold out her portion until the others' have been retired and price has gone up. In equilibrium, the buyback price has to make the marginal lender indifferent and thus all benefits from risk reduction will be ultimately captured by lenders.⁸ If holdouts are possible when lenders face an acceleration at par, the maximization problem in equation (3) should be subject to individual lenders' participation constraints: $Q(b'(1 - \lambda - \tilde{\alpha})b) \leq 1$. However, there is no need to worry about such constraints because debt restructurings require the collective action of lenders. Principal acceleration is a material amendment to the credit agreement, which in practice requires unanimous lender consent. Any lender's holdout shall cause a failure of the restructuring and thus is inferior if the outcome improves average debt value. In other words, the "holdout effect" breaks down.

2.4 Equilibrium and Characterizations

Definition 1. A Markov Perfect Equilibrium is given by (i) equity holders' policy function $b'(\cdot)$ with associated value function $V^e(\cdot)$, default set D and covenant violation set H ; (ii) a debt pricing function $Q(\cdot)$ and associated value function $V^b(\cdot)$; (iii) a payment accelera-

⁷If I assume shareholders continue to operate the firm when their value equals zero, the only ex ante meaningful modification it will bring to the model is that lenders now expect a smaller default loss. Overall, how I treat these cases makes a fairly small quantitative difference: the fraction of defaults that can be avoided by a more lenient acceleration is about 2% in the simulated sample generated from the full model.

⁸Pitchford and Wright (2011) investigate the role of individual holdouts in an extensive-form sovereign debt bargaining model.

tion function $\alpha(\cdot)$ such that (i) given $Q(\cdot)$ and $\alpha(\cdot)$, equity holders' decisions are optimized; (ii) given equity holders' decisions and $\alpha(\cdot)$, $Q(\cdot)$ and $V^b(\cdot)$ satisfy debt holders' zero profit condition in equation (2); (iii) given $V^b(\cdot)$, $\alpha(\cdot)$ solves the restructuring problem in equation (3).

I now proceed to develop some key insights of the paper. Since my goal here is to characterize the equilibrium rather than to establish a general theory, I assume the existence of an interior optimum and the validity of first-order conditions. Section 2.4.1 discusses the long-term debt problem without financial covenants, i.e. imposing $\alpha(\cdot) \equiv 0$ or equivalently $\kappa = -\infty$. I introduce them in later sections and illustrate how the problem is reshaped. Section 2.4.2 focuses on the realization effect given equilibrium functions. Section 2.4.3 discusses how the anticipation effect influences equilibrium policies and firm values.

2.4.1 Long-term debt and shareholders' time inconsistency

To understand the role of time inconsistency, let's start with a case where shareholders borrow long-term debt but can commit to future issuances. Here, I preserve the assumption that there is no commitment to repayment in order to isolate the issuance friction created specifically by debt maturity. Consider a "Ramsey" planner who maximizes the un-levered shareholder value by choosing a borrowing stream $\{b_t\}$ conditional on no previous default. The allocation has to satisfy shareholders' optimal default rule and the zero-profit condition for lenders. As derived in Appendix B.1, her first-order condition is given by:

$$\beta \mathbb{E}_t \left[\int_{z_{d,t+1}}^{\bar{z}} \tau c d\Phi(z_{t+1}) \right] = -b_{t+1} \frac{\partial Q_{t+1}}{\partial b_{t+1}}, \quad (4)$$

where

$$\frac{\partial Q_{t+1}}{\partial b_{t+1}} = -\beta \mathbb{E}_t \left\{ [(c + \lambda) + (1 - \lambda)Q_{t+2}] \phi(z_{t+1}^d) \frac{\partial z_{t+1}^d}{\partial b_{t+1}} \right\}. \quad (5)$$

The left-hand side (LHS) of equation (4) is the present value of tax shields while the right-hand side (RHS) the impact of issuances on the price of the *entire* debt stock b_{t+1} . Last-period debt b_t is not a state variable and there is no endogenous debt persistency. Debt choices adjust only in response to the potential evolution of investment opportunities, if exists. Move to equation (5). Because the choice of b_{t+1} has no effect on b_{t+2} and thus

Q_{t+2} , the impact of issuances is reflected only by an increase in the contemporaneous default probability: $\partial z_{t+1}^d / \partial b_{t+1}$. The following proposition summarizes these results:

Proposition 1 (Ramsey Equilibrium). *When able to commit to future issuances, shareholders internalize the impact of issuances on the entire debt. Equilibrium debt choice at any point in time does not depend on the amount of legacy debt.*

Proof. See Appendix B.1 □

Corollary 1. *Policies in a long-term debt Ramsey equilibrium are time-inconsistent.*

Proof. See Appendix B.2 □

In contrast to the Ramsey equilibrium, without commitment on issuances, shareholders re-optimize the debt structure period by period conditional on the legacy debt $\tilde{b}$. The first-order condition of shareholders in the competitive equilibrium without covenants is given by:

$$\beta \mathbb{E} \left[\int_{z'_d}^{\bar{z}} \tau c d\Phi(z') \right] = -(b' - \tilde{b}) \frac{\partial Q(b')}{\partial b'}, \quad (6)$$

where

$$\frac{\partial Q(b')}{\partial b'} = -\beta \mathbb{E} \left\{ [(c + \lambda) + (1 - \lambda)Q(b'')] \phi(z'_d) \frac{\partial z'_d}{\partial b'} - \int_{z'_d}^{\bar{z}} \left[(1 - \lambda)^2 \frac{\partial Q(b'')}{\partial b''} \frac{\partial b''}{\partial \tilde{b}'} \right] d\Phi(z') \right\}. \quad (7)$$

Compared to (4), the RHS of equation (6) becomes the marginal impact of issuances on the price of *new* debt $b' - \tilde{b}$. Under the same pricing schedule, shareholders would like to borrow more than what the Ramsey planner would do simply due to a partial ignorance of the negative price impact. Of course, in a rational expectation equilibrium, the pricing schedule does adjust as lenders price these expropriative behavior in. The price impact in equation (7) now incorporates in addition the expectation of future leverage ratchet $\partial b'' / \partial \tilde{b}'$. Debt in equilibrium becomes history-dependent and endogenous dynamics emerge. Shareholders slowly adjust their debt even without any ad-hoc issuance cost.

Proposition 2 (Competitive Equilibrium). *When unable to commit to future issuances, shareholders internalize the impact of issuances on the new debt. In a long-term debt competitive equilibrium, shareholders with more legacy debt find it marginally beneficial to carry a larger amount of debt, i.e. $\partial b' / \partial \tilde{b} > 0$.*

Proof. See Appendix B.3 □

Since shareholders fail to internalize the price impact of issuances on past debt demand, they effectively compete against themselves across time in issuances. With equilibrium debt choices deviating from the Ramsey allocation, they lose some of their rents from the ex ante perspective.⁹

A natural question to ask here is that whether trading one-period debt, i.e. $\lambda = 1$, implements¹⁰ the Ramsey allocation since shareholders are also forced to fully internalize price impacts period by period. The answer is negative. After the limited issuance commitment has been neutralized, there still exists no commitment to repayment. Thus different debt maturities, even all with full issuance commitment, generate distinctive allocations as the rollover arrangement influences shareholders' strategic decisions on whether to bear rollover gains/losses, i.e. $[Q(b') - 1]\lambda b$, at each point in time. Because there are only i.i.d. shocks, I am able to get some relatively clear results:

Proposition 3. *A long-term debt Ramsey equilibrium where debt is always sold at (below/above) par yields an identical (higher/lower) firm value compared to a short-term debt competitive equilibrium with an identical coupon rate.*

Proof. See Appendix B.4 □

In Figure 2a, I plot equilibrium debt policies in respectively the Ramsey equilibrium, the long-term debt competitive equilibrium without covenants, and the one-period debt

⁹DeMarzo and He (2017) analyze a long-term corporate debt model in the continuous time, where all the monopoly rents are dissipated (Coase, 1972). Imposing fixed-length periods enables the seller to make limited commitment about the path of durable stock (Stokey, 1981).

¹⁰I am able to show that the Ramsey allocation can be implemented by the following incurrence covenant. Shareholders making a debt choice b' with legacy debt $\tilde{b}$ are required to transfer $\tilde{b}[Q(b^{RE}) - Q(b')]$ to existing lenders, where b^{RE} is the debt choice in the Ramsey Equilibrium. Such a required compensation eliminates the incentive of shareholders to over-borrow. Apparently, writing down an “optimal contract” of this sort is infeasible in reality as it requires the knowledge of a counterfactual quantity. As will become clear in the next section, debt punishment resulting from payment acceleration in high-risk states captures some flavor of this “optimal contract”.

Figure 2: Long-Term Debt and Time Inconsistency. *Notes:* This figure presents the impact of time inconsistency on equilibrium firm policies and values. The Ramsey policy is solved via constructing an equivalent recursive problem. Parameter and functional choices are $\beta = 0.99, \tau = 0.3, \xi = 0.25, Y = 0.012, \bar{z} = 1, \phi(z) = 3(1 - z^2)/4, c = 1/\beta - 1 + 0.0019$ and $\lambda = 1/25$ for long-term debt cases.

model from numerical solutions. Consider an un-levered firm. In the Ramsey case, it immediately issues debt up to approximately 0.2. In contrast, debt policy is upward sloping without issuance commitment (consistent with my marginal characterization in Proposition 2), resulting in the firm being initially under-leveraged. Debt will be gradually accumulated and ultimately the firm levers up more aggressively compared to the Ramsey planner under the chosen parameters. Meanwhile, with the policy function being upward sloping, a larger $\tilde{b}$ means a higher default risk in the future. Figure 2b plots continuation firm values: $J(\tilde{b}) + Q(b'(\tilde{b}))\tilde{b}$. For the case of long-term debt without covenants, it is downward sloping. Of particular interests are un-levered firm values, representing shareholder welfare under different financing arrangements, where the negative impact of lack of commitment becomes apparent. A relatively high coupon rate makes one-period debt more appealing than the Ramsey benchmark, although the difference here is small.

2.4.2 Realization of debt restructuring

Now I introduce financial covenants. This section characterizes the realization of a covenant violation given equilibrium policy and pricing functions. The ex post impact of debt acceleration can be decomposed into two parts. First, it transfers resources between debt and equity holders (*redistribution effect*). More specifically, holding shareholders' choice of b' fixed, an acceleration forces them to retire debt at face value, which might be different from the reissuing price in the market. Thus an additional loss/gain of $[Q(b') - 1]\alpha b$ is generated. Second, when debt becomes history-dependent without commitment, a shifting in the state $\tilde{b}$ changes shareholders' future debt choices and thus a firm's continuation risk and value (*efficiency effect*).

On the margin, shareholders are affected by the redistribution effect. To see this, differentiate the equity value with respect to $\tilde{\alpha}$ and then utilize the envelope condition to substitute out the derivative of the value function:

$$\frac{\partial V^e(b, z; \tilde{\alpha})}{\partial \tilde{\alpha}} = -b - b \frac{\partial J(\tilde{b})}{\partial \tilde{b}} = b[Q(b'(\tilde{b})) - 1], \quad (8)$$

where $\tilde{b} = (1 - \lambda - \tilde{\alpha})b$. Shareholders find the acceleration of an additional unit of debt beneficial if they are able to reissue it at a higher price in the market and thus get a rollover gain.

Different from shareholders, lenders are marginally affected by both the redistribution and efficiency effects. Again, consider the derivative of debt value before repayment:

$$\frac{\partial V^b(b; \tilde{\alpha})}{\partial \tilde{\alpha}} = b \left\{ [1 - Q(b'(\tilde{b}))] - \tilde{b} \frac{\partial Q(b')}{\partial b'} \frac{\partial b'}{\partial \tilde{b}} \right\}. \quad (9)$$

The first term in the bracket is the transfer realized through the additional unit of debt accelerated, mirroring what is laid out in equation (8). The second term captures how the value of remaining debt is marginally impacted because of the changes in legacy debt. If the upward-sloping debt policy and downward-sloping pricing function preserve after the introduction of covenants, i.e. $\partial b' / \partial \tilde{b} > 0$ and $\partial Q(b') / \partial b' < 0$, the second term is positive. Debt holders in this case always benefit from the efficiency effect as the un-retired debt becomes safer.

Lenders decide on how much debt to accelerate based upon the signs and relative strengths

Figure 3: State-Contingent Debt Restructuring. *Notes:* This figure illustrate for three different pre-violation debt levels the debt restructuring outcomes and equity/debt payoffs. Parameter and functional choices are $\beta = 0.99, \tau = 0.3, \lambda = 1/25, \xi = 0.25, Y = 0.012, \bar{z} = 1, \phi(z) = 3(1 - z^2)/4, c = 1/\beta - 1 + 0.0019, \kappa = 2$.

of these two effects. Figure 3 reveals a key determinant – the pre-violation debt level. Figure 3a presents a restructuring taking place when debt is little and new debt will be traded above par even without any acceleration. From lenders' perspective, since default risk is fairly low, any further reduction via raising $\tilde{\alpha}$ becomes second order and is dominated by the debt-to-equity transfer. They are uniformly worse off and choose not to take any action.

Covenant violations in these states end up being inactive, meaning no credit amendment takes place.

As the pre-violation debt level increases, acceleration starts to take place. Figure 3b demonstrates a scenario where debt is moderate and will be re-traded at par without any acceleration. Even though an acceleration still generates an undesirable redistribution to shareholders, debt holders are willing to implement it as the potential efficiency gain becomes relatively pronounced.¹¹ Debt relief is achieved in which equity and debt holders share such an efficiency gain. Going back to my discussions in Section 2.3, such a scenario is made possible by the break-down of the “holdout effect”. For instance, although at $\tilde{\alpha} = 0.2$ the re-traded debt price is already above 1, further acceleration can still be sustained since individual lenders’ holdouts are not feasible.

Finally, in Figure 3c, existing debt is abundant and will be traded far below par without acceleration. Lenders in this case have a strong desire to accelerate. First, the redistribution effect flips its sign and starts to benefit lenders in the beginning of acceleration. Second, with the default risk being severe, the efficiency gain becomes fairly pronounced. Lenders keep raising $\tilde{\alpha}$ until the firm’s continuation default risk becomes so small that on the margin the benefit from further risk reduction is offset by the transfer to equity holders. Equity holders end up receiving a debt punishment because they have to retire debt at a higher average price.

Although higher pre-violation debt levels lead to stronger accelerations, firms’ legacy capital structures turn out to be identical after active restructurings. Readers might have already noticed that the $\tilde{b}$ equating (9) with zero does not depend on b . The following proposition formalizes this result:

Proposition 4. *Define restarting legacy debt $\tilde{b}^R = \operatorname{argmax}_{\tilde{b}} [Q(b'(\tilde{b})) - 1]\tilde{b}$. Consider a firm violating covenants with debt b . If $b \leq \tilde{b}^R/(1 - \lambda)$, no debt will be accelerated, i.e. $\alpha = 0$. If $b > \tilde{b}^R/(1 - \lambda)$, shareholders are required to pay down the legacy debt to $\tilde{b}^R$, i.e. $\alpha = (1 - \lambda) - \tilde{b}^R/b$.*

Proof. The proof requires rewriting equation (3) in $\tilde{b}$. □

¹¹These cases arise because the default risk is nonlinear in leverage, typical for this class of models. The sensitivity of debt price with respect to leverage grows much faster than the debt price itself in response to an increase in leverage.

Remark 1. When c is small enough such that $\forall b', Q(b') < 1$, a full acceleration follows every violation, i.e. $\tilde{b}^R = 0$. If further $\kappa = \infty$, covenants implement short-term debt.¹²

Remark 1 describes an extreme case where covenants implement one-period debt. A more general characterization of violation consequences requires knowing global properties of $b'(\tilde{b})$ and $Q(b')$ since accelerations create jumps in state variables. For such a purpose, I numerically demonstrate a capital structure restart in Figure 4 with less extreme parameters. The upward slope of debt policy and downward slope of pricing schedule preserve after the introduction of covenants. Consider a firm with a stationary capital structure. Following a violation, shareholders have to repay more and get the firm restarted with a legacy debt of $\tilde{b}^R$. Leverage experiences a persistent decline while continuation firm value rises.

Figure 4: Covenant Violations and Capital Structure Restart. *Notes:* This figure presents dynamics of a firm violating covenants under the stationary debt level. Parameter and functional choices are $\beta = 0.99, \tau = 0.3, \lambda = 1/25, \xi = 0.25, Y = 0.012, \bar{z} = 1, \phi(z) = 3(1 - z^2)/4, c = 1/\beta - 1 + 0.0019, \kappa = 2$.

¹²Recall Proposition 3. If debt is always traded below par, one-period debt is inferior to long-term debt with full commitment. In other words, when c is low enough, shareholders who can commit to future issuances become worse off when creditors impose state-contingent acceleration.

2.4.3 Anticipation and ex ante implications

Moving from ex post to ex ante, the anticipation effect becomes no less important – equilibrium policies are influenced by the anticipation of future violations. Shareholders behave differently with the presence of covenants even if no violation is realized ex post. The key determinant is again the state-dependent restructuring payoff to shareholders. Covenant inclusions discourage debt issuances conditional on the pricing schedule when leverage is high. In those states, shareholders find debt punishment painful and thus will try to avoid breaching covenants. Because the violation threshold is written on inverse leverage, the way to achieve such a goal is to issue new debt less aggressively. In contrast, covenants encourage debt accumulation in states where leverage is moderate because shareholders are likely to get a debt relief upon back shocks.

Because the key friction is shareholders' temptation to over-lever, the value of covenants stems from the leverage discipline they introduce. Debt relief, being always value enhancing ex post¹³, can instead harm commitment production—shareholders become less cautious about leverage adoption when anticipating their possibility. Sometimes, that additional debt can be costlessly retired through a realization of relief, meaning shareholders successfully extract extra tax shields without paying a default loss. However, for the non-relief paths, the firm steps into the high-leverage region more rapidly and therefore experiences a default earlier.

Firm value will be enhanced by covenants only if the realization and anticipation effects together create strong discipline in future debt issuances. Factors that affect the conditional distribution of future states, such as properties of covenants and the state in which they are evaluated, all matter.¹⁴ Figure 5 shows the solutions to a model without covenants and two with covenants but under different violation thresholds. Covenants increase firm value under high $\tilde{b}$'s as they significantly reduce future leverage no matter whether a violation actually realizes or not. Moving towards the left, debt reliefs kick in and make the conclusion ambiguous. In the benchmark case where $\kappa = 2$, including covenants overall increases debt capacity and firm value at $\tilde{b} = 0$. The initial borrowing and un-levered firm value increase.

¹³Although a realization of debt punishment always increases firm value conditional on repayment, it triggers default under certain shocks and is thus not necessarily ex post beneficial.

¹⁴Features of the debt contract to which covenants are attached also play a role. Unreported numerical experiments suggest that under small coupon rates, covenants are always value enhancing. Under such parameterization, debt generally trades below par and only the disciplining force is operative ex ante.

Figure 5: Impact of Covenant Inclusions. *Notes:* This figure presents how equilibrium policy and value functions are influenced by covenants. Parameter and functional choices are $\beta = 0.99, \tau = 0.3, \lambda = 1/25, \xi = 0.25, Y = 0.012, \bar{z} = 1, \phi(z) = 3(1 - z^2)/4, c = 1/\beta - 1 + 0.0019$.

However, in the other case, the expectation of debt relief quantitatively dominates debt disciplines and thus the inclusion of covenants introduces a large expropriation risk from the ex ante perspective. Initial borrowing and firm value are lower than their counterparts in the covenant-free model.

To clearly know the sign and magnitude of the ex ante value of covenants as well as their implications for equilibrium firm behavior, one needs to quantify the strengths of the realization and anticipation effects. With all the demonstrated forces in mind, I now turn to the quantitative part of the paper.

3 The Full Model

This section presents the full model for quantitative analyses. In addition to the model outlined in the last section, I introduce (risky) investment, persistent shocks as well as a more realistic violation/default treatment. I will carry most of the notations and economize on descriptions of ingredients that have already appeared.

3.1 The Environment

The firm operates two types of capital with different risk profiles: high (k_H) and low (k_L). A constant returns to scale (CRS) technology generates the following profit:

$$\sum_{i \in \{L, H\}} (e^x + \nu_i z) k_i$$

where the persistent income follows a standard AR(1) process: $x' = (1 - \rho)\bar{x} + \rho x + \sigma \tilde{\epsilon}$, $\tilde{\epsilon} \sim N(0, 1)$. z shocks, which can be interpreted as extraordinary items, capital quality shocks, accounting noises, etc., are i.i.d.

Capital k_H has a larger exposure to i.i.d. z shocks: $\nu_H = \nu \times \nu_L$ where $\nu > 1$. I normalize $\nu_L = 1$ without loss of generality. The p.d.f. of z is symmetric with support $[-\bar{z}, \bar{z}]$, and thus the risk choice is mean-preserving. Total capital stock $k = k_H + k_L$. Investment is given by $k' - (1 - \delta)k$, where δ stands for depreciation rate. Adjusting capital incurs a cost:

$$\Psi(\cdot) = \sum_{i \in \{L, H\}} \frac{\gamma}{2} \left(\frac{k'_i - k_i}{k} \right)^2 k,$$

where a capital reallocation friction is embedded in the quadratic form – buying one type of capital and selling an equal amount of the other is costly.

Define k_H share $s = k_H/k$ and further a firm's exposure to z shocks $a(s) = s\nu + (1 - s)$. Conditional on α , the value of the firm to its shareholders is given by:

$$\begin{aligned} V^e(b, k, s, x, z; \alpha) &= (1 - \tau)[e^x + a(s)z]k - [(1 - \tau)c + \lambda + \alpha]b \\ &\quad + J((1 - \lambda - \alpha)b, k, s, x). \end{aligned}$$

A firm violates financial covenants when $\frac{V^e(b, k, s, x, z; 0)}{b} \leq \kappa$. A default happens if equity value becomes negative, i.e. $V^e(b, k, s, x, z; \alpha) - fk \leq 0$ where f is the resource cost associated with debt restructurings. Continuation value $J(\cdot)$ is given by

$$\begin{aligned} J(\tilde{b}, k, s, x) &= \max_{b', k', s'} Q(b', k', s', x)[b' - \tilde{b}] - [k' - (1 - \delta)k] + \tau\delta k - \Psi(\cdot) \\ &\quad + \beta \mathbb{E} \left[\int_{z'_b}^{\bar{z}} V^e(b', k', s', x', z'; 0) d\Phi(z') + \int_{z'_d}^{z'_b} [V^e(b', k', s', x', z'; \alpha') - fk] d\Phi(z') \right], \quad (10) \end{aligned}$$

where $z_v(b, k, s, x)$ and $z_d(b, k, s, x; \alpha)$ are cutoffs representing violation and default.

Move to the debt holders. Conditional on an additional payment of αb , their value is:

$$V^b(b, k, s, x; \alpha) = (c + \lambda + \alpha)b + (1 - \lambda - \alpha)b \\ \times Q\left(b'((1 - \lambda - \alpha)b, k, s, x), k'((1 - \lambda - \alpha)b, k, s, x), s'((1 - \lambda - \alpha)b, k, s, x), x\right),$$

where $b'(\cdot)$, $k'(\cdot)$ and $s'(\cdot)$ are equity policies solving equation (10).

Upon default, lenders recover the contemporaneous output, $(1 - \tau)[e^x + a(s)z]k$, together with the un-depreciated capital $(1 - \delta)k$. However, a liquidation cost of ξk is incurred. Compared to the simplified model, a positive default recovery creates a dilution problem – for a given amount of recovered resources, having more creditors means that each one of them ends up with less.

As usual, debt holders' zero profit condition pins down the pricing schedule:

$$Q(b', k', s', x)b' = \beta \mathbb{E} \left\{ \int_{z'_v}^{\bar{z}} V^b(b', k', s', x'; 0) d\Phi(z') + \int_{z'_d}^{z'_v} V^b(b', k', s', x'; \alpha') d\Phi(z') \right. \\ \left. + \int_{\underline{z}}^{z'_d} [(1 - \tau)[e^{x'} + a(s')z'] + (1 - \delta) - \xi] k' d\Phi(z') \right\}. \quad (11)$$

For the convenience of carrying out counterfactual analyses on alternative covenants, I consider a more general debt restructuring problem where equity holders might have some bargaining strength. Acceleration schedule $\alpha(b, k, s, x, z)$ is given by

$$\alpha(b, k, s, x, z) = \operatorname{argmax}_{\tilde{\alpha} \in [0, 1 - \lambda]} \theta V^e(b, k, s, x, z; \tilde{\alpha}) + (1 - \theta) V^b(b, k, s, x; \tilde{\alpha}). \quad (12)$$

where θ stands for the bargaining power of equity holders in restructurings.

It is easy to show that the property stated in Proposition 4 extends to the full model: There exists a restarting legacy debt level $\tilde{b}^R(k, s, x)$ from which I can back out $\alpha(b, k, s, x, z)$. In this richer environment, $\tilde{b}^R$ for a firm is no longer state-invariant but depends on its capital stocks, k and s , as well as persistent cash flow x . Again, covenant violators with identical asset-side characteristics will have to pay off part of their debt such that shareholders regain controls with the same amount of legacy debt.

3.2 Discussions of New Features

The full model is scale-invariant with one exogenous state variable x and two endogenous ones – legacy leverage $\tilde{\omega} \equiv \tilde{b}/k$ and s . It is achieved by the construction of production technology, adjustment costs, violation/default thresholds, and restructuring objectives. The equilibrium concept is again Markov-perfect. (Equilibrium definition of the full model and an linearity proof can be found in Appendix C.)

3.2.1 Risk shifting and debt overhang

The existence of k_H introduces a risk-shifting motive. When shareholders fully internalize the impact of their asset choices, a heavy investment in k_H can hardly be beneficial as it simply increases expected default losses.¹⁵ It is no longer the case if shareholders optimize over s' with the presence of some legacy debt. Even though a huge s' destroys the total firm value, when the legacy leverage $\tilde{\omega}$ becomes high enough relative to x , shareholders can find it privately profitable because of an increase in the equity value (Leland, 1998). Lenders are sensitive to the downside risk and debt value falls sharply in consequence.

A limited commitment problem arises on the asset side. At the time of borrowing, long-term debt holders have to price shareholders' incentive not only to over borrow ex post but also to raise the firm's exposure to z shocks under an excessive leverage. Of course, the link between these two commitment problems is not just one-way: A heavy allocation on k_H also exacerbates the conflict of interests on the financing side. Consequences of shareholders' incentive to extract further tax rents become more detrimental to debt values under a riskier technology because defaults are more likely ceteris paribus. Because of such feedback, equilibrium debt policies without commitment become more nonlinear.

The rate of total investment, $i \equiv k'/k - 1 + \delta$, is pinned down by capital adjustment costs as the production technology is CRS. As long as shareholders are not able to commit to repayment, which is also the case in the Ramsey equilibrium and short-term debt model, a debt overhang problem naturally emerges. Specific to this competitive setting is the interaction between commitment friction and corporate investment along various dimensions.

¹⁵It can be shown analytically that $s' \equiv 0$ if i) there is no capital reallocation friction, i.e. $\Psi(.) = \frac{\gamma}{2}(\frac{k'-k}{k})^2 k$; and ii) debt is one-period. In addition to the risk-shifting channel, equity holders would like to diversify their assets to a certain extent because of the capital adjustment costs. However, such a motivation enhances firm value and creates no agency problem on its own.

Consider a firm violating covenants with a large amount of debt and k_H . As legacy leverage falls due to acceleration, shareholders start to internalize a larger fraction of the price impact. Persistent declines in ω and s alleviate expected default losses and in turn facilitate investment. However, when adjustment costs prevent a perfect and instant capital reallocation from happening, the drop in k_H might not be completely unwound by the increase in k_L . Depending on quantitative strengths of these two forces, the rate of total investment can be temporarily suppressed or boosted in the short run by violations. On contrary, in full commitment scenarios, future investment behavior only responds to shocks to x and will thus not be altered by a shift in states.

From an unconditional perspective, if covenant inclusions can partly resolve the over-borrowing and risk-shifting problems, one should expect the under-investment to be milder on average. Firm values will be further enhanced. Overall, the existence of risk choices and debt overhang makes commitment more treasured compared to a model with only capital structure problems.

3.2.2 Time variation of commitment frictions

Upon an introduction of persistent shocks, the severity of commitment problem starts to exhibit rich dynamics. First, time inconsistency becomes relatively more detrimental when a deterioration in x is expected. For instance, when inspecting the numerical solution to the full model, I find that for a given s , the equilibrium debt policies are much steeper and the under-leveraging is more severe for smaller x 's. Such "counter-cyclicity" roots deeply in the intrinsic property of defaults, which also underlines the feedback between risk shifting and over-borrowing that I discussed in the last section. Defaults are highly nonlinear and require complementary workings of large negative shocks and high leverage. A high x means not only a handsome profit in the current period but more importantly also a huge continuation equity value. At that time, a default is unlikely even with leverage and asset risks further inflated. When x worsens, shareholders' tax shield extraction becomes more harmful from the lenders' perspectives. Furthermore, shareholders are more willing to increase the volatility of income and shift the risk to lenders. Therefore in those scenarios, lack of commitment to future investment and financing becomes more value destructive.

Second, endogenous firm characteristics— ω and s —also influence the severity of the commitment problem. When existing leverage is high, shareholders will only internalize a small

fraction of the default loss. Conflict of interests in those scenarios becomes relatively critical. Meanwhile, for shareholders who have already invested heavily in k_H , the convexity of capital adjustment costs means that it is less costly for them to adopt an extremely high asset riskiness and thereby destroy debt holders' value once such a strategy becomes appealing.

Overall, the severity of commitment frictions varies across time together with fundamentals of the firm. In a given state, it depends on the conditional distribution of future cash flows as well as balance sheet characteristics ω and s .

3.2.3 Restructuring cost

In reality, debt restructurings are costly. One can interpret such costs in a similar fashion to those incurred during payment defaults. There are direct charges by attorneys and accountants for rewriting contracts and in addition some indirect losses such as reputation damages caused by disclosure and the opportunity cost of time (Beneish and Press, 1993). In the full model, these are all summarized by the term fk in the covenant violation region of equation (10). A one-time incurrence of f does not affect motions of state variables and therefore dynamics of non-defaulted firms. It is always ex post undesirable for shareholders and the firm as a whole since some resources are simply taken away.

However, the anticipation of f , similar to that of debt punishments, contributes to disciplining shareholders' extrapolative behavior. Overall, f is ex ante value destructive if the commitment problem is far from being severe and thus the anticipation effect fails to generate enough merit to justify painful realizations. For instance, in a short-term debt model where shareholders fully internalize the consequences of their choices, possible incurrences of f in certain parts of the state space shall reduce ex ante welfare. In contrast, an enhancement in firm value may be witnessed if time inconsistencies are quantitatively severe enough.

4 Quantitative Exploration

I now proceed to quantitative analyses of the full model. This section mainly presents my calibration and model assessments.

4.1 Calibration

I solve the model with value function iterations (details can be found in Appendix C.3). I adopt a quadratic approximation to the probability density function of z shocks:

$$\phi(z) = \eta_0 + \eta_1 z^2.$$

After imposing $\phi(\bar{z}) = 0$ and $\int_{-\bar{z}}^{\bar{z}} \phi(z) = 1$, $\bar{z}$ is sufficient to pin down η_1 and η_0 .

To make the model quantitatively more realistic, I impose an upper limit on the degree of risk shifting: $s' \leq \bar{s}$. One can interpret this boundary as other restrictions on firm behavior that are not in my model: regulations, career concerns, etc. Consistent with the claim made by Leland (1998), without reallocation friction, optimal s' becomes bang-bang as the marginal return to risk shifting turns out to be increasing. Bounding s' helps deliver quantitatively reasonable i.i.d. shocks without resorting to an unrealistically large capital adjustment cost.

Parameters	Values	Description
β	0.99	discount rate
δ	0.025	depreciation rate
τ	0.3	tax rate
$1/\lambda$	12	maturity
κ	1/2	covenant tightness
θ	0	bargaining power
$\rho, \sigma, \bar{x}$	0.9, 0.15, $\ln(0.0353)$	x process
$\bar{z}$	0.9	z distribution
c	$1/\beta - 1 + 0.001538$	coupon rate
ξ	0.25	bankruptcy cost
f	0.0025	restructuring cost
γ	3	capital adjustment cost
$\nu, \bar{s}$	13.65, 0.02	risk shifting

Table 1: Parameters

The model is calibrated under a quarterly frequency with all parameters listed in Table 1. I first directly parametrize discount rate $\beta = 0.99$, depreciation rate $\delta = 0.025$ and corporate tax rate $\tau = 0.3$. Repayment rate is set to $\lambda = 1/12$ so that debt maturity is equal to the

median value reported by [Chava and Roberts \(2008\)](#) for Dealscan loans. Admittedly, public firms typically have some corporate bonds, which tend to have a longer maturity. Choosing a 3-year maturity provides a conservative estimate of the covenant value. Tightness of equity-to-debt covenant κ is close to what has been documented by [Chava, Fang and Prabhat \(2015\)](#). Again, as covenants typically shift all the discretion to lenders after they have been violated, I set $\theta = 0$.

All parameters left are calibrated. The boundary of idiosyncratic shocks $\bar{z}$ targets at default probability. Coupon rate c is set such that on average debt goes out at par in simulations, consistent with how bank loans are observed. Bankruptcy cost ξ is set to match median book leverage. Restructuring cost f is identified via covenant violation frequency. As a standard practice, capital adjustment cost curvature γ is set to match the investment volatility. Parameters governing income dynamics and risk-shifting behavior— ρ, σ, ν and $\bar{s}$ —jointly target pre- and post-violation differences in net debt issuances and investment rates between violators and non-violators. The mean of persistent income $\bar{x}$ is chosen to match the median investment rate.

Moments	Model	Median	10%	90%
debt/assets*	0.279	0.245	[0.013	0.810]
volatility of debt/assets	0.029	0.100	[0.013	0.554]
investment/assets*	0.022	0.024	[0.005	0.083]
volatility of investment/assets*	0.024	0.022	[0.006	0.084]
market-to-book	0.999	1.934	[1.068	10.020]
volatility of market-to-book	0.314	0.693	[0.144	8.511]
income/assets	0.038	0.018	[−0.279	0.057]
volatility of income/assets	0.013	0.039	[0.012	0.360]
covenant violation frequency*	0.017	0.015		
default frequency*	0.002	0.003		

Table 2: Unconditional Moments. *Notes:* This table presents unconditional moments calculated from simulated sample (model median) and data (median, 10% and 90% percentiles). I simulate 2,000 firms for 1,000 quarters. Data sample spans from 1996Q1 to 2011Q4. * denotes moments used in calibration. Details about data construction are presented in [Appendix D](#).

4.2 Simulation Results

Table 2 compares unconditional sample moments generated from simulated series and their data counterparts. Inspired by a large body of empirical research on covenant violations, discussions in this section will be focused on results in that regard.

4.2.1 Covenant violators

Why are financial covenants violated? There are two plausible explanations. First, violations might simply be driven by bad luck – the likelihood goes up after negative shocks to x even though ω and s are not particularly high. For example, if covenants are imposed in a short-term debt model, this explanation will lie behind every violation. Second, it can also be a boom-bust story with the “leverage ratchet effect”. After a sequence of positive shocks, leverage has been piled up. At that moment, having experienced an erosion of x , shareholders respond slowly in terms of buying back their long-term debt because of their resistance to transfer resources to debt holders. Meanwhile, shareholders also start to find it beneficial to increase the loading on k_H when leverage becomes excessive relative to x . Under a high ω , a large exposure to z shocks and a lower level of x , violations also become more likely.

Figure 6: Pre-violation Dynamics. Notes: This picture presents dynamics of state variables before covenant violations and compare them to those of the whole simulated sample. I simulate 2,000 firms for 1,000 quarters.

Which scenario is more typical for violators is a quantitative question, the answer to

which depends on both the dynamics of exogenous uncertainty and how endogenous behavior responds over time. More specifically, the likelihood of receiving a sequence of bad shocks and the degree of asymmetry in leverage adjustment both matter. Figure 6 plots the dynamics of state variables averaged across firms before violating their financial covenants, which lends support to the boom-bust explanation. As the persistent income erodes after a long boom, shareholders reduce their debt slowly. Right before violations, violators still have a larger amount of debt on their balance sheet although their persistent cash flow is already lower than their counterpart's. They also double their positions in k_H . The reversal in persistent income is arguably mild—less than 1/2 of a standard deviation.

In Table 3, I provide some supporting empirical evidence. Violators tend to have higher income-, net debt issuance- and investment-to-assets ratios compared to others two years before the actual breaches. These differences either revert or disappear when approaching violations. For moments that the model misses their magnitudes, correct signs are produced.

Moments	Model		Data		
	vio	<i>diff</i>	<i>diff</i> (FE)	[5%	95%]
$t - 1 \rightarrow t$					
income/assets	0.035	−0.002	−0.013	[−0.018	−0.008]
net debt issuance/assets*	−0.002	−0.002	0.000	[−0.002	0.003]
investment/assets*	0.022	−0.000	−0.000	[−0.001	0.001]
$t - 8 \rightarrow t - 7$					
income/assets	0.039	0.002	0.004	[−0.002	0.009]
net debt issuance/assets	0.001	0.001	0.006	[0.003	0.008]
investment/assets	0.025	0.003	0.003	[0.002	0.004]

Table 3: Pre-Violation Moments. *Notes:* This table presents pre-violation differences between covenant violators (vio) and non-violators (*diff* = violators−non-violators). I simulate 2,000 firms for 1,000 quarters and calculate the means. Empirical moments are fixed-effect regression coefficients and associated 95% confidence interval calculated from data between 1996Q1 and 2011Q4. * denotes moments used in calibration. Details about data construction are presented in Appendix D.

4.2.2 After covenant violations

How firms change their behavior after covenants are violated? Table 4 reports firm statistics averaged over two quarters after violations. Covenant violators stay with a lower income-to-

assets ratio as x is persistent. Moreover, they experience declines in net debt issuances and k_H share. Because of these risk-reduction measures, their default probability going forward becomes much smaller compared to non-violators albeit x is relatively inferior. The rate of total investment drops.

In the last column, I utilize my model to isolate the causal impacts of the realization of a violation – how firm dynamics would have been different if covenant violators were not forced to retire an additional $\alpha(\cdot)$. (Recall that the payment of f should have no impact.) Within the model, I am able to fix the state variables right before violations and subsequent shocks, and thus not bothered by the fact that distinctions between violators and non-violators are also affected by differences in economic fundamentals.

Moments	Model		Data	Violation	
	vio	diff	diff (FE)	[5%	95%]
$t + 1 \rightarrow t + 2$					
income/assets	0.035	−0.003	−0.004	[−0.010	0.001]
net debt issuance/assets*	−0.012	−0.012	−0.011	[−0.013	−0.010]
investment/assets*	0.012	−0.010	−0.010	[−0.011	−0.009]
k_H share	0.004	−0.001			
default frequency	0.000	−0.002			

Table 4: Post-Violation Moments. *Notes:* This table presents post-violation differences between covenant violators (vio) and non-violators ($diff = violators - non-violators$). The last column presents causal impacts of violations on endogenous behavior. I simulate 2,000 firms for 1,000 quarters. Empirical moments are fixed-effect regression coefficients and associated 95% confidence interval calculated from data between 1996Q1 and 2011Q4. * denotes moments used in calibration. Details about data construction are presented in Appendix D.

In all, 97.8% of violations in the simulated sample are active.¹⁶ My results suggest that covenant violations are responsible for around half of the decline in net debt issuances. The deterioration in x are equally powerful in explaining such a decline. This piece of result is quantitatively in line with what has been concluded by Roberts and Sufi (2009a) with a difference-in-difference design. The capital reallocation friction turns out to be quantitatively large enough to generate a drop in investment rate, even though the overhang problem has

¹⁶Roberts (2015) documents that more than 75% of covenant violations in his sample lead to restructurings. Roberts and Sufi (2009a) analyze voluntary reports of covenant violation outcomes in a random sample of 10-K or 10-Q filings and conclude a lower bound of 32.2%.

been alleviated by declines in debt flows and risk shifting. The economic magnitude is a bit smaller compared to [Chava and Roberts \(2008\)](#).¹⁷ The erosion of investment opportunities accounts for the majority of the decline in investment. Consistent with the evidence in [Gilje \(2016\)](#), violations cause disengagement in risk-shifting activities.

5 Counterfactual Experiments

This section carries out counterfactual experiments to isolate the ex ante implications of covenants. In Section 5.1 I quantify the impact of existing covenants ($\kappa = 0.5$ and $\theta = 0$) on shareholder values and unconditional firm behavior. Section 5.2 examines how shareholder welfare responds to alternative calibrations of covenants.

5.1 Impact of Covenant Inclusions

I first focus on the impact of covenants on ex ante firm value, or welfare, and unconditional moments. In the second section, I present how the covenant value evolves across time.

5.1.1 Welfare and unconditional moments

Table 5 demonstrates the impact of covenant inclusions by comparing results from the benchmark model and those from a covenant-free long-term debt model, i.e. $\kappa = -\infty$.¹⁸ The welfare metric I use across different models is the shareholder value under zero debt, zero risky capital and mean level of persistent income, i.e. $j(\tilde{\omega} = 0, s = 0, x = \bar{x}) \equiv J(0, k, 0, \bar{x})/k$.

Let's first focus on columns 1 to 3. If shareholders have access to the covenants specified in the benchmark analyses, their welfare improves from 0.9532 to 0.9657. Covenants indeed produce commitment. In terms of magnitudes, such a 1.31% gain in shareholder/firm value

¹⁷For tractability, the model abstracts from frictions such as equity issuance costs, which might help produce a larger short-run negative impact of acceleration on investment. Incorporating additional costs equity holders have to bear ex post in restructuring is likely to further inflate the covenant value because of a strengthening in the anticipation effect.

¹⁸Under my choice of f , shareholders never find it beneficial to voluntarily propose a debt restructuring to lenders. As a result, the results will be identical if I instead compare i) a long-term debt model with costly debt renegotiability where $\theta \equiv 1$ and ii) one where covenants are present and $\theta = 0/1$ when violated/unviolated.

Moments	w/o covenants		with covenants		
	w/o recalib	recalib	benchmark	$f \equiv 0$	$\alpha(.) \equiv 0$
welfare	0.9532	0.9582	0.9657	0.9529	0.9671
debt/assets	0.2511	0.2589	0.2789	0.2470	0.2809
investment/assets	0.0175	0.0191	0.0218	0.0172	0.0222
k_H share	0.0089	0.0087	0.0053	0.0089	0.0056
default frequency	0.0023	0.0022	0.0017	0.0019	0.0019

Table 5: Impacts of Covenant Inclusions and Decompositions. *Notes:* This table presents simulated medians and shareholder welfare $j(0, 0, \bar{x})$ for alternative models. Column 1 reports results for a model without covenants where coupon rate is fixed to that in Table 1. Column 2 reports results for a model without covenants where coupon rate is recalibrated so that debt on average goes out at par ($c = 1/\beta - 1 + 0.002228$). Column 3 reports results for the benchmark model with covenants. Column 4 reports results for a model with covenants where f is fixed to 0. Column 5 reports results for a model with covenants where $\alpha(.)$ is fixed to 0. I simulate 2,000 firms for 1,000 quarters.

is arguably significant considering at that point firms hardly have any default risk.¹⁹ Even if I increase the coupon rate for the covenant-free model such that debt still on average goes out at par, shareholder value still improves by approximately 0.78% after the introduction of covenants.

Because of the alleviation of time inconsistency, lenders are less worried about the leverage ratchet effect and risk shifting. Prices adjust and in equilibrium firms are able to borrow more on average. Firm default probability falls sharply even though leverage becomes higher. As the expected default loss shrinks, the debt overhang problem is also alleviated, leading to a higher investment rate in the long run. To give those magnitudes some context, by a rough calculation, a firm with access to covenants will have about 20% more capital and 30% more debt after 10 years in a deterministic environment.

Now move to the last two columns. Having acknowledged the positive value of covenants, I conduct a value decomposition. More specifically, covenant violations have two consequences: endogenous debt acceleration $\alpha(.)$ and the incurrence of a restructuring cost f . How does each component contribute to the commitment production? First, although debt acceleration always improves post-violation total firm value, it turns out to be value destruc-

¹⁹Welfare equals 0.9265 if shareholders are restricted to only equity financing. Simulations of my model deliver a net benefit of debt that is in line with the estimates by e.g. Korteweg (2010) and van Binsbergen, Graham and Yang (2010).

tive from the ex ante perspective. By comparing shareholder welfare in columns 1 with that in column 4, one can see a decrease after introducing costless control right shifting into a covenant-free model. In addition, by looking at welfare in columns 3 and 5, one can see that covenants can become better if lenders commit not to restructure the debt, which of course is time-inconsistent. Recall discussions in Section 2.4.3—this result suggests that the current restructuring setup and contractual calibration produce a pretty strong expectation of debt relief in the eyes of a newborn firm without debt and risky capital. A bad incentive ex ante is provided and a speedy risk accumulation is encouraged. In contrast, although f is never desirable when ex post realized, it significantly improves welfare as the anticipation effect turns out to be considerably strong. In other words, such a “burn the boats” strategy becomes highly valuable when the incentive problem is severe.

5.1.2 Time variation of covenant values

The value of covenants varies across time. There are two contributing factors. First, as pointed out in Section 3.2.2, the severity of commitment frictions varies with economic fundamentals. Second, as the default risk and commitment frictions change, outcomes of debt restructurings can also become different.

Figure 7a demonstrates the time variation of covenant values by presenting the empirical c.d.f.s calculated from my simulated sample. The first thing to notice is that the average covenant value is 50% larger than the ex ante welfare. This is because firms in the simulated sample on average have accumulated a positive amount of debt and risky assets, which have driven up the default risk and severity of commitment frictions. Moreover, the covenant value varies significantly across time. Approximately 10% of the time, covenants account for more than 2% of the firm value.

Again, I decompose all firm-quarter covenant values and plot the empirical c.d.f.’s of contributions made by, respectively, the acceleration scheme and restructuring costs. Figure 7b shows that as the potential restructuring outcome evolves, the payment acceleration component sometimes does help enhance firm values. For instance, when a highly levered firm applies for a loan from banks, imposing state-contingent debt accelerations can be beneficial even if they are costless. In those states with a considerable amount of risk and suppressed debt prices, debt reliefs are unlikely in the near future and quantitatively dominated by the discipline generated by debt punishment. A frequency of 0.6% is not completely trivial as

Figure 7: Time-Variation of Covenant Values and Decompositions. *Notes:* This figure presents empirical cumulative density functions for covenant values and associated decomposition. Figure 7a (7b/7c) measures the contribution of covenants (debt restructuring/restructuring cost) to firm values by computing the differences between simulated firm values and counterfactual firm values under identical states but without covenants (with covenants but fixing $\alpha(\cdot) \equiv 0$ /with covenants but fixing $f \equiv 0$). I simulate 2,000 firms for 1,000 quarters.

it is around half of the probability of violation and three times that of default. Moreover, Figure 7c suggests that the existence of f plays a dominant role in commitment production and always enhances firm values under the existing calibration of covenants.

Figure 8 takes a closer look at those observations where covenant values go beyond 0.02 by tracing the dynamics of firm states 20 quarters before. It confirms that covenants become valuable when economic fundamentals deteriorate. However, different from violations (Figure 6), covenant values are most pronounced at the onset of a sudden deterioration. At that point, firms have a strong tendency to start increasing risk shifting and act slowly in debt buyback. Covenants generate significant merits by nipping severe expropriation in the bud.

5.2 Contractual Efficiency

In this section, I move a step further and evaluate the efficiency of existing covenants in addressing time inconsistency. For such a purpose, I fix the current contracting structure and experiment on how shareholder welfare responds to adjustments in covenant calibration.

Figure 8: When Do Covenants Become Highly Valuable? *Notes:* This picture presents dynamics of state variables for firms before the value of their covenants goes beyond 0.02 and compare them to those of the whole simulated sample. I simulate 2,000 firms for 1,000 quarters.

5.2.1 Covenant tightness

I first study the choice of violation threshold κ , which governs how frequently violations take place ex post. Figure 9a suggests that welfare is hump shaped with respect to violation frequency and the maximum is achieved under a much tighter calibration: $\kappa = 1/1.15$ and the violation probability exceeds 4%. (Recall that the benchmark violation frequency is 1.5% and $\kappa = 1/2$.) If I adjust downward the coupon rate so that debt on average trades at par, there still exists a potential efficiency improvement, though both the required tightening and the resulting gain become much smaller.²⁰

Behind the hump shape lies the efficient allocation of covenant violations. Suppose lenders have only one chance of costly debt restructuring, it is ex ante ideal to allocate that opportunity to a state where fundamentals are highly undesirable. Ex post, lenders have a strong incentive to accelerate principal payment at shareholders' expenses. Such an allocation is thus able to generate not only a huge risk-reduction surplus ex post but also strong discipline ex ante.

²⁰The coupon rate determines the tax shield and thus plays a crucial role in the interest conflict between lenders and borrowers. When coupons are reduced, the ex post incentive of shareholders in pushing up leverage and thus default risks of legacy debt becomes milder. Therefore, when I adjust downward the coupon rate along the increase in κ , the time inconsistency becomes less harmful and thus the value of financial covenants is weakened. Expected restructuring costs start to be dominant more quickly, resulting in a lower turning point in ex ante firm value.

Figure 9: Covenant Tightness and Efficiency. *Notes:* This figure presents results from alternative models with different κ 's but all the other parameters fixed to those in Table 1. Figure 9a shows shareholder welfare $j(0, 0, \bar{x})$. Figure 9b shows the average deviation in firm value one quarter before violations (number of standard deviations below mean). Figure 9c (9d/9e) presents causal impacts of violations on the net debt issuances-to-assets ratio (investment rate/ k_H share) averaged over two post-violation quarters. I simulate 2,000 firms for 1,000 quarters.

As the violation threshold increases, lenders have more and more opportunities to implement a restructuring. However, violations locating in low-risk states start to account for a larger and larger proportion. This phenomenon is again driven by the nonlinearity of the defaultable debt problem. To loosely understand what happens, first send κ to a positive value fairly close to 0. Covenant violations in that case overlap with defaults—large negative shocks together with high firm riskiness are required. It is thus unlikely to expect a violation in the “good” time. Now consider an infinitely tight covenant, i.e. $\kappa = \infty$, instead. In this

scenario, one shall expect to see a large chunk of violations happening when fundamentals are fairly good. Indeed, Figure 9b shows that, as covenants get tighter, on average violations happen under a relatively higher level of firm value.

While the benefit of future restructurings is decreasing in violation frequency, the resource cost f is constant. These two forces combined lead to the existence of an inner optimum. Under an extremely tight covenant, many violations take place when the lack of commitment is far from being problematic. Frequent realizations of f losses impose huge harm to shareholders and the firm, which cannot be justified by the value of the commitment produced by restructurings.

Figures 9c, 9d and 9e together demonstrate that covenant violations become less consequential along the raise of κ , which happens again because violations become relatively more likely when default risk is small. Such a negative relationship between the tightness of covenants and the severity of violation consequences is consistent with the cross-sectional evidence in Demiroglu and James (2010). The activeness of violations also declines.

5.2.2 Allocation of control rights

The second structural parameter governing covenants is the bargaining power θ . When lenders hold all the control right in restructurings, firm value ex post is not maximized in lots of states—after default risk has been reduced to a certain level, lenders find a further retirement not privately beneficial because of the transfer to equity holders. Does this mean that equity holders should get some bargaining power and be able to enforce some transfers from lenders?

Figure 10a suggests a negative answer. Indeed, Figure 10b shows that when shareholder bargaining power is increased from 0 to 0.5, covenant violations lead to larger improvement in firm value ex post – a stronger realization effect. However, the anticipation effect changes as well. In Section 5.1.1, I have shown that the acceleration mechanism fails to improve welfare exactly because equity holders tend to get too much via debt relief. With a larger bargaining power, shareholders will on average get even more, as suggested by Figure 10c, and sometimes at the expense of debt holders. My analyses suggest that the weakening of the anticipation effect quantitatively dominates the improvement of the realization effect.

Figure 10: Control Right Allocation and Efficiency. *Notes:* This figure presents results from alternative models with different θ 's conditional on κ . Figure 10a shows shareholder welfare $j(0, 0, \bar{x})$. Figure 10b (10c) presents causal impacts of violations on firm (equity) value net of the restructuring cost f . I simulate 2,000 firms for 1,000 quarters.

6 Conclusion

This paper proposes a quantitative theory of financial covenants. By state-contingently introducing costly creditor intervention, these contract clauses serve as a potential solution to shareholders' time inconsistency problem associated with long-term debt financing. My quantitative analyses show that financial covenants significantly increase debt capacity and investment, restrict asset substitution and improve ex ante shareholder welfare. Furthermore, I quantify the tradeoff underlying the calibration of covenants and demonstrate a potential improvement in efficiency.

Considering the recent boom in covenant-lite loans, it is interesting to explore whether making the covenant tightness state-contingent can significantly improve contractual efficiency further. Such an extension should be straightforward. Moreover, it is valuable to incorporate this model into a general equilibrium framework and quantify the implications of these contingency contracts for macroeconomic quantities and fluctuations. I leave these to future work.

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Appendix

A An Example of Financial Covenants

The following paragraph presents a typical description of financial covenants in corporate financial reports and is taken from the 10-K of the HealthSouth Corporation for the fiscal year ended December 31, 2004:

“Non-compliance with these financial covenants under our credit facilities—our interest coverage ratio and our leverage ratio—could result in the lenders requiring us to immediately repay all amounts borrowed. Any such acceleration could also lead the investors in our public debt to accelerate their maturity. In addition, if we cannot satisfy these financial covenants in the indenture governing the credit agreements, we cannot engage in certain activities, such as incurring additional indebtedness, making certain payments, acquiring and disposing of assets.”

B Proofs in Section 2

As noted in the main text, since my propositions are mainly for characterization purposes, I assume the existence of an inner optimum and the validity of first-order conditions.

B.1 Proposition 1

Consider a “Ramsey” problem where shareholders lack commitment to repay but could commit to the path of debt $\{b_s\}_s$. The planning objective at time 0 is:

$$\max_{\{Q_s\}, \{b_s\}, s \geq 1} J_0^c$$

where J_s^c is recursively defined as:

$$\begin{aligned} J_s^c &= Q_{s+1}[b_{s+1} - (1 - \lambda)b_s] \\ &+ \beta \mathbb{E}_s \left\{ \int_{z_{s+1}^d}^{\bar{z}} \left[(1 - \tau)(Y + z_{s+1}) - [(1 - \tau)c + \lambda]b_{s+1} + J_{s+1}^c \right] d\Phi(z_{s+1}) \right\}, \end{aligned} \quad (13)$$

subject to the participation constraint of lenders:

$$Q_{s+1} = \beta \mathbb{E}_s \left\{ \int_{z_{s+1}^d}^{\bar{z}} \left[(c + \lambda) + (1 - \lambda)Q_{s+2} \right] d\Phi(z_{s+1}) \right\}, \quad (14)$$

and the ex post optimal default rule of shareholders: $(1 - \tau)(Y + z_{s+1}^d) - [(1 - \tau)c + \lambda]b_{s+1} + J_{s+1}^c = 0$. Initial conditions are given by $b_0 = 0$ and $Q_0 = 1$.

Define Π_{t_i, t_j} as the time- t_i expected probability of no default before time- t_j . The first-order condition with respect to (w.r.t.) b_{t+1} is given by

$$\begin{aligned} \beta^t \Pi_{0,t} Q_{t+1} + \beta^{t+1} \Pi_{0,t} \mathbb{E}_t \left\{ \int_{z_{t+1}^d}^{\bar{z}} \left[-[(1 - \tau)c + \lambda] - (1 - \lambda)Q_{t+2} \right] d\Phi(z_{t+1}) \right\} \\ - \mu_t \beta \mathbb{E}_t \left\{ [(c + \lambda) + (1 - \lambda)Q_{t+2}] \phi(z_{t+1}^d) \frac{\partial z_{t+1}^d}{\partial b_{t+1}} \right\} = 0, \end{aligned} \quad (15)$$

and that w.r.t. Q_{t+1} for all $t \geq 1$

$$\beta^t \Pi_{0,t} [b_{t+1} - (1 - \lambda)b_t] - \mu_t + \mu_{t-1} (1 - \lambda) \beta \Pi_{t-1,t} = 0, \quad (16)$$

and that w.r.t. Q_1 :

$$b_1 - \mu_0 = 0, \quad (17)$$

where μ_t is the Lagrangian multiplier.

From (17) and (16) and the fact that $\Pi_{0,0} = 1$, we know: $\beta^t \Pi_{0,t} b_{t+1} = \mu_t$. Plug this together with (14) into (15), we get:

$$\begin{aligned} \mathbb{E}_t \left[\int_{z_{t+1}^d}^{\bar{z}} \tau c d\Phi(z_{t+1}) \right] &= b_{t+1} \mathbb{E}_t \left\{ [(c + \lambda) + (1 - \lambda)Q_{t+2}] \phi(z_{t+1}^d) \frac{\partial z_{t+1}^d}{\partial b_{t+1}} \right\} \\ &= - \frac{b_{t+1}}{\beta} \frac{\partial Q_{t+1}}{\partial b_{t+1}}. \end{aligned} \quad (18)$$

Optimal choices b_{t+1} and Q_{t+1} no longer depend on legacy debt b_t . The persistence of $\{b_t\}$ inherits that of $\{z_t\}$, if it exists.

B.2 Corollary 1

Consider the planner's re-optimization at time $1 - \{b_s^{re}, Q_s^{re}\}_{s>1}$ – with legacy debt $(1-\lambda)b_1 > 0$. First-order condition w.r.t. Q_2^{re} evolves into:

$$b_2^{re} - (1-\lambda)b_1 = \mu_1^{re}.$$

Plugging this into the first-order condition w.r.t. b_2^{re} results in a condition different from equation (18).

B.3 Proposition 2

The first part of the proposition is self-evident from the first-order condition in equation (6). To derive it, first ignore the covenants, and then shareholders' problem becomes:

$$J(\tilde{b}) = \max_{b'} \left\{ (b' - \tilde{b})Q(b') + \beta \mathbb{E} \left\{ \int_{z'_d}^{\bar{z}} \left[(1-\tau)(Y+z) - [(1-\tau)c + \lambda]b' + J((1-\lambda)b') \right] d\Phi(z') \right\} \right\}, \quad (19)$$

subject to

$$Q(b') = \beta \mathbb{E} \left\{ \int_{z'_d}^{\bar{z}} \left[(c + \lambda) + (1-\lambda)Q(b''((1-\lambda)b')) \right] d\Phi(z') \right\}, \quad (20)$$

and $(1-\tau)(Y+z_d) - [(1-\tau)c + \lambda]b + J((1-\lambda)b) = 0$.

The first-order condition derived from equation (19) is given by:

$$Q(b') + \frac{\partial Q(b')}{\partial b'}(b' - \tilde{b}) - \beta \mathbb{E} \left\{ \int_{z'_d}^{\bar{z}} \left[[(1-\tau)c + \lambda] - (1-\lambda) \frac{\partial J(\tilde{b}')}{\partial \tilde{b}'} \right] d\Phi(z') \right\} = 0. \quad (21)$$

The envelope theorem gives $\frac{\partial J(\tilde{b})}{\partial \tilde{b}} = -Q(b')$. After plugging this equality together with the pricing function of equation (20) into (21), I get

$$\frac{\partial Q(b')}{\partial b'}(b' - \tilde{b}) + \beta \mathbb{E} \left[\int_{z'_d}^{\bar{z}} \tau c d\Phi(z') \right] = 0. \quad (22)$$

Differentiation of the pricing function is straightforward and thus omitted here. Now move to the second part of the proposition. Denote the LHS of equation (22) as H . Use the implicit function theorem:

$$\frac{\partial b'}{\partial \tilde{b}} = - \left[\frac{\partial H}{\partial b'} \right]^{-1} \frac{\partial H}{\partial \tilde{b}} = \left[\frac{\partial H}{\partial b'} \right]^{-1} \frac{\partial Q(b')}{\partial b'}.$$

As I always focus on an inner solution, the second-order derivative at the optimum should be negative, i.e. $\partial H / \partial b' < 0$. As a result:

$$\frac{\partial b'}{\partial \tilde{b}} \frac{\partial Q(b')}{\partial b'} < 0.$$

By equation (7), we know $\partial Q(b') / \partial b' < 0$. Therefore $\partial b' / \partial \tilde{b} > 0$.

B.4 Proposition 3

To see the difference between long-term debt under full commitment and one-period debt, consider an alternative recursive problem (subscripted with a) which replicates the constrained-efficient allocation under i.i.d. shocks. Conditional on no previous default, shareholders are forced always to maximize the total firm value F_a when issuing debt:

$$F_a = \max_{b'} Q_a(b') b' + \beta \mathbb{E} \left\{ \int_{z'_d}^{\bar{z}} \left[(1 - \tau)(Y + z) - [(1 - \tau)c + \lambda]b' + F_a - Q_a(b'')(1 - \lambda)b' \right] d\Phi(z') \right\}, \quad (23)$$

where

$$Q_a(b') = \beta \mathbb{E} \left\{ \int_{z'_d}^{\bar{z}} \left[(c + \lambda) + (1 - \lambda)Q_a(b'') \right] d\Phi(z') \right\}, \quad (24)$$

and $(1 - \tau)(Y + z_d) - [(1 - \tau)c + \lambda]b + F_a - Q_a(b')(1 - \lambda)b = 0$.²¹

²¹The first-order condition of this problem is given by:

$$\beta \mathbb{E} \left[\int_{z'_d}^{\bar{z}} \tau c d\Phi(z') \right] = b' \beta \mathbb{E} \left\{ \left[(c + \lambda) + (1 - \lambda)Q_a(b'') \right] \phi(z'_d) \frac{\partial z'_d}{\partial b'} \right\}.$$

One can easily prove the equivalence by conjecture-and-verify.

Plug (24) into (23):

$$F_a = \max_{b'} \beta \mathbb{E} \left\{ \int_{z'_d}^{\bar{z}} \left[(1 - \tau)(Y + z) + \tau c b' + F'_a \right] d\Phi(z') \right\}, \quad (25)$$

subject to $(1 - \tau)(Y + z_d) - [(1 - \tau)c + 1]b + F_a + [1 - Q_a(b')](1 - \lambda)b = 0$. Now consider the firm value for a one-period debt problem:

$$F_1 = \max_{b'} \beta \mathbb{E} \left\{ \int_{z'_d}^{\bar{z}} \left[(1 - \tau)(Y + z) + \tau c b' + F'_1 \right] d\Phi(z') \right\}, \quad (26)$$

where $(1 - \tau)(Y + z_d) - [(1 - \tau)c + 1]b + F_1 = 0$.

Since there are no persistent shocks here, allocations become time-invariant under full commitment. This is also true for one-period debt. Now consider a Ramsey equilibrium $\{(b_a^*)', F_a^*\}$ and a competitive equilibrium with one-period debt trading $\{(b_1^*)', F_1^*\}$.

I) If $Q_a(b_a^*) = 1$, $(b_1^*)' = (b_a^*)'$ and $F_1^* = F_a^*$. The proof is by conjecture and verify. Start with the conjecture $(F_a^*)' = (F_1^*)'$ and plug it into the RHS of equation (25). The maximization objectives become identical, and thus $(b_a^*)'$ that solves (25) should also be a solution to (26), i.e. $(b_1^*)' = (b_a^*)'$. As a result, $F_1^* = F_a^*$.

II) If $Q_a(b_a^*) \leq 1$, $F_a^* \geq F_1^*$. Again start with $(F_a^*)' \geq (F_1^*)'$. Take the solution to (26) and plug it into (25), I have $F_a[(b_1^*)'] \geq F_1^*$. Since F_a^* maximizes (25), $F_a^* \geq F_a[(b_1^*)']$. Therefore I have $F_a^* \geq F_a[(b_1^*)'] \geq F_1^*$ and can thus verify the conjecture.

III) If $Q_a(b_a^*) \geq 1$, $F_a^* \leq F_1^*$. The proof resembles that in case II).

C The Full Model

C.1 Equilibrium definition

Definition 2. A Markov Perfect Equilibrium of the full model is given by (i) equity holders' policy function $b'(\cdot)$, $k'(\cdot)$, $s'(\cdot)$ with associated value function $V^e(\cdot)$, default set D and covenant violation set H ; (ii) a debt pricing function $Q(\cdot)$ and associated value function $V^b(\cdot)$; (iii) a repayment acceleration function $\alpha(\cdot)$ such that (i) given $Q(\cdot)$ and $\alpha(\cdot)$, equity holders' policies are optimized; (ii) given equity holders' decisions and $\alpha(\cdot)$, $Q(\cdot)$ and $V^b(\cdot)$ satisfy debt holders' zero profit condition in equation (11); (iii) given $V^e(\cdot)$, equity holders'

decisions and $V^b(\cdot)$, $\alpha(\cdot)$ solves the restructuring problem in equation (12).

C.2 Proof of linear homogeneity

This section sketches the proof of the linear homogeneity of the full model. First, I conjecture repayment schedule $\alpha(b, k, s, x, z)$ is homogeneous of degree 0 (HOD0) to k and b (conjecture i). Further, debt policy $b'(\tilde{b}, k, s, x)$ and capital policy $k'(\tilde{b}, k, s, x)$ are homogeneous of degree 1 (HOD1) to k and $\tilde{b}$, while risk-shifting policy $s'(\tilde{b}, k, s, x)$ is HOD0 to k and $\tilde{b}$ (conjecture ii).

Conjecture the default cutoff is HOD0 to k and b (conjecture iii). Because default recovery is linear in k , based on conjectures i, ii, iii and the fact that the violation cutoff z_v is HOD0 to k and b by construction, pricing function $Q(b', k', s', x)$ can be shown to be HOD0 to k' and b' . Moreover, based on conjectures i, ii and iii, HOD0 of z_v and z_d and HOD0 of Q , I can show that the equity value function $J(\tilde{b}, k, s, x)$ is HOD1 to k and $\tilde{b}$. Together with conjecture i, it implies that $V^e(b, k, s, x, z; \alpha)$ is HOD1 to k and b , which in turn verifies conjecture iii.

With conjecture i, HOD1 of V^e , HOD0 of z_v and z_d , and HOD0 of Q , one can utilize the linearity of argmax operator and verify conjecture ii.

Because of conjecture i and the HOD0 of Q , it can be shown that $V^b(b, k, s, x; \alpha)$ is HOD1 to b and k . Combining with the HOD1 of $V^e(\tilde{b}, k, s, x)$, I can finally verify conjecture i.

C.3 Computational algorithm

Thanks to the linear homogeneity, I only have to solve the scaled version of the model. State space consists of $\tilde{\omega}$, s and x , which are discretized respectively with 183, 7 and 5 points. I set the upper bound for $\tilde{\omega}$ so that it never binds in simulations. I iterate over four functions in two loops: investment policy function $i(\tilde{\omega}, s, x)$, leverage restart policy $\tilde{\omega}^R(s, x)$, equity value function $j(\tilde{\omega}, s, x)$ and debt pricing schedule $q(\omega', s', x)$.

[1] Inner loop: Conditional on $i(\cdot)$ and $\tilde{\omega}^R(\cdot)$, iterate over $j(\cdot)$ and $q(\cdot)$ until convergence. In each step, I simultaneously update these two functions.

[2] Outer loop: Given $i(\cdot)$, $q(\cdot)$ and the policy functions $\omega'(\tilde{\omega}, s, x)$ and $s'(\tilde{\omega}, s, x)$ obtained in the inner loop, update $i(\cdot)$ analytically by the first-order condition and $\tilde{\omega}^R(\cdot)$ numerically by searching over a grid of 9150 points. Iterate until convergence.

D Data Construction

The main dataset I use combines i) COMPUSTAT North America Fundamental Quarterly between 1996Q1 and 2011Q4 and ii) an extended version of the covenant violation data constructed by [Roberts and Sufi \(2009a\)](#). Corporate default rates are taken from Exhibit 30 of Moody’s *Annual Default Study: Corporate Default and Recovery Rates, 1920-2015*.

I drop observations that are i) duplicated; ii) within the following industries – agriculture (sic ∈ [0000, 999]), utilities (sic ∈ [4900, 4999]), financial business (sic ∈ [6000, 6999]), foreign government (sic = 8888) and international affairs & non-operating establishments (sic ∈ [9000, 9999]); and iii) non-US. Table 6 presents how variables are constructed within the model and in the data. All variables in the data are winsorized at top and bottom 1%.

Moments	Model	Data
debt	b	dlttq + dlcq
net debt issuance	$b' - b$	...
assets	k	atq
investment	$k' - (1 - \delta)k$	capxy - sppey
market-to-book	$[J(\tilde{b}) + (1 - \lambda)\tilde{b}Q(b')]/k$	(atq + (prccq × cshoq) - ceqq - txdbq) / atq
income	$e^x k$	oibdpq

Table 6: Data Construction